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Errors of Constant Rotation Angle Fast Orthogonal Transforms Used for Fixed-Point Arithmetic DSP Applications: Preliminary Results

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Introduction

In the digital signal processing unitary transforms are widely used [1]. For simplicity, here we will operate only with the real orthonormal transforms and one-dimensional signals. It is well known that for the calculation of direct transform (spectrum) usually the following matrix expression is used:

$$\mathbf{y} = \mathbf{\Phi} \cdot \mathbf{x}\,, \tag{1}$$

where $\mathbf{\Phi}$ is orthonormal matrix with size N by N, but $\mathbf{x}$ and $\mathbf{y}$ – the input and output column-vectors (with size N by 1), respectively. To perform the inverse transform we need the transposed matrix $\mathbf{\Phi}^{\mathbf{T}}$ instead of $\mathbf{\Phi}$:

$$\mathbf{x} = \mathbf{\Phi}^{T} \cdot \mathbf{y}\,. \tag{2}$$

We use also a well-known and practical restriction for N:

$$N = 2^{n}, \quad n = 1, 2, \ldots \tag{3}$$

If necessary, our results related to the estimation of errors can be adopted also to the real orthogonal transforms. Briefly, in such a case we should use some scaling constant(s) for the final correction of error(s).

Additionally, within this paper only normalized input signal has been used:

$$x_{norm}(k) = x(k) / \sqrt{\sum_{m=1}^{N} x^{2}(m)}\,, \tag{4}$$

where $x(k)$ is the k-th element of the vector $\mathbf{x}$. Further in the text we will skip the index "*norm*" and consider the vector $\mathbf{x}$ as normalized.

The fast algorithms and transforms cover the huge area for research [2]. The large amount of papers and books deal mainly with well-known fast transforms (FT) like FFT, Hadamard FT, Haar FT, wavelets, and so on. In our investigation we focus only on the very narrow class of real fast orthogonal transforms – CRAFOT (see explanations below). We were not able to find a term as "constant rotation angle transform" in the literature [3]. We suppose that this is a novel class of orthogonal functions, and we are ready to discard our results (fully or partly) immediately if we plagiarize some results published by anybody else.

The final goal of our work is the implementation of CRAFOT into chip and DSP systems with fixed-point arithmetic (FPA). The accuracy of calculations by FPA DSP depends on the wordlength of processor because we have a quantization error. Additionally, the size of transform matrix (the length of input vector) impacts the accuracy of results.

The estimation of errors depends on the application of CRAFOT. Currently we work on several applications of the mentioned algorithm. This paper covers only a small part of our work, mainly the estimation of the maximal MSE of restoration of signal after the subsequent encoding and decoding by the CRAFOT.

At the time of submission of the paper we consider only "preliminary results". That is because we present only the results for the limited number of wordlengths and the limited sizes of operands. And we give only a limited number of details about the CRAFOT basis functions. On the other hand, we are not able to highlight all details in such a short paper.

Basics of Fast Orthogonal Transforms

This section is only for illustrative purposes and cannot be considered as a strong mathematical description. The main goal of this section is to highlight some points which are related to FT, and possibly can be useful to the readers who are not so familiar with the subject of this section. On the other hand, this section is the base for understanding of the content of subsequent sections.

The basic idea is the representation of orthonormal (orthogonal) matrix by the product of sparse matrices (factorization) $\mathbf{B_i}$:

$$K \cdot \mathbf{\Phi} = k_1 \cdot \mathbf{B_1} \cdot k_2 \cdot \mathbf{B_2} \cdot \ldots \cdot k_n \cdot \mathbf{B_n}\,, \tag{5}$$

where

$$n = \log_2 N\,, \tag{6}$$

but K – some scaling factor (the product of scalar constants k_1, k_2, etc.). The subject of the present work is a particular case of (5), when we operate only with equal matrices:

$$\mathbf{B} = \mathbf{B_1} = \mathbf{B_2} = ... = \mathbf{B_n}. \tag{7}$$

The main result of factorization is the principal reduction of multiplications and additions for the calculation of spectrum (in the algebraic sense) by (1) and (2). The mentioned reduction is possible because we operate with sparse matrices that contain mainly zeros. There are usually only two elements (in some algorithms can be more) per column and per row also which are not equal to zero (see below).

