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Causes of Collapse of Q-factor and Consequences for DSP Applications

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Introduction

[1] describes the effect of the collapse of Q-factor (further - collapse) for the digital resonator (DR), obtained from the analog prototype by using a bilinear transformation (BT). The mentioned side-effect makes using of DR very problematic in DSP applications for frequencies, which lie between the Nyquist frequency and half the Nyquist frequency.

The next figure is taken from [1] to demonstrate the effect of collapse for the reader unfamiliar with the subject of the paper.

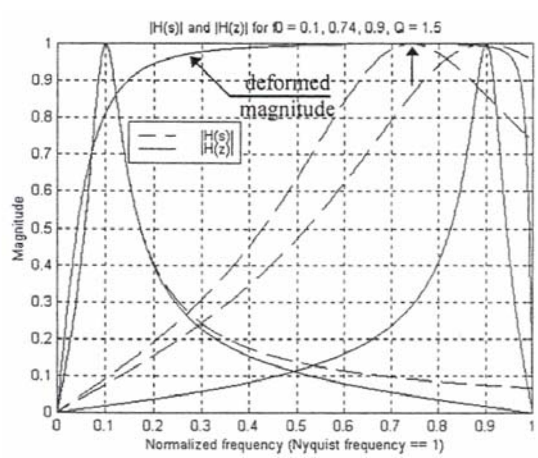

Fig. 1. Deformation of magnitude response of DR. "Normal" magnitude for f_0 = 0.1, "bypass filter" (Q-collapse) for f_0 = 0.74, "narrow band filter" (Q-explosion) for f_0 = 0.9. Taken from [1]

We have made further investigations related to the collapse. What is new? First of all, we now understand why does the collapse appears.

Causes of Collapse

The main cause of the collapse is the method used for the calculation of bandwidth frequencies. This method is widely used for calculations of digital filters [2]. If we have an analog prototype, we can get an associated digital filter. To compute coefficients for digital filters designers widely (and usually) use the BT.

The bilinear transformation has many advantages for lowpass, highpass, bandpass un stopband filters [2]. For the synthesis of differentiator BT is not useful [2]. A well-known disadvantage is the warping of frequencies by arctangent rule. Designers use the recalculation (pre-warping) of critical frequencies of analog prototype (for DR – bandwidth frequencies) by tangent rule to escape the warping of frequencies for the digital filter. The mentioned solution is well-known. Many engineers use the mentioned approach for synthesis of DR also.

The transfer function of analog prototype of DR can be written as [3]:

$$H(s) = \frac{(\omega_2 - \omega_1)\cdot s}{s^2 + (\omega_2 - \omega_1)\cdot s + \omega_0^2}, \qquad (1)$$

where $\omega_0 = 2\cdot\pi\cdot f_0,\ \omega_2 - \omega_1 = \frac{\omega_0}{Q}$ - resonant frequency and bandwidth specified by bandwidth frequencies ω_1, ω_2 (defined at level $1/\sqrt{2}$ of maximal value of magnitude) or Q-factor.

Well known from the circuit theory are expressions for bandwidth frequencies (bandedges):

$$\begin{cases} \omega_1 = 2\cdot\pi\cdot f_0\cdot\left(\sqrt{\frac{1}{16\cdot Q^2}+1} - \frac{1}{2\cdot Q}\right), \\ \omega_2 = 2\cdot\pi\cdot f_0\cdot\left(\sqrt{\frac{1}{16\cdot Q^2}+1} + \frac{1}{2\cdot Q}\right). \end{cases} \qquad (2)$$

We should recalculate all critical frequencies before using the bilinear transformation:

$$H_t(s) = \frac{(\omega_{2t} - \omega_{1t})\cdot s}{s^2 + (\omega_{2t} - \omega_{1t})\cdot s + \omega_{0t}^2}, \qquad (3)$$

where transformed frequencies can be expressed as:

$$\omega_{it} = 4\cdot\tan(\frac{\omega_i}{4}) = 4\cdot\tan(\frac{\pi}{2}\cdot f_i),\ i = 0, 1, 2\cdot \qquad (4)$$

The next step in getting the transfer function of DR is to perform a well-known substitution of s in the transformed function (1):

$$s = 2\frac{z-1}{z+1}. \tag{5}$$

Formulas (4), (5) are true only for normalized frequencies (Nyquist frequency is equal to 1). We will not present here the discrete transfer function for DR because a reader can find it out in many books and papers about signal processing (also in [2], [3]).

