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Advanced Processing of Nonuniformly Sampled Non-Stationary Signals

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Introduction

A signal is stationary if its statistical characteristics do not change with time. Signals of practical interest often do not comply with this requirement [1]. It has been quit difficult to satisfactorily handle non-stationary signals using conceptualizations based on stationarity, as it is assumed, for example, by classical Fourier transform. Non-stationary signals justify the need for joint time-frequency analysis and representation.

Non-stationary signals may be divided into two types: momentarily transient and persistent. The momentarily transient signal has a brief, finite duration. The persistent non-stationary signal has continuous time-varying behavior. In practice the time-frequency representation is characterized by points on a time-frequency gram with a finite duration time axis and finite bandwidth frequency axis.

Time-frequency analysis typically deals with signals for which the instantaneous frequency bandwidth is considerably narrower than the whole bandwidth of signal spectral characteristics [2]. As examples can be quoted chirps, Doppler signals, frequency tracking etc. To process signals digitally they should be sampled. The Nyquist criterion gives us a theoretical limit to what rate we have to periodically sample a signal that contains data at a certain maximum frequency. Once we sample below the Nyquist rate we get the spectral analysis results, which have corrupting artifacts – so called "aliases". A dilemma concerning the choice of sampling rate arises: on the one hand the maximum signal frequency defines sampling frequency according to Nyquist, while on the other hand the narrow instantaneous bandwidth of signal at each time moment allows a considerably lower sampling density. One possible course of action in such a case is to use a nonuniform sampling technique. The proper application of nonuniform sampling suppresses the frequency aliasing and allows the use of a sampling density below the Nyquist rate [3].

It should be stated that nonuniformly taken signal samples require the focusing of more attention on the signal processing algorithm. The benefit achieved by suppression of frequency aliasing could translate into some other corrupting artifact, for example, the increased noise floor of spectrogram as it is usually for the standard spectral estimation algorithms. In this paper the advanced signal processing method will be discussed, which will provide high frequency and time resolution in a wide dynamic range of analysis.

Typical Time-frequency representations

The classical method for analyzing non-stationary signals is short time Fourier transform (STFT). It was proposed by Gabor in 1946. STFT is based on the well known Fourier transformation

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\exp(-j\omega t)dt\,. \qquad (1)$$

From (1) follows that signal $x(t)$ is integrated over all time. It means that one does not need to worry about time after transformation is applied. There is no attention to when the signal components of different frequencies act. The basic idea of STFT is to introduce the time window, which is moved along the signal, and in such a way time indexed spectrum can be calculated:

$$STFT(t,\omega) = \int x(\tau)g(\tau - t)\exp(-j\omega\tau)d\tau\,. \qquad (2)$$

It is obvious from (2) that the time-frequency analysis result depends on time window $g(t)$ choice. Long time windows provide good frequency resolution, but poor time resolution. Short time windows provide good time resolution, but poor frequency resolution [4]. STFT for signals sampled nonuniformly at time instants t_k can be expressed as:

$$STFT(\tau,\omega) = \sum_{k:(t_k-\tau)\in T_g} x(t_k)g(t_k-\tau)\exp(-j\omega t_k)\,, \qquad (3)$$

where summation involves the samples located within the selected time window with length T_g. The basic drawback of STFT is its resolution limitation. It can be improved replacing Fourier transform with high-resolution spectral estimate techniques, for example, autoregressive (AR) modeling [1, 5].

The Wigner distribution (WD) has been employed as an alternative to overcome resolution drawback of the STFT [4]. WD in general is expressed as

$$W_x(t,\omega) = \int x(t+\tau/2)x^*(t-\tau/2)\exp(-j\omega\tau)d\tau\,. \qquad (4)$$

WD provides high-resolution representation in time and in frequency for monocomponent signals. However, if the signal consists of several subcomponents, additional interference or cross-terms appears [4, 6].

A discrete form of the WD can be expressed as

$$WD(\tau,\omega) = 2\sum_{k=-\infty}^{\infty} x(\tau+t_k)x^*(\tau-t_k)\exp(-j2\omega t_k)\,. \qquad (5)$$

Note the necessity to know signal values at time instants $\tau + t_k$ and $\tau - t_k$ for all k that leads to the WD application only for uniformly sampled signals. Moreover, to avoid the distortion due to frequency aliasing, the signal $x(t)$ has to be sampled at twice the Nyquist frequency for real valued signal.

