ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2012. No. 1(117)
ELEKTRONIKA IR ELEKTROTECHNIKA

SIGNAL TECHNOLOGY
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SIGNALŲ TECHNOLOGIJA

Non-Stationary Signal Reconstruction from Level-Crossing Samples using Akima Spline

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http://dx.doi.org/10.5755/j01.eee.117.1.1044

Introduction

The signal-driven data acquisition method for non-stationary signals, such as a level crossing sampling (LCS) [1, 2], allows data sampling at a rate significantly below the Nyquist rate. It can be achieved for the burst signals due to the following LCS advantages: no data sampling if a signal remains constant and a data sampling only at the time instants when the signal is crossing the predetermined reference levels. The implementation of LCS advantages is giving an opportunity to sample less data without losing significant signal data. The desirable result can be achieved using the appropriate LCS algorithms and the signal recovery methods.

The selection of the appropriate LCS algorithms and the signal recovery methods is equally important. In [3] the usage of LCS with pre-adapted limited number of reference levels and the cubic spline for the signal recovery has demonstrated a low volume of the sampled data for the non-stationary input signal. However the approach from [3] applied to the burst signal is providing a poor quality of the recovered signal. The cause is a signal reconstruction with the cubic spline from the LCS data sampled using limited number of the reference levels. To improve the recovery of the signal and to keep low sampled data volume at the same time a reconstruction with the Akima spline [4] is introduced and evaluated. The evaluation of the signal reconstruction with the different splines including the Akima spline is performed for the data sampled with two different LCS algorithms. The first LCS algorithm is using limited number of the reference levels. The second LCS algorithm is sampling data close to the local extreme values of the signal. The quality of the recovered signals is estimated using the selected evaluation measures. The reduction of the signal data volume can be useful in certain signal processing applications including a signal separation, a biomedical signal processing and etc.

Sampling algorithms and signal reconstruction

Two different LCS algorithms were used for the burst signal sampling. Both LCS algorithms are sampling data at consecutive time instants t_i where $i \in (1....n)$ according to sampling conditions. The sampling condition for the first LCS algorithm LCS1 is following

$$(x(t_i) \le r_i^- \in R(N)) \vee (x(t_i) \ge r_i^+ \in R(N)) \Rightarrow d(t_i), \quad (1)$$

where $x(t_i)$ is an original signal, r_i^+ is a crossed reference level at the rising signal slope in the time instant t_i, r_i^- is a crossed reference level at the falling signal slope in the time instant t_i, $R(N)$ is the set of reference levels and $d(t_i)$ is a digitized time interval value. All sampled values are saved.

The second LCS sampling algorithm LCS2 has two stages: the initial sampling and the decision making. The only exception is the start step that includes only the initial sampling satisfying the sampling condition (1) with the following saving. Each next step includes the initial sampling according to the condition (1) and the decision making (2) based on the compared values of the crossed reference levels for 2 successive samples at the time instants t_{i-1} and t_i

$$(r_{i-1}^- = r_i^+) \vee (r_{i-1}^+ = r_i^-) \Rightarrow (d(t_{i-1}), d(t_i)), \quad (2)$$

where r_{i-1}^+ is the crossed reference level at the rising signal's slope, r_{i-1}^- is the crossed reference level at the falling signal slope in the time instant t_{i-1} and $d(t_{i-1})$ is the digitized time interval value.

The LCS1 algorithm is saving all data samples that are satisfying the sampling condition (1). The LCS2 algorithm is saving selected data samples that are the closest to the local extreme values of the signal. The sampling condition is (1) and the saved samples are fulfilling the condition (2). It results in the smaller volume of sampled data than in case of the algorithm LCS1.

The reconstruction of the signal is performed using data samples obtained with the algorithm LCS1 or LCS2. Three different interpolation splines are used for the signal recovery: the cubic spline, the cubic Hermite spline and the Akima spline. All three splines are based on cubic piecewise polynomials. The difference between the splines is in the conditions imposed at the data points. The Akima spline is built from piecewise third order polynomials and only data from the next neighbor points is used to determine the coefficients of the interpolation polynomial.

