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V-Characteristics Analysis of Non-Salient Pole Synchronous Electrical Machines

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Introduction

The property of synchronous electrical machines to supply reactive power to a power network or to consume the power from there ensures the reliable stabilisation of the network's voltage. This function may be carried out by synchronous machine, operating in motor, generator or compensator mode. The power exchanges between a synchronous machine and a network are illustrated graphically by V-characteristics. It would be desirable that the analytical expressions for V-characteristics would be in disposition in the case of excitation current's automatic control usage for a synchronous machine. There is stated in [1] that the V-characteristics have the shape of ellipses. The weakness of this statement is discussed hereinafter. It is proved in [2] that the V-characteristics form the branches of hyperbola. The vector diagrams of synchronous machines and the geometrical analysis of the diagrams are used in [3] when it touches upon some properties of V-characteristics. After the assessment of positive properties of synchronous machines in [4, 5, 6], the necessity arises to make comprehensive analysis of the machine's V-characteristics by submitting the algorithms for the analytical relationship's formation of V-characteristics as well as for the functions' determination of stability line and isogonic line.

Main equations

The vector diagram of synchronous machine is shown in the Fig. 1, where I_a – phase current of synchronous machine's armature; E – electromotive force of armature phase; U_T – phase voltage of mains; $\underline{U} = -\underline{U}_T$ – complex voltage of synchronous machine's phase; R_a – phase resistance of armature; $X = X_\sigma + X_a$ – synchronous inductive reactance; X_σ – leakage inductive reactance of armature phase; X_a – inductive reactance of armature's phase reaction; $\rho = \arctan\dfrac{R_a}{X}$; θ – load angle; $\alpha = \dfrac{\pi}{2} + \rho + \varphi$.

From the triangle OAB we find that

$$E^2 = U^2 + I_a^2\left(X^2 + R_a^2\right) + 2UI_a\sqrt{X^2 + R_a^2}\sin(\varphi + \rho). \quad (1)$$

The equation (1) corresponds to the right branches of V-curves in the case when $\varphi > 0$, and it corresponds to the left branches in the case when $\varphi < 0$. In both these cases the invariant $\delta = -\left(X^2 + R_a^2\right) < 0$ of the equation (1) determines the graph of the equation (1) as a hyperbola.

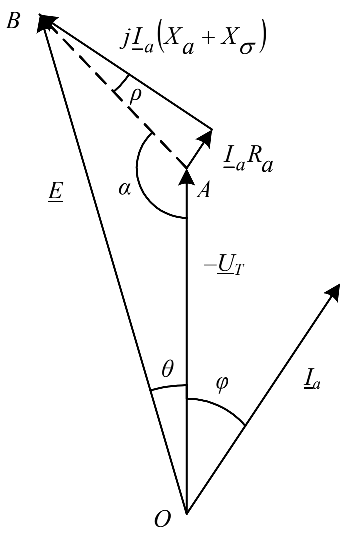

Fig. 1. Vector diagram of non-salient pole synchronous machine

Making use from rewriting of equation (1), we receive for the right branch of V-characteristics:

$$E^2 = U^2 + I_a^2\left(X^2 + R_a^2\right) + 2PR_a + 2X\sqrt{U^2 I_a^2 - P^2}\,; \quad (2)$$

where P – active power of synchronous machine.

For the left branch we receive

$$E^2 = U^2 + I_a^2\left(X^2 + R_a^2\right) + 2PR_a - 2X\sqrt{U^2 I_a^2 - P^2}\ . \quad (3)$$

It was already proved earlier [2], that the graphs of V-characteristics are hyperbolae.

The analysis of V-characteristics, presented in the paper [1], should be taken into consideration. The angle *ABO* (Fig.1) of the vector diagram is equated to the angle ρ in the paper. However, this angle is expressed as

$$\angle ABO = \frac{\pi}{2} - \theta - \varphi - \rho\ .$$

The definition mistake of the angle *ABO* has determined the following relation on the bases of the vector diagram in the Fig.1:

$$U^2 = E^2 + I_a^2\left(X^2 + R_a^2\right) - 2EI_a\sqrt{X^2 + R_a^2}\cos\rho\ . \quad (4)$$