Interpretation of Fast Orthogonal Transforms by Rotation of Planes

In the literature exist different approaches how to interpret the matrix **B**. Unfortunately, we had difficulties to find out a clear geometrical interpretation of FT in the available books and IEEE papers [3]. Sophisticated explanations may be found but they require advanced knowledge of matrix algebra to be understood. One of the authors of this paper uses comparatively simple geometrical interpretation of FT for teaching students over many years.

The idea about the rotation of planes in the Euclidian space is not original. This approach is well known, for example, from the QR-algorithm [5], [6]. Unfortunately, none uses the geometrical interpretation for the explanation of FT. However, several viewpoints and approaches can be the base for additional benefits. We will try to demonstrate some consequences of the present approach.

Rotation Matrixю From the linear algebra [5], [6] is well known Given's rotation matrix that contains the basic structure:

$$\mathbf{G}_+(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}, \tag{8}$$

where ϕ - rotation angle, but "+" means "clockwise". Further we use the shortcuts for the cosine (c), sine (s) and rotation matrices (**G**, **R**). Multiplication by $\mathbf{G}_+$ means a clockwise rotation of plane or Cartesian coordinate system by angle ϕ, if we consider geometrical interpretation. This multiplication leads to the changes of two coordinates of vector **x** and we obtain the new coordinates for vector **y**:

$$\begin{bmatrix} y_i \\ y_j \end{bmatrix} = \mathbf{G}_+ \cdot \begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} c \cdot x_i - s \cdot x_j \\ s \cdot x_i + c \cdot x_j \end{bmatrix}, \tag{9}$$

where x_i, x_j – "old" coordinates of vector, but y_i, y_j – coordinates of vector in the rotated coordinate system.

The structure (8) is not a sole elementary rotation matrix. There are several possibilities for the representation of matrices like $\mathbf{G}_+$ – we may consider the anticlockwise rotation, reflection or more sophisticated manipulations with plane [5]. For example, the reflection matrix:

$$\mathbf{R}_+ = \begin{bmatrix} s & c \\ c & -s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \mathbf{G}_+ \tag{10}$$

ensures the clockwise rotation with the subsequent permutation of coordinates (briefly – the reflection versus bisector (between coordinate axes)). The elementary Hadamard matrix is a good example for the interpretation by reflection matrix:

$$\frac{\sqrt{2}}{2}\mathbf{H_2} = \frac{\sqrt{2}}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \mathbf{R}_+(\frac{\pi}{4}) \tag{11}$$

In the N-dimensional Euclidian space we can define the infinite number of orthonormal basis. Each pair of basis vectors represents one plane, and the basis can be interpreted as a set of ***N/2*** independent planes (for N defined by (3)). There are a lot of combinations for the choice of pairs of vectors, but it is not so important for this paper.

For the rotation of each pair of basis vectors (or each plane) we may use a rotation matrix like (8), (10) or other. Since we have *N/2* independent planes it is possible to rotate all of these planes simultaneously. In the matrix notation that means the using of:

$$\mathbf{B_R} = \left.\overbrace{\begin{bmatrix} s_0 & c_0 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & s_1 & c_1 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & 0 & 0 & s_2 & c_2 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 0 & 0 & ... & s_{\frac{N}{2}-1} & c_{\frac{N}{2}-1} \\ c_0 & -s_0 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & c_1 & -s_1 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & 0 & 0 & c_2 & -s_2 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 0 & 0 & ... & c_{\frac{N}{2}-1} & -s_{\frac{N}{2}-1} \end{bmatrix}}^{N}\right\} N, \tag{12}$$

where c_i and s_i represents the cosine and sine values for the i-th rotation. The index "**R**" means that we use the rotation matrix defined by (10) as the basic structure for $\mathbf{B_R}$. But we do not limit the structure of elementary rotation matrix only by (10). Other rotation structures are also allowed in (12). Generally we can use N/2 different angles for definition of $\mathbf{B_R}$. The next steps of generalization are for the papers in the future.

The main feature of this sparse matrix is that each row/column contains only two elements that are not equal to zero. The second important feature is the "stairs-like" placement of rows of elementary rotation matrices in the upper and in the lower part of $\mathbf{B_R}$.