In the case when the upper bandwidth frequency reaches the value 1

$$f_0 \cdot \left(\sqrt{\frac{1}{16\cdot Q^2}+1}+\frac{1}{2\cdot Q}\right) \to 1,\ \omega_{2t} \to \infty\ , \tag{6}$$

the transformed upper bandwidth frequency reaches the infinite value, but the bandwidth of resonator has "explosion" and Q-factor fall down to 0. This is because we have the collapse.

From the condition (5) follows that the collapse frequency can be calculated as:

$$f_{col} = \frac{4\cdot Q}{2+\sqrt{16\cdot Q^2+1}} \cong \frac{2\cdot Q}{1+2\cdot Q}. \tag{7}$$

Formula (7) has a good correlation with simulation results obtained in [1].

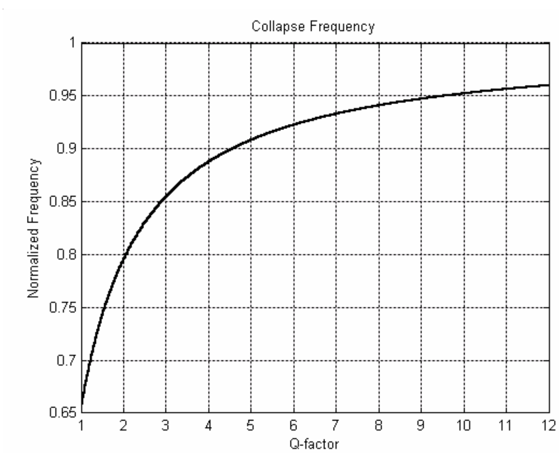

Fig. 2. Normalized collapse frequency in dependence on Q-factor

The transformed bandwidth is negative, if we look for frequencies above the collapse frequency:

$$\omega_{2t} - \omega_{1t} < 0 \tag{8}$$

and that causes the instability of DR. This result is obvious because tangent function for argument values ($\pi/2$, π) is negative (see (4)). The magnitude from Fig. 1 which corresponds to (8) is senseless from the practical point of view, and it is named an "explosion of Q-factor".

Behaviour of Magnitude of DR Near the Collapse

In [1] we demonstrated that for frequencies above the half the Nyquist frequency, the magnitude of DR has a deformation. Simultaneously with the deformation of magnitude the Q-factor of DR decreases also. That happens if we calculate bandwidth frequencies using (4).

We cannot explain the behaviour of magnitude of DR for the range of frequencies below the collapse frequency so straightforward as in the case of collapse or instability of DR. We should look at the next figure for better understanding. The figure shows the magnitude of analog resonator and the magnitude of DR without collapse.

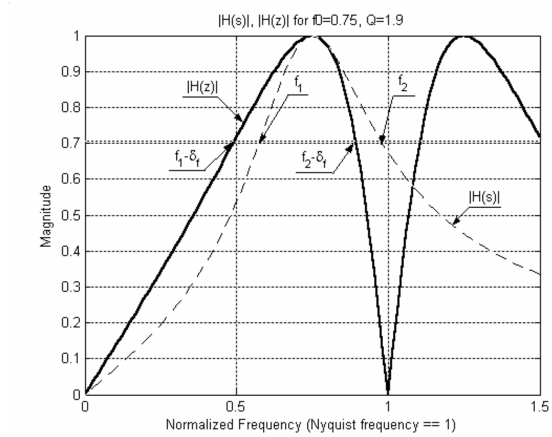

Fig. 3. Magnitudes for the collapse-free DR (|H(z)|) and its analogue prototype **(|H(s)|). |H(z)|** is obtained by MATLAB function **iirpeak**

In Fig. 3 we can see that even the curve of magnitude for the collapse-free DR has serious deformation. That follows from the nature of BT. One of the BT properties ensure the equivalence to zero of the value of magnitude at Nyquist frequency. That forces the magnitude of DR to be deformed. We have the shifting of bandwidth frequencies to left. Here are no alternatives, if we use BT and do not allow the change of resonant frequency. In [3] we can find out more sophisticated transformation (an alternative to BT) that keeps values of both bandwidth frequencies but needs the additional guard-filter to avoid aliasing.

From Fig. 3 follows that bandedges must be calculated as:

$$\omega_{it} = 4\cdot\tan(\frac{\omega_i-\delta_\omega}{4}) = 4\cdot\tan(\frac{\pi}{2}\cdot(f_i-\delta_f)),\quad i=1,2\ ; \tag{9}$$

where δ_ω, δ_f – unknown frequency shifting. We do not consider here how we should estimate or find out the value of frequency shifting, and we use δ for illustration only.