Evaluation metrics

The following evaluation measures are employed to evaluate the signal reconstruction quality applying different interpolation splines: the cross correlation (CC), the root mean square error (RMS), the percentage root mean square difference (PRD) and the compression ratio (CR). CC, RMS and PRD is used to evaluate the performance of the signal recovery with different interpolation splines as well as to compare results with LCS1 and LCS2 algorithm. CR is used only for LCS1 and LCS2 comparison.

CC is used to evaluate the similarity between the original signal and its reconstruction from the data samples

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}, \tag{4}$$

where x_i are re-sampled values of the original signal, and y_i are re-sampled values of the reconstructed signal, $\bar{x}$ and $\bar{y}$ are the sample means of X and Y, s_x and s_y are the sample standard deviations of X and Y.

RMS is used to measure the signal distortion of the reconstructed signal for the chosen LCS algorithm and the signal reconstruction method

$$RMS = \sqrt{\frac{\sum_{i=1}^{n}(x_i - y_i)^2}{n}}. \tag{5}$$

PRD is used to appreciate the distortion in a reconstructed signal with respect to the original for the chosen LCS algorithm and the signal reconstruction method

$$PRD = 100 \times \sqrt{\frac{\sum_{i=1}^{n}(x_i - y_i)^2}{\sum_{i=1}^{n}(x_i)^2}}. \tag{6}$$

CR is a ratio between the data samples representing the original signal and LCS samples

$$CR = \frac{S_{orig}}{S_{lcs}}, \tag{7}$$

where S_{orig} and S_{lcs} represent samples of the original signal and LCS respectively.

Simulation results

Both LCS algorithms and the signal reconstruction with different interpolation splines are simulated using several normalized data sets of the electroencephalogram (EEG) signals [5]. Here we are using two random selected data sets as an illustration. The performance of LCS1 and LCS2 algorithms and the following signal recovery is illustrated using 10s and 5min long EEG signals at the different sets of the reference levels. The reference levels are spaced equidistantly. Initially the evaluation of the applicability of the three different interpolation splines was performed using the cross correlation metric by correlating values of the original and the reconstructed signal. 10s EEG signal was sampled using several sets of the reference levels $R\{6,7...16\}$. EEG signal was recovered with the Akima, the Hermite (PCHIP is Piecewise Cubic Hermite Interpolating Polynomial) and the cubic interpolation splines. The evaluation of the reconstruction of EEG signal (Fig. 1) with three different splines shows that the cubic spline is not applicable for EEG signal interpolation.

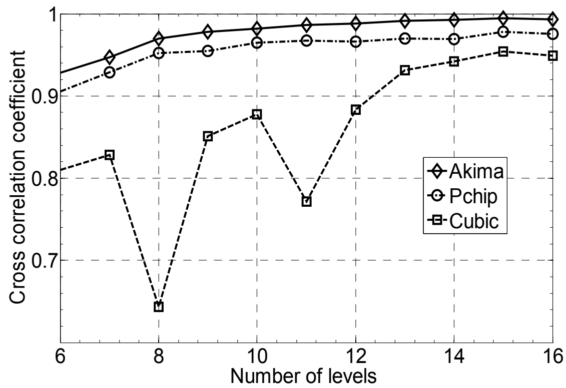

Fig. 1. Comparison of 3 EEG signal reconstruction methods

The values of the cross correlation curve *Cubic* are essentially behind the values of cross correlation curves *Akima* and *Pchip*. Besides, the cross correlation curve *Cubic* is inconsistent having gaps at $R\{8\}$ and $R\{11\}$.

Further simulation of the signal recovery is performed using only the Akima and the Hermite splines as the interpolation splines for the signal reconstruction. The comparison of the results obtained with the Akima and the Hermite spline (Fig. 2) gives us that the Akima spline interpolation is performing better than the Hermite spline in all cases for both LCS algorithms. The values of cross correlation coefficient curves *Pchip1* and *Pchip2* are always below the curves *Akima1* and *Akima2* respectively. *Pchip1* and *Akima1* are for LCS1. *Pchip2* and *Akima2* are for LCS2. The same tendency is remaining in the case of 5 min signal (Fig. 3).