Constant Rotation Angle Fast Transform. In this paper we limit our efforts to the narrow and novel class of orthonormal basis functions (BF). We assume that all angles for the definition of (12) are equal. In this case:

$$c = c_1 = c_2 = ... = c_{\frac{N}{2}-1}, \quad s = s_1 = s_2 = ... = s_{\frac{N}{2}-1} \tag{13}$$

Now from (5), (7), (12) and (13) we get:

$$\mathbf{\Phi} = K \cdot \underbrace{\mathbf{B} \cdot \mathbf{B} \cdot ...\mathbf{B}}_{n} = \mathbf{B^n} \tag{14}$$

We ignore the index "**R**" and assume that generally for the building of **B** there can be used different rotation matrices like (8) or (10), or other. Further we will talk about CRAFOT without the strong restrictions on the kind of elementary rotation matrix. In some cases (if necessary) we will add the index that corresponds to the "basic brick" of **B**.

We use the term CRAFOT within this paper only. Possibly, we can look at the presented algorithm as a particular case of the unified approach for the generation of fast unitary transforms [7]. That is not so principal question because the main goal of this work is the estimation of errors but not so much the strong mathematical definition or presentation of CRAFOT in details.

Constant Rotation Angle Orthogonal Transform

The ***C**onstant **R**otation **A**ngle **O**rthogonal **T**ransform* (CRAOT) can be presented in the factorized form (14) or as the result of the product of matrices **B**. Talking about FT we mean CRAFOT, but using of CRAOT can be handy in other cases. The factorized form (14) is more practical for the implementation, but the matrix $\mathbf{\Phi}$ better describes the properties of CRAOT. It is like the mutual relation between DFT and FFT.

We save a detailed investigation of properties of CRAOT for the future. Here we present only the introductory description of properties without strong mathematical proof, and we suppose that our description is only a preliminary description. Some properties come from the basics of linear algebra. For example, an orthogonality follows from the mathematical fact that the product of orthogonal matrices is also an orthogonal matrix.

The shape of CRAOT BF depends on the chosen rotation angle. For $\phi = 0°$ we have the well-known set of orthonormal δ-functions, but for $\phi = 45°$ we get the Hadamard matrix:

$$\mathbf{\Phi}_{\mathbf{R}_+}(\frac{\pi}{4}) = (\sqrt{2}\cdot\mathbf{B}_{R_+}(\frac{\pi}{4}))^n = \mathbf{H}_N = \mathbf{H}_{2^n}, \qquad (15)$$

where $\mathbf{H}_N$ – the Hadamard matrix of order N. If we represent Hadamard matrix as the product of rotation matrices (terms) $\mathbf{B}_\mathrm{R}$, we must provide the scaling constant for each factor. That is necessary to avoid multiplications.

For other rotation angles we have the rich diversity of the shapes of BF. Next figure shows a typical BF. Possibly, each BF we may characterize as a "pulse-like self-similar sequence". On the other hand we can treat the shown BF as a pulse sequence with echo (for N/2 samples). Maybe the name "echo functions" is the best and concise characteristic of CRAOT, however there is a well-known and similar term in telecommunications [8]. When we increase the rotation angle (near to 45°), we obtain "damaged Hadamard functions".

From one of the theorems proven by Good [9] it follows that the Kronecker power of matrix (the Kronecker product of equal matrices) can be expressed as the power of sparse matrices (12):

$$\underbrace{\begin{bmatrix} s & c \\ c & -s \end{bmatrix} \otimes \ldots \otimes \begin{bmatrix} s & c \\ c & -s \end{bmatrix}}_{n} = \underbrace{\mathbf{R}_+ \otimes \ldots \otimes \mathbf{R}_+}_{n} = \mathbf{B}_\mathbf{R}^\mathbf{n}. \qquad (16)$$

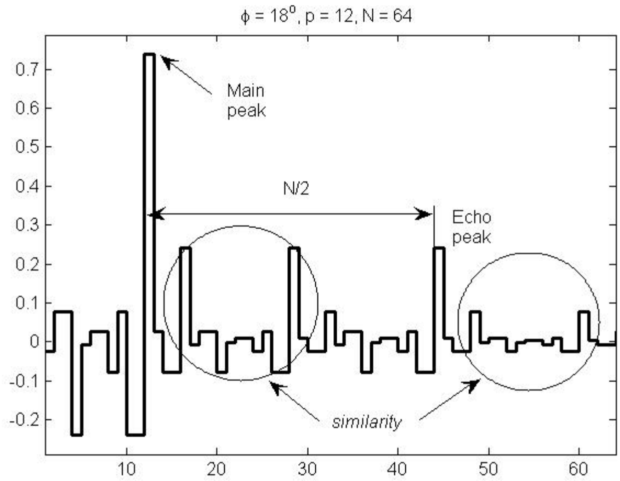

Fig. 1. Basis function with the index p=12 for the rotation angle ϕ=18° and N=64

We may use the **G** instead of **R** and $\mathbf{B_G}$ instead of $\mathbf{B_R}$ if necessary. From (16) it follows that the value of CRAOT BF can be written also as the product of the power of sine and cosine values:

$$\Phi(p,t) = \prod_{i=0}^{n-1} (-1)^{a_i} \cdot s^{m_i(p,t)} \cdot c^{k_i(p,t)}, \qquad (17)$$

where

$$\begin{cases} a_i(p,t) = p_i \cdot t_i,\ p,t \in [0, N-1], \\ m_i(p,t) = p_i \cdot t_i + \bar{p}_i \cdot \bar{t}_i, \\ k_i(p,t) = \bar{p}_i \cdot t_i + p_i \cdot \bar{t}_i, \\ \sum_{i=0}^{n-1} m_i(p,t) + k_i(p,t) = n, \quad i \in [0, n-1], \end{cases} \qquad (18)$$

p – the index of row of $\mathbf{\Phi}$ (the index of BF), t – the index of column (the index of sample of BF), p_i, t_i – the value of the i-th bit of p and t, respectively. The "roof" label $\bar{x}$ means the binary inversion of bit. Formulas (18) are useful only for the rotation matrix $\mathbf{R}_+$ and follow from the Kronecker product (16) and the next truth table:

Table 1. "Selector" of multipliers (for $\mathbf{R}_+$)

$p_i \backslash t_i$	0	1
0	s	c
1	c	-s

For the other kinds of rotation matrices we should rewrite the table accordingly.

From (17) it follows that <u>CRAOT BF has the maximum</u> (for the module of BF). <u>The position of the maximum is at $t = p$</u>. An exception is the Hadamard functions. The maximum is more remarkable for small rotation angles and decreases by increasing the angle.

The use of (14) provides the simplest and fastest way to generate the full (N) set of BF. In some cases we need only one or few BF. Then formula (17) comes in very handy.

Sources and definitions of Errors

Quantization and Quantization Errors (QE). We deal only with the QE in this paper. We use the well-known quantization rule for the scalars and operands:

$$e_q = \begin{cases} e_{q\,\min}, & for\ e \le e_{q\,\min}, \\ round\left(\dfrac{e - e_{q\,\min}}{q}\right) * q + e_{q\,\min}, & for\ e \in (e_{q\,\min}, e_{q\,\max}), \\ e_{q\,\max}, & for\ e \ge e_{q\,\max}, \end{cases} \quad (19)$$

where

$$q = (e_{q\max} - e_{q\min})/(2^{nbits} - 1) \quad (20)$$

– quantization step, *nbits* – the wordlength of quantized value, but e – the value of x, y, s, c $\in$ [–1, 1] and $\phi \in [0, \pi/2]$ (or [0, 90°]). The index "q" indicates that we use the quantized value. Readers should remember that we talk only about the digital system. It means that the quantization of input signal makes sense only in the case if we use the quantization with the reduced number of bits. Additionally, the rotation angle must be chosen from the finite set of values (depends on the wordlength of quantized angle).

The QE can be written as

$$\delta\mathbf{e} = \mathbf{e}_q - \mathbf{e}, \quad (21)$$

where $\mathbf{e}$ – the name of selected scalar or vector (s, c, $\mathbf{x}$, $\mathbf{y}$).

Figure 2 shows the changes of the normalized (to the quantization step) QE for trigonometric functions versus the rotation angle. Such picture is typical for the QE. The frequency of oscillations of QE increases by the wordlength of angle. We observe also a mutual mirror-symmetry between the sine and cosine QE, and the central symmetry for the squared error around 45°.

Primary and Secondary Errors. We can distinguish here the primary and secondary errors. The primary errors are caused by the direct corruption (quantization) of scalars and operands. This kind of error is defined above by (21).

The secondary errors appear as the consequences of calculation of transforms. The definition of the secondary error is the following:

$$\Delta\mathbf{e} = \hat{\mathbf{e}} - \mathbf{e}. \quad (22)$$

In this case the corrupted value or operand is marked by "^".