From the first look, if we have a collapse-free solution (for example, MATLAB function **iirpeak**) for the DR synthesis, we do not need special investigations of collapse. But, if we look deeper, we see problems with pre-warping of critical frequencies for digital filters derived from analog prototypes.

General conclusion is that we must not perform the pre-warping of critical frequencies blindly. We should estimate the possibility and the quality of fitting of magnitude of digital filter to the magnitude of analog prototype before pre-warping of critical frequencies. If values of critical frequencies are important, the magnitude of digital filter must interpolate the magnitude of analog prototype at critical frequencies. But, if the mentioned fitting is impossible, we must use more sophisticated transformation instead of BT.

We have not yet been able to define universal and exact criteria for "estimation of the possibility and the quality of fitting of magnitude ...". That needs some additional investigations. For example, in some applications of DR the accuracy of resonant frequency and bandwidth is very important but it is not so much for bandedge frequencies.

Second Order Sections

It is a well-known approach – the presentation of the transfer function of IIR filters by the chain of second order section [2]. We can interpret each second order section as a digital resonator. This can lead to serious consequences, if we take into account the possibility of collapse for separate sections of filter.

Here we need additional serious investigations in the immunity of IIR digital filter design algorithms to the collapse. It means not only novel filter design algorithms, but also the preparation of existing algorithms, which include the BT.

Compensation of Collapse

The [1] recommended two ways for the compensation of the collapse:

- using of well-known zero-pole placement method;
- pre-distortion of Q-factor before operating. An iterative algorithm is used.

Here we suggest the approach that is simpler than that described in [1]. Our method is based on the different approach to calculate critical frequencies.

We use the tangent function to calculate the pre-warped resonant frequency ω_{0t} (see (4)). However, for the calculation of bandwidth frequencies we use some first terms of Taylor series. As the basepoint we choose the pre-warped resonant frequency. If we take only one term of Taylor series, we get relative simple formulas:

$$\begin{cases} \omega_{1t} = \omega_{0t} + (1 + (\frac{\omega_{0t}}{4})^2) \cdot (\omega_1 - \omega_0), \\ \omega_{2t} = \omega_{0t} + (1 + (\frac{\omega_{0t}}{4})^2) \cdot (\omega_2 - \omega_0), \end{cases} \quad (10)$$

where pre-warped resonant frequency ω_{0t} can be obtained using (4).

The next figure demonstrates examples of magnitudes of DR for cases when pre-warped bandwidth frequencies are calculated by using the different number of Taylor series.

We see that the difference between bandwidths obtained by using one or two terms of Taylor series is insignificant for the practical goal.

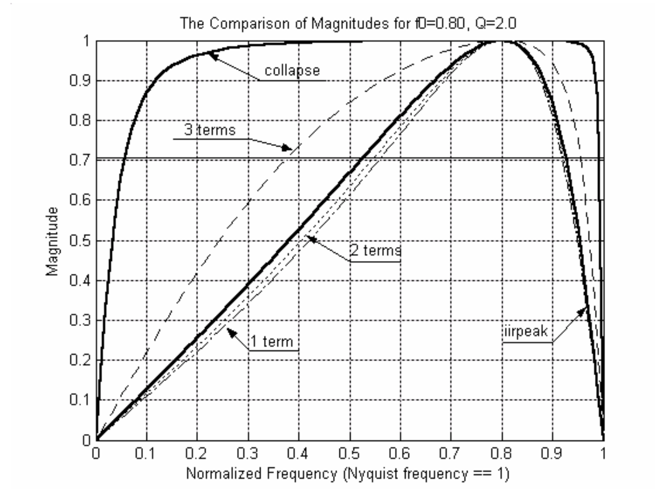

Fig. 4. Comparison of magnitudes of DR for the case of collapse, for the different number of terms of Taylor series and for the MATLAB **iirpeak**

The difference between shapes of magnitudes lies in the range from several percents (for Q>**3**) up to 20-30% (Q=**1**, f_0=**0.9**). The mentioned difference increases by increasing the resonant frequency. This means that for the practical use formulas (10) are more than sufficient. In the case of three terms we get the increasing of error of magnitude. This is because transformed bandwidth frequencies are more close to frequencies calculated by (4).

For Students

We developed the special interactive program to demonstrate the collapse of DR for our students. The program demonstrates the behavior of magnitude and the moving of poles of DR transfer function.