The usage of general theory of quadratic curves [7] enables to answer the question what quadratic curve is presented by the equation (4). The following invariants of the function (4) are analysed herein:

$$\Delta = \begin{vmatrix} 1 & -\sqrt{X^2 + R_a^2}\cos\rho & 0 \\ -\sqrt{X^2 + R_a^2}\cos\rho & X^2 + R_a^2 & 0 \\ 0 & 0 & -U^2 \end{vmatrix} =$$

$$= -U^2\left(X^2 + R_a^2\right)\sin^2\rho < 0\ ;$$

$$\delta = \begin{vmatrix} 1 & -\sqrt{X^2 + R_a^2}\cos\rho \\ -\sqrt{X^2 + R_a^2}\cos\rho & \left(X^2 + R_a^2\right) \end{vmatrix} =$$

$$= \left(X^2 + R_a^2\right)\sin^2\rho > 0\ ;$$

$$S = 1 + X^2 + R_a^2\ .$$

The equation (4) represents an ellipse because it fulfils the conditions $\delta > 0$ and $\Delta\rho < 0$. In the paper [1] the conclusion is made about the shape of V-curves on the bases of the above mentioned conditions.

Since the angle

$$\angle ABO = \frac{\pi}{2} - \theta - \varphi - \rho \neq \rho\ ,$$

the equation (4) may be transformed into

$$U^2 \approx E^2 + I_a^2\left(X^2 + R_a^2\right) -$$

$$-2X\left[\frac{PX}{U^2}\sqrt{U^2 I_a^2 - P^2} + \frac{EP}{U}\left(1 - \frac{1}{2}\frac{P^2 X^2}{E^2 U^2}\right)\right]. \quad (5)$$

The invariant of the equation (5) is

$$\delta = \begin{vmatrix} 1 & 0 \\ 0 & X^2 + R_a^2 \end{vmatrix} = X^2 + R_a^2 > 0.$$

However, that is the feature of hyperbola.

The results of experiments, presented in [3, 6] also deny the opinion, given in [1], that the V-characteristics of synchronous machine have the shape of ellipses. Hence, the further investigations are performed herein assuming that the V-characteristics are the branches of hyperbola.

The locus cosφ = 1 of synchronous machine's V-characteristics

From the equation (1) it follows

$$\sin(\varphi + \rho) = \frac{U^2 - E^2 + I_a^2\left(X^2 + R_a^2\right)}{2UI_a\sqrt{X^2 + R_a^2}}. \quad (6)$$

The medium and high power synchronous machines comply with the requirement

$$R_a << X\ . \quad (7)$$

In this case from the equality (6) it follows that

$$\sin\varphi = \frac{U^2 - E^2 + I_a^2 X^2}{2XUI_a} \quad (8)$$

and when $\varphi = 0$, we receive

$$U^2 - E^2 + I_a^2 X^2 = 0\ . \quad (9)$$

The above relation can be rewritten as

$$E = \sqrt{U^2 + I_a^2 X^2}\ . \quad (10)$$

That is the analytical expression for the case when $\cos\varphi = 1$.

After the evaluation of electromotive force's dependence on excitation current I_f, we receive

$$E = f_1\left(I_f\right); \quad (11)$$

where the excitation current I_f is expressed using the relative units – in the same way as electromotive force.

The no-load characteristic (11), normalized for all non-salient pole synchronous machines, is used hereinafter (Fig. 2).

The function (11) is approximated using the piecewise-linear function: *OA, AB, BC,* etc. The equation of no-load characteristic, approximated by piecewise-lines, is

$$E = a_k I_f + b_k\ ; \quad (12)$$

where $a_k = tg\alpha_k$; b_k – the initial ordinate of a line when k = 1, 2, … (Fig. 2).

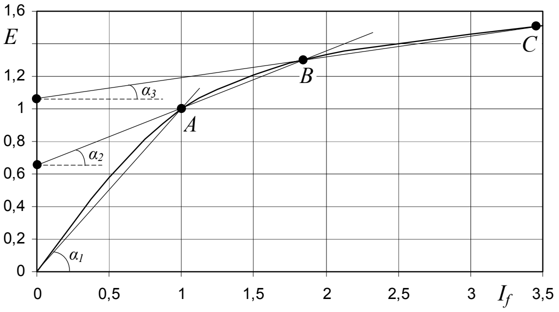

Fig. 2. No-load characteristic of non-salient pole synchronous machine

The graph *AB* of the function (10) is presented in Fig. 3, assuming that $U = 1, X = 1,\ R_a = kX\ ,\ k \geq 1$.