Corruption Scheme. The next scheme-formula illustrates the corruption of operands for each stage of signal transformation if we implement the subsequent direct and inverse transforms:

$$\mathbf{x} + \delta\mathbf{x} \Rightarrow (\mathbf{\Phi} + \Delta\mathbf{\Phi}) \cdot \mathbf{x}_q \Rightarrow \mathbf{y} + \Delta\mathbf{y} \Rightarrow$$

$$\Rightarrow \hat{\mathbf{y}} + \delta\hat{\mathbf{y}} \Rightarrow (\mathbf{\Phi}^T + \Delta\mathbf{\Phi}^T) \cdot \hat{\mathbf{y}}_q \Rightarrow \mathbf{x} + \Delta\hat{\mathbf{x}}. \quad (23)$$

This scheme is simplified and symbolic enough. For example, the corruption of $\mathbf{\Phi}$ is complicated and that cannot be interpreted as only additive by its nature.

If we use the CRAFOT for the encoding of signal with the subsequent decoding of encoded signal, the error of restoration of input signal can be expressed as:

$$\Delta\hat{\mathbf{x}} = \hat{\mathbf{\Phi}}^T \cdot (\hat{\mathbf{\Phi}} \cdot \mathbf{x}_q)_q - \mathbf{\Phi}^T \cdot (\mathbf{\Phi} \cdot \mathbf{x}) =$$

$$= (\underbrace{\mathbf{B}_q \cdot \mathbf{B}_q \cdot \ldots \cdot \mathbf{B}_q}_{n})^T \cdot \hat{\mathbf{y}}_q - \mathbf{x}, \quad (24)$$

where

$$\hat{\mathbf{y}}_q = \underbrace{\mathbf{B}_q \cdot \mathbf{B}_q \cdot \ldots \cdot \mathbf{B}_q}_{n} \cdot \mathbf{x}_q \quad (25)$$

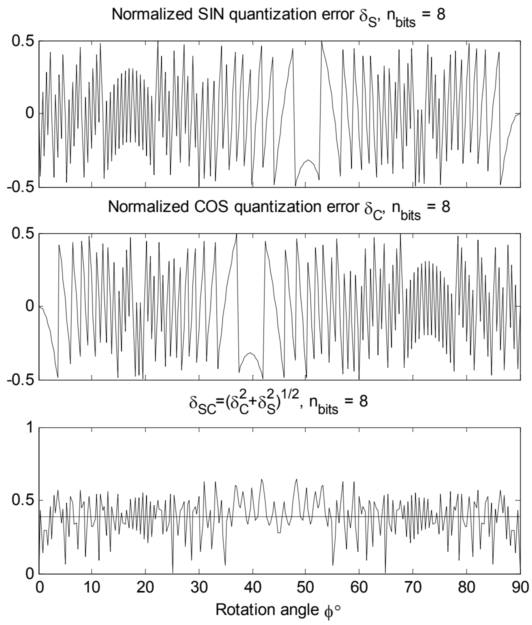

Fig. 2. QE (normalized to the quantization step) for the sine and cosine functions versus the rotation angle

Definition of Errors. Further we will use the maximal normalized (to quantization step) MSE (more precisely, Euclidean norm):

$$\varepsilon_{norm}^{\max} = \max_{p \in [1,\,M]} \left(\frac{1}{q} \sqrt{\sum_{i=1}^{N} \Delta^2 \hat{x}_p(i)} \right) \quad (26)$$

and the upper limit of this error:

$$\varepsilon_{norm}^{\max,\max} = \max_{\phi \in [0,\,\pi/4]} (\varepsilon_{norm}^{\max}), \quad (27)$$

where $\Delta\hat{x}_p(i)$ is the i-th element of $\Delta\hat{\mathbf{x}}_p$. For simulation we use a set of M random input signals. The index "p" means the index of trial. We ignore the division by N in the formula (26) because the vector $\mathbf{x}$ is normalized before (see (4)).

Numerical Results

The present results are useful for **Q1.x** FPA [10].

CRAFOT Simulation. The basic tool for the calculation of errors is the CRAFOT simulator together with some additional functions. This tool is an interactive program coded in MATLAB, containing approximately 1000 lines.