Conclusions

General conclusion is that we must not perform the pre-warping of critical frequencies blindly if we use BT for the synthesis of digital filters derived from the analog prototypes. We must revise the form of magnitude of analog prototype before pre-warping of critical frequencies. This means that we must also revise the values of critical frequencies for analog prototype. But, if the mentioned revision is impossible, we should use more sophisticated transformation instead of BT.

References

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2. **Cunningham E. P.** Digital Filtering. An Introduction. - Houghton Mifflin Company, Boston, Toronto, 1992.

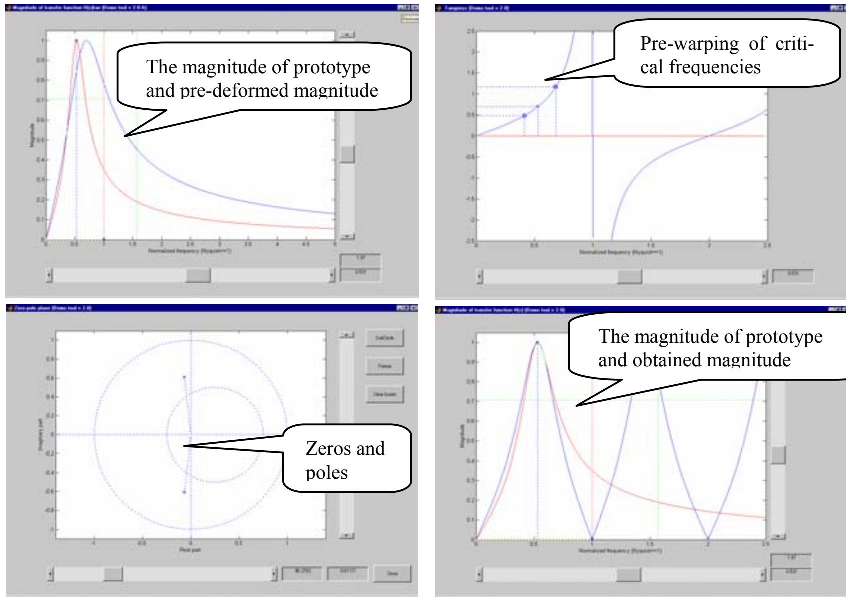

Fig. 5. The interactive tool for the demonstration of collapse effect

3. **Moorer J.A.** The Manifold Joys of Conformal Mapping: Applications to Digital Filtering in Studio // J. Audio Eng. Soc. - Vol. 31. - No.11. - 1983. - P. 826-840.

Pateikta spaudai 2005 04 12

P. Misans, A. Hauka. Skaitmeninio rezonatoriaus kolapso kokybės tyrimai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 5(61). – P. 9–12.

Skaitmeninio rezonatoriaus realizacija naudojant analoginį prototipą bei bilinijinę transformaciją leidžia pasiekti rezonatoriaus kolapso efekto kokybę. Nagrinėjama kolapso priežastis ir pateiktos formulės kolapso dažniui nustatyti. Pasiūlytas supaprastintas metodas kolapso kompensavimui. Siūlomo būdo esminis privalumas yra tas, kad perskaičiuojant analoginių prototipų kritinius dažnius lengviau išvengti skaitmeninių filtrų klaidingos analizės. Ill. 5, bibl. 3 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

P. Misans, A. Hauka. Causes of Collapse of Q-factor and Consequences for DSP Applications // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 5(61). – P. 9–12.

Implementation of digital resonator by using of analog prototype and bilinear transformation leads to the effect of collapse of Q-factor. This paper describes the cause of the collapse and presents the formula for calculation of collapse frequency. We also present a simplified method for the compensation of collapse. The main consequence of collapse effect is that the formal using of pre-warping of critical frequencies of analog prototype can be dangerous for the synthesis of digital filters. Ill. 5, bibl. 3(in English; summaries in Lithuanian, English and Russian).

П. Мисанс, А. Хаука. Причины коллапса добротности и последствия для применений в ЦОС // Электроника и электротехника.– Каунас: Технология, 2005. – № 5(61). – С. 9–12.

Реализация цифрового резонатора с использованием аналогового прототипа и билинейного преобразования приводит к эффекту коллапса добротности резонатора. Описывается причина коллапса и представляются формулы для определения частоты коллапса. Приводится также упрощенный метод для компенсации коллапса. Главное следствие упомянутого эффекта это то, что при пересчете крнтических частот аналоговых прототипов, может возникнуть опасность некорректного синтеза цифровых фильтров. Ил. 5, библ. 3 (на английском языке; рефераты на литовском, английском и русском яз.).

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