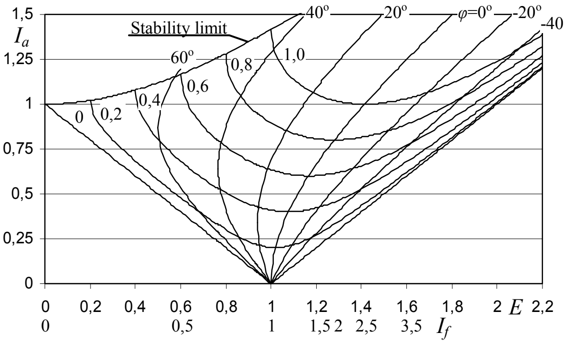

Fig. 3. The dependence of armature's phase current I_a from electromotive force *E* and excitation current I_f (the line *AB*)

The difference of the graph ordinates does not exceed 0,2 % when the resistance of armature changes in the range of $0 \leq R_a/X \leq 0{,}1$. Consequently, the functions (2), (3) and (10) for medium and high power synchronous machines may be analyzed in the case when $R_a = 0$.

The stability limit of synchronous machines, operating in parallel

Using the equation (3), written for the left branches of V-characteristics when $R_a = 0$, we receive

$$X^4 I_a^4 + \left[2\left(U^2 - E^2\right)X^2 - 4X^2U^2\right]I_a^2 +$$

$$+\left(U^2 - E^2\right)^2 + 4X^2P^2 = 0\,. \qquad (13)$$

The discriminant of the equation (13) shall fulfil the following condition

$$\frac{X^2P^2}{E^2U^2} \leq 1\,. \qquad (14)$$

By inscribing the limitary value of the active power $P = EU/X$ to the equation (13) the following relation is received

$$I_{a\,rib} = \frac{\sqrt{E^2 + U^2}}{X}\,. \qquad (15)$$

If the above given limitary value of the armature's phase current is exceeded, the synchronous machine loses a synchronism (stability). The current $I_{a\,rib}$ has one extremum (minimum):

$$\frac{dI_{a\,rib}}{dE} = \frac{E}{X\sqrt{E^2 + U^2}} = 0 \ \Rightarrow E = 0\,. \qquad (16)$$

Applying (16) to the equation (15) we have the following relation

$$I_{a\,rib\,min} = \frac{U}{X}\,. \qquad (17)$$

The curve *CD*, represented by the equation (15), limits the left branches of V-characteristics from above (Fig. 3).

The family of V-characteristics, presented in Fig. 3, is described by the equations

$$I_a = \frac{1}{X}\sqrt{E^2 + U^2 - 2\sqrt{E^2U^2 - X^2P^2}}\ , \qquad (18)$$

derived from the relationships (2) and (3), assuming that $R_a = 0$. The V-curves are given in Fig. 3 for the case when $U = 1, X = 1,\ R_a = 0$.

The trajectories of isogonic lines (*φ* = constant)

The interdependence between isogonic lines and V-characteristics is presented in the investigation of reactive current's optimization [4] for synchronous machine. When *φ* = constant, the dependencies

$$E = f\left(I_a, U\right) \qquad (19)$$

have great importance in the exploitation of synchronous machines. They represent the family of isogonic curves, described by the equations (1). The angle *φ* is treated herein as the parameter of the curves family.

The medium and high power synchronous machines fulfil the condition $R_a << X$, therefore the equation (1) may be simplified to

$$E^2 = U^2 + X^2I_a^2 + 2UI_aX\sin\varphi\,. \qquad (20)$$

Thus, the family of isogonic curves is described by the equation

$$E = \sqrt{U^2 + X^2I_a^2 + 2UI_aX\sin\varphi}\ ; \qquad (21)$$

where $\varphi < 0$ corresponds to the family of isogonic curves on the left side of the line *AB* (Fig. 3), and $\varphi > 0$ corresponds to the family of isogonic curves on the right side of the line *AB*.

It is evident that the family of isogonic lines presents the regulation characteristics of synchronous machine. In this case the mesh-graph consisting of V-characteristics and regulation characteristics may be called mesh-nomogram. The mesh-nomogram simplifies the assessment of the excitation current when the load P is constant and the requested character of reactive load $\cos\varphi$ is available. The abscissa of the point, in which the required isogonic line and V-characteristic's curve intersect, is matched by the corresponding values of electromotive force or excitation current (Fig. 3). It is easy to make sure that the dots of the plane, formed by the coordinate axes E, I and located on the left side from the line AB (Fig. 4), specify the reactive power, which is consumed from the network by a synchronous machine

$$Q_1 = UI_a \sin\varphi < 0\,; \tag{22}$$

where $\varphi < 0$.