We performed simulation of CRAFOT for rotation angles within the range from 0 to 45 degrees. Next figure demonstrates the typical behavior of error depending on the rotation angle and the size of operands N for the fixed value of wordlength. We see the significant oscillations of error defined by (26) versus the rotation angle.

Such behavior of error is caused by the nature of QE. The QE of sine and cosine has serious oscillations as we see below (fig. 3).

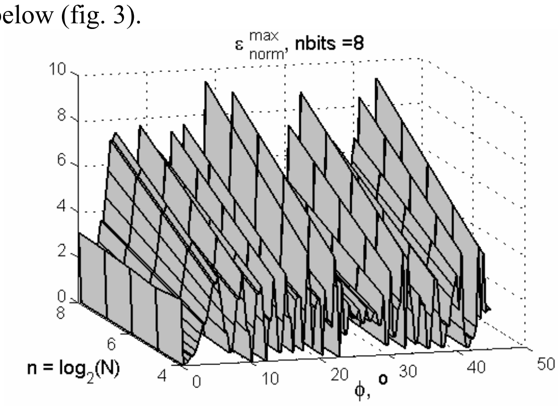

Fig. 3. The maximal normalized MSE depending on the rotation angle and the size of operands for *nbits* = 8

Our experiments show that the errors (26) differ by about 10-20 times (also for the large number of trials) depending on the rotation angle. This means that we need also the detailed investigation of the lower limit and distribution of MSE (supplementary to (27)). That can help in the choice of optimal angles (from the viewpoint of error) for the implementation of CRAFOT.

We would like also to add some words about the conditions of simulation. We used randomly generated input signal vectors. The number of trials lies in the range from 50 to 100 for each point of calculations. There are different results for the input signals with uniform probability density function (PDF) and normal PDF (the variance equal to 1 before normalization). This paper presents only results related to normal PDF.

Behavior of Upper Limit of Error. The oscillation of error is the reason for the use of upper limit (27) of MSE (further simply - error). Next figures show the behavior of error (calculated by (27) and marked as “all sources of QE“) versus the wordlength of operands.

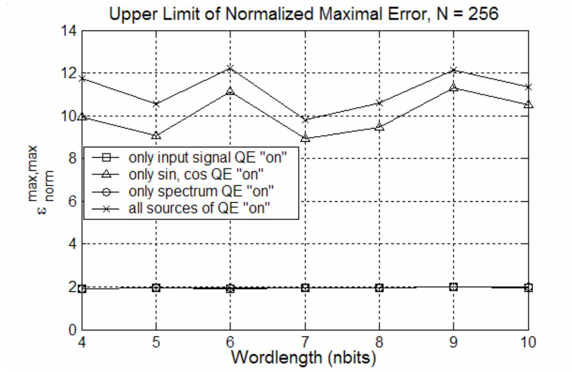

Fig. 4. The upper limit of maximal normalized MSE for different kind of sources of QE versus the wordlength of operands. $N = 256$

The oscillation of upper limit around 11 seems strange only at first sight. We should remember that we operate with the error normalized to quantization step q. This means that we have the decreasing of the value of “unnormalized” error by the increasing of wordlength of operands in reality. We estimate the value of error as a constant (*12q,* if *nbits*=**8**) for the practical needs.

As shown in figure 5, the error increases logarithmically depending on the size of operands. For the practical needs the logarithmic relationship between the N and error can be useful:

$$\varepsilon_{norm}^{\max,\max} \equiv k(nbits) \cdot n + b(nbits)\,, \tag{28}$$

where n is defined by the logarithm (6), $k \in$ **[1, 1.4]** and $b \in$ **[0, 2]**. The values of k and b are the functions of the wordlength of operands. For example, $k(8) =$ **1.1** and $b(8) =$ **1.7**. Unfortunately, the changes of k and b are not monotonous, and we are not able to present simple formulas for that. The accuracy of (28) depends on the model used for fitting (on the number of terms). The linear model ensures the accuracy within the range 5-15%. Formula (28) is useful only for the case when all the sources of QE are taken into account.