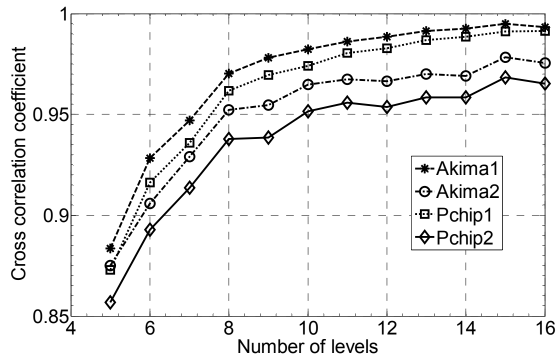

Fig. 2. Cross correlation of EEG 10 s reconstructed signal versus number of reference levels for Akima and Hermite splines, and LCS algorithms

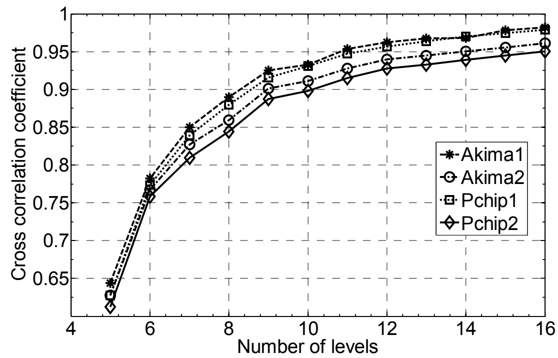

Fig. 3. Cross correlation of EEG 5 min reconstructed signal versus number of reference levels for Akima and Hermite splines, and LCS algorithms

Also the Akima spline interpolation is demonstrating the better results in the signal reconstruction according to the metrics as RMS (Fig. 4) and PRD (Fig. 5) compared to the Hermite spline interpolation. The Akima spline interpolation has smaller error values. The difference between the Akima spline and the Hermite spline interpolation is better noticeable for 10s signals and for PRD (Fig. 5).

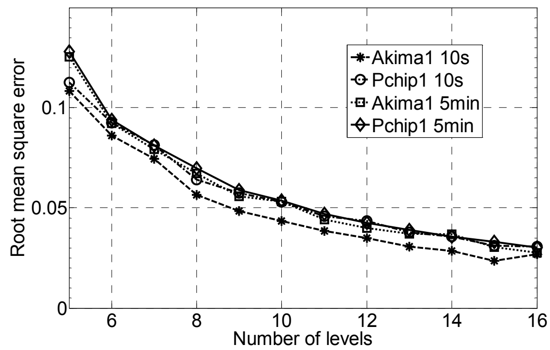

Fig. 4. Root mean square error for EEG reconstructed signal versus number of reference levels for Akima and Hermite splines

The quality of the Akima spline interpolation is depending from the choice of the set of reference levels. However any increase of the number of levels is causing the decrease of the compression ratio (Fig. 6). It is valid for LCS1 as well as for LCS2 algorithm. Therefore equilibrium between the method's performance and the compression ratio should be observed. It means that any selected compression ratio can provide only the limited set of all metrices. For example, selecting the compression ratio value equal to 10 (Fig. 6) the applicable number of the values of the reference levels will be 6, 10 and 12. From (Fig. 7) we can select PRD values that are equal 20, 8 or 6. The implementation depends on the choice of the selected cross correlation coefficient, PRD, RMS values, and the value of the desirable compression ratio.

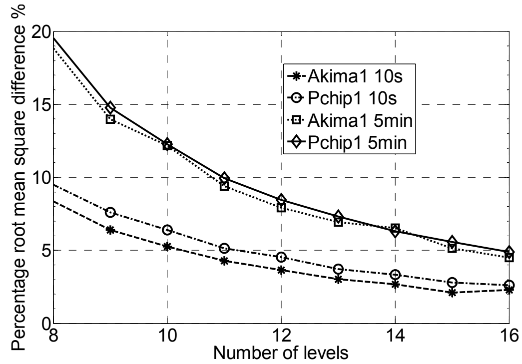

Fig. 5. Percentage root mean square difference for EEG reconstructed signal versus number of reference levels for Akima and Hermite splines