The dots of the plane E, I_a, located on the right side from the line AB, determine the reactive power, which is supplied to a network by a synchronous machine

$$Q_2 = UI_a \sin\varphi > 0\,; \tag{23}$$

where $\varphi > 0$.

From the equation (1) it follows that

$$E^2 = U^2 + I_a^2\left(X^2 + R_a^2\right) +$$

$$+ 2UI_a\sqrt{X^2 + R_a^2}\left(\sin\varphi\cos\rho + \cos\varphi\sin\rho\right);$$

or

$$E^2 = U^2 + I_a^2\left(X^2 + R_a^2\right) + 2UI_a X\sin\varphi + 2UI_a R_a\cos\varphi\,.$$

After inserting the substitutions $UI_a\sin\varphi = Q$,

$UI_a\cos\varphi = \pm UI_a\sqrt{1 - \sin^2\varphi} = \pm\sqrt{U^2 I_a^2 - Q^2}$, we obtain

$$E^2 = U^2 + I_a^2\left(X^2 + R_a^2\right) + 2XQ \pm 2R_a\sqrt{U^2 I_a^2 - Q^2}\,. \tag{24}$$

Finally, assuming that $R_a << X$, the following relation is received

$$E^2 = U^2 + I_a^2 X^2 + 2XQ\,. \tag{25}$$

The family of the curves Q = constant (25) is plotted in Fig. 4 and together with the family of V-characteristics it forms the mesh-nomogram which is suitable for the determination of synchronous machine's excitation current according to the reactive power Q under demand.

The relation between reactive power Q and voltage deviation ΔU is described [5] in the given point as follows

$$Q = k\frac{U}{X_1}\Delta U\,; \tag{26}$$

where X_1 – inductive reactance of transmission line and transformer between the given point and the point in which $\Delta U = 0$; U – nominal voltage; ΔU – voltage deviation in the given point; k = constant – coefficient; $\Delta U > 0$ – when the voltage in the given point is lower than the nominal voltage; $\Delta U < 0$ – when the voltage in the given point is higher than the nominal voltage.

Hence, according to the formula (26) we find $\pm Q$, which is adequate to the corresponding line of the Fig. 4. The abscissa of the point, in which the lines Q and P intersect, presents the excitation current I_f, determining the equality $\Delta U = 0$.

The active load P of synchronous machine shall be reduced in the case the excitation current exceeds the permissible value. When such possibility is not available, the line of reactive power, determined by the marginal value of excitation current, shall be selected. It is obvious that the deviation of voltages will be reduced in this case but it will not become zero.

The mesh-nomogram, shown in the Fig. 4, is convenient for the use in the nods of distribution networks, which synchronous motors or compensators are connected to.

The excitation current I_f and the phase electromotive force E, set on the axis of abscissa (Fig. 4), correspond to the no-load characteristic (Fig. 2).

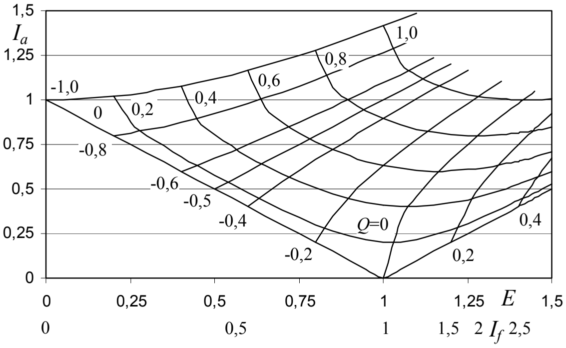

Fig. 4. The mesh-nomogram of synchronous machines, operating in parallel

Conclusions

1. The analytical expressions are offered for the calculation of synchronous machines V-characteristics. It is proved that the form of V-characteristics match up to the form of hyperbola, but not to an ellipse as it is claimed in [1].

2. The stability limit and its analytical expression are determined for V-characteristics of synchronous machines.

3. The definition for the isogonic lines as well as the mesh-nomogram of the isogonic lines and V-characteristics of synchronous machines are presented.

4. The mesh-nomogram P = constant, Q = constant is formed in the coordinate system $\left(E, I_a\right)$ and $\left(I_f, I_a\right)$ for the determination of excitation current when the voltage deviation ΔU in the node of synchronous machine's connection is known.

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P. Kostrauskas, A. Kalvaitis, A. Degutis, L. Andriušienė. Neryškiapolių sinchroninių mašinų V charakteristikų analizė // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 5(61). – P. 18–22.