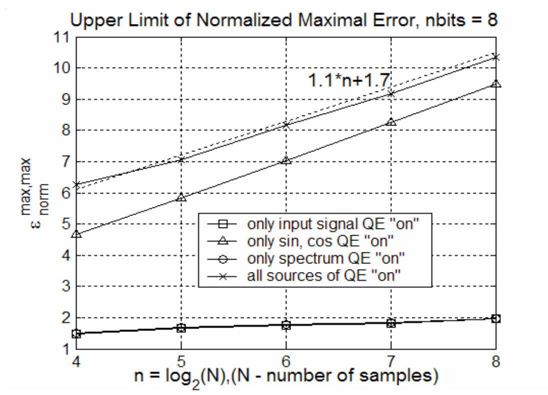

Fig. 5. The upper limit of maximal normalized MSE for different kind of sources of QE versus the size of operands. *nbits* = 8

Impact of Different Sources of Errors. We simulated the impact of different sources of QE, which correspond to corruption scheme (23). The figures 4 and 5 show the behavior of error when we observe or ignore different sources of errors. If we use the same wordlength for all the operands, the impact of quantization of sine and cosine dominates. We can ignore the impact of quantization of input vector and spectrum for N > 32, if we allow the inaccuracy up to 15%. On the other hand, obtained results indicate that we must use more accurate quantization for the trigonometric functions than is needed for the signal vector or spectrum.

Conclusions

The main conclusions:

- The upper limit of MSE of restoration of signal is practically constant depending on the wordlength of CRAFOT operands and practically linear (in the logarithmic scale) versus the size of operands.
- The upper limit of MSE of restoration of signal is overestimated for many rotation angles. We need more realistic and detailed estimation in the near future.
- Potential applications of CRAFOT could be signal compression systems and "echo signal systems" (audio signal processing, biology etc.)
- We need a further and more detailed investigation of the properties of CRAOT.

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Pateikiamas trumpas greito ortogonalinio keitiklio su pastoviu sukimosi kampu (GOKPSK) aprašymas. Pagrindinis dėmesys skiriamas GOKPSK klaidų, kurias sąlygoja įėjimo signalo kvantavimas ir ilgio, bei keitimo matricos dydis, analizei. Gauti rezultatai rodo žymias klaidų osciliacijas priklausomai nuo sukimosi kampo. Pateikiama viršutinė signalo kvantavimo vidutinės kvadratinės paklaidos atkuriant signalą riba (po tiesioginio ir atgalinio keitimo). Atkūrimo paklaidą galima laikyti pastovia priklausomai nuo GOKPSK operandų pateikimo žodžio ilgio, ir praktiškai tiesine (logaritminaime mastelyje) priklausomai nuo GOKPSK operandų dydžio. Il. 5, bibl. 8 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

P. Misans, M. Terauds. Errors of Constant Rotation Angle Fast Orthogonal Transform Used for Fixed-Point Arithmetic DSP Applications: Preliminary Results // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 4(60). – P. 17–22.

A brief description of the constant rotation angle fast orthogonal transform (CRAFOT) is presented. The main goal of the present work is the investigation of the errors of CRAFOT algorithm caused by the quantization and the size of operands (the input signal vector and the transform matrix). The obtained results show significant error oscillations that depend on the rotation angle. The paper presents the upper limit of the normalized to the quantization step the maximal mean-squared error (MSE) of restoration of signal (after the subsequent direct and inverse transform). The restoration error is approximately constant depending on the wordlength of CRAFOT operands and practically linear (in the logarithmic scale) versus the size of operands. Ill. 5, bibl. 8 (in English, summaries in Lithuanian, English, Russian).

П. Мисанс, М. Тэраудс. Ошибки быстрого ортогонального преобразования с постоянным углом вращения для использования в ЦОС применениях с фиксированной запятой: Предварительные результаты // Электроника и электротехника. – Каунас: Технология, 2005. – № 4(60). – С. 17–22.

Приводится краткое описание быстрого ортогонального преобразования с постоянным углом вращения (БОППУВ). Центральное место в работе занимает исследование ошибок БОППУВ, вызванных квантованием и конечной длиной входного сигнала и размером матрицы преобразования. Полученные результаты показывают существенные осцилляции ошибок в зависимости от угла вращения. В статье представляется верхний предел нормированной к шагу квантования максимальной среднеквадратической ошибки восстановления сигнала (после прямого и обратного преобразования). Ошибку восстановления можно считать примерно постоянной в зависимости от длины слова представления операндов БОППУВ и практически линейной (в логарифмическом масштабе) в зависимости от размера операндов БОППУВ. Ил. 5, библ. 8 (на английском языке; рефераты на литовском, английском и русском яз.).