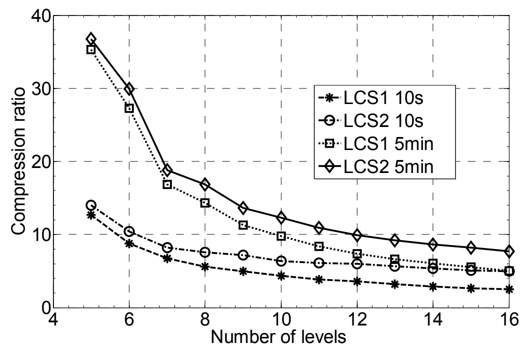

Fig. 6. Compression ratio versus number of reference levels for LCS1 and LCS2 algorithms

We can start with the selection of the desirable cross correlation coefficient's value, and to continue with the selection of the value of the compression ratio and PRD or another metrics.

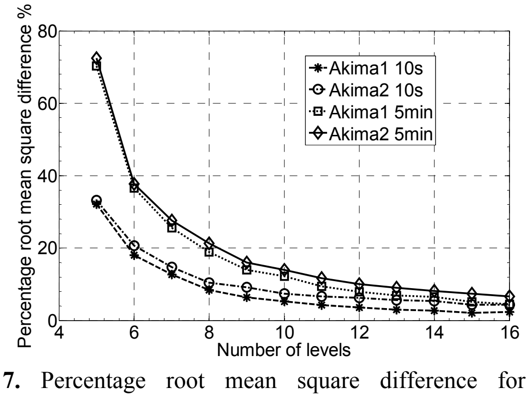

Fig. 7. Percentage root mean square difference for EEG reconstructed signal samples versus number of reference levels for LCS1 and LCS2 algorithms

Conclusions

The non-stationary signal reconstruction with the Akima spline interpolation from the level-crossing samples is proved to be the most effective for both LCS algorithms. The Hermite spline interpolation is demonstrating a good performance for all metrics but is not exceeding the Akima spline interpolation. The cubic spline interpolation is not applicable for the reconstruction purposes of EEG signals.

The quality of reconstructed signals is depending from the number of the reference levels. Any number increase is causing the improvement in all metrics. The exception is the compression ratio. Therefore equilibrium between the method's performance and the compression ratio should be observed.

Acknowledgements

This work has been supported by a funding from the European Social Fund within the grant No. 2009/0219/1DP/1.1.1.2.0/09/APIA/VIAA/020.

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Received 2011 02 17
Accepted after revision 2011 10 12

U. Grunde. Non-Stationary Signal Reconstruction from Level-Crossing Samples using Akima Spline // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 1(117). – P. 9–12.

The modification of the level crossing sampling algorithm is proposed in order to decrease the volume of sampled data. The reconstruction of the signal based on sampled data is discussed. The performance of the signal reconstruction based on different interpolation splines is evaluated. The evaluation of the signal recovery is based on metrics including the cross correlation coefficient, the root mean square error and the percentage root mean square difference. It is demonstrated that compression coefficient's value 10 is achievable for the cross correlation coefficient 0.92. Ill. 7, bibl. 5 (in English; abstracts in English and Lithuanian).

U. Grunde. Nestatcionaraus signalo atkūrimas naudojant Akima splainą // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 1(117). – P. 9–12.

Pasiūlyta lygio apdorojimo algoritmo modifikacija duomenų kiekiui sumažinti modifikacija. Aptartas signalo atkūrimas iš paimtų duomenų. Įvertintas signalo atkūrimo naudojant skirtingus interpoliavimo splainus našumas. Signalo atstatymo įvertinimas atliktas naudojant tarpusavio koreliacijos koeficientą, taip pat vidutinio kvadratinio nuokrypio ir procentinio kvadratinio nuokrypio skirtumą. Parodyta, kad suspaudimo koeficiento vertę 10 galima gauti, kai tarpusavio koreliacijos koeficientas yra 0,92. Il. 7, bibl. 5 (anglų kalba; santraukos anglų ir lietuvių k.).