Sinchroninių elektros mašinų, dirbančių lygiagrečiuoju režimu, techninių ir ekonominių rodiklių optimizavimas – aktualus sinchroninių mašinų eksploatavimo uždavinys. Analiziškai nustatyta neryškiapolės sinchroninės mašinos darbo statinio stabilumo ribos išraiška. V charakteristikoms apskaičiuoti pateiktos lygtys: viena lygtis aprašo V charakteristikos, esančios kairėje nuo linijos cosφ = 1, išraišką, o antra – V charakteristikos, esančios dešinėje nuo linijos cosφ = 1, išraišką. Patvirtinta, kad V charakteristikų šakas sudaro hiperbolės. Pateikta V charakteristikų izogonalių linijų definicija; kiekvieną V charakteristikų plokštumos (E, I_a) tašką atitinka viena kampo φ vertė. Pateikta plokštumos (E, I_a) tinklinė nomograma, įgalinanti, esant kiekvienai žadinimo srovės vertei, nustatyti sinchroninės mašinos aktyviajai galiai reaktyviosios galios rezervą, kurį sinchroninė mašina gali tiekti į tinklą arba iš tinklo gauti. Ši informacija įgalina optimaliai išnaudoti sinchroninės mašinos galimybes, optimizuojant elektros perdavimo linijų ir tinklų reaktyviosios galios cirkuliaciją. Pateiktoji grafinė medžiaga papildo V charakteristikas atitinkančias linijas P = const naujomis linijomis: Q = const, φ = const bei statinio stabilumo linija. Il. 4, bibl. 7 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

P. Kostrauskas, A. Kalvaitis, A. Degutis, L. Andriušienė. V-Characteristics Analysis of Non-Salient Pole Synchronous Electrical Machines // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 5(61). – P. 18–22.

The optimisation of technical and economic indices of synchronous electrical machines, operating in parallel, is the urgent problem in the exploitation of synchronous machines. The analytical expression of the static stability limit is determined for the operation of non-salient pole synchronous machine. The equations are offered for the calculation of V-characteristics: one of the equations describes the expression of V-characteristic, situated on the left side of the line cosφ = 1, and the other equation describes the expression of V-characteristic, situated on the right side of the line cosφ = 1. There is proved, that the branches of V-characteristics have the shape of hyperbola. The definition for the isogonic lines of V-characteristics is given. Every point of the plane (E, I_a) of V-characteristics is matched by one value of angle φ. The mesh-nomogram of the plane (E, I_a) is presented. The mesh-nomogram enables, according to the given active power of synchronous machine, to define for any value of excitation current the reserve of reactive power, which may be supplied to or may be consumed from the power network. This information enables to use optimally the possibilities of synchronous machine by optimising the circulation of reactive power in electricity transmission lines and networks. The presented graphical materials supplement the lines P = const, corresponding to V-characteristics, by additional lines: Q = const, φ = const as well as by the line of static stability. Ill. 4, bibl. 7 (in English; summaries in Lithuanian, English, Russian).

П. Костраускас, А. Калвайтис, А. Дегутис, Л. Андрюшене. Анализ V-образных характеристик неявнополюсных синхронных электрических машин // Электроника и электротехника. – Каунас: Технология, 2005. – № 5(61). – С. 18–22.

Оптимизация технико–экономических показателей синхронных машин, работающих в параллельном режиме – важная эксплуатационная задача современной электроэнергетики. В работе представлено определение и аналитическое выражение линий статической устойчивости неявнополюсной синхронной машины. Для вычисления V-образных характеристик представлены два уравнения: первое – для левых ветвей, второе – для правых. Доказано, что V-образные характеристики представляют собой гиперболы. Дан вывод уравнения cosφ = 1, а также изогональных линий φ = const. Представлена сетчатая номограмма в плоскости электродвижущей силы E и тока якоря I_a для определения активной и реактивной мощности, отдаваемой синхронной машиной в сеть или забираемой из сети, и для определения коэффициента мощности. Данная информация позволяет оптимизировать циркуляцию реактивной мощности с целью стабилизации напряжения в рассматриваемом узле. Представленный аналитический и графический материал дополняет V-образные характеристики, изображающие семейство линий P = const, дополнительными линиями Q = const и φ = const, а также линией статической устойчивости синхронной машины. Ил. 4, библ. 7 (на английском языке; рефераты на литовском, английском и русском яз.).