To overcome the disadvantages of the cross-terms of the Wigner distribution and the resolution limitations of the STFT, the wavelet transform (WT) is an alternative [7]. The continuous wavelet transform of a signal $x(t)$ is defined as

$$WT(t, a) = \frac{1}{\sqrt{a}} \int x(\tau) h^* \left(\frac{\tau - t}{a} \right) d\tau , \qquad (6)$$

where a is the scaling factor and $h(t)$ is the so-called analyzing wavelet. The time-frequency version is obtained by making the substitution $a = f_0 / f$. The analysis can be viewed as a filter bank comprising bandpass filters with bandwidths proportional to frequency. The multiresolution nature of wavelet analysis leads to some limitations. Wavelet transform techniques use a scaling profile such that frequency resolution decreases at high frequencies, while temporal resolution decreases at low frequencies. While this choice of scaling leads to nice mathematical structures and algorithms, there is no physical reason to assume that, it is contrary to natural structure behavior. In addition, the time- and scale-sampling grid should usually be considerably oversampled, in order to get the best performance of WT analysis. This oversampling introduces redundancy in the time-scale representation.

Proposed time-frequency analysis approach

The approach developed in this paper is based on the idea of keeping the valuable features of the above mentioned classical approaches and to minimize the impact of its drawbacks. Several authors consider a promising advancement of Wigner distribution, which allows the suppression of cross-terms and the improvement of resolution. The basic idea is to obtain a signal dependent kernel instead of simple kernel selection without any reference to signal features [8]. The approach featured here is based on a signal dependent transformation [9], which is used instead of windowed exponential functions in the expression (3) for discrete STFT. In the general form the proposed transformation could be expressed as

$$S(\tau, \omega) = \sum_{k:(t_k - \tau) \in T_s} x(t_k) s_k^{(\tau)}(\omega) , \qquad (7)$$

where $\left\{ s_k^{(\tau)}(\omega) \right\}$ is a set of transformation functions for time moment of analysis τ. T_s is assumed as time interval of signal's quasi-stationarity. From (7) it follows that the proposed transformation is applicable to arbitrarily distributed signal samples. The signal dependent transformation functions set is chosen in such a way that the nature of time-frequency representation corresponds to the nature of short time Fourier transform. In this case it is possible to reconstruct the signal from its time-frequency representation by inverse STFT.

The construction of $\left\{ s_k^{(\tau)}(\omega) \right\}$ is based on Minimum Variance (MV) filter idea to minimize variance of the selective filter output [5, 10]. The frequency response of such a filter adapts to the input signal on each frequency of interest. The variance of the output is $\rho = \mathbf{s}^H \mathbf{R} \mathbf{s}$, where $\mathbf{s}$ is vector of filter coefficients and $\mathbf{R}$ is signal autocorrelation matrix. To guarantee that sinusoid with frequency ω_0 passes through the filter designed for this frequency without distortion the following condition have to be considered

$$\mathbf{e}^H(\omega_0)\mathbf{s} = 1 , \qquad (8)$$

where $e_i(\omega_0) = \exp(j\omega_0 t_i)$. The coefficients $\mathbf{s}$ under condition (8) can be calculated as

$$\mathbf{s} = \frac{\mathbf{R}^{-1}\mathbf{e}}{\mathbf{e}^H \mathbf{R}^{-1} \mathbf{e}} . \qquad (9)$$

To obtain the whole time-frequency representation of the signal the calculation of coefficient vector (9) should be performed for each grid point (ω, τ) of time-frequency representation.

Simulation results

The performance of the proposed signal dependent time-frequency transformation has been compared with classical approaches. The 256 uniformly distributed (sampling period $T = 1$) samples of test-signal have been used. The test-signal consists of two components: one is a rising chirp, which rises from middle frequency (normalized frequency - 0.3) to high frequency (normalized frequency - 0.45) and the second is a frequency modulated signal in low frequency region (sin modulation with period $128T$, central frequency 0.125 and modulation range from 0.05 to 0.2 of normalized frequency):

$$\begin{aligned} x_n &= \exp(j2\pi(0.3 + (0.15/256)n)n) + \\ &+ \exp(j2\pi(0.125 + 0.075\sin(2\pi\, n/128))n) . \qquad (10) \end{aligned}$$

The trace of frequencies changes of test-signal along a time axis is shown in Fig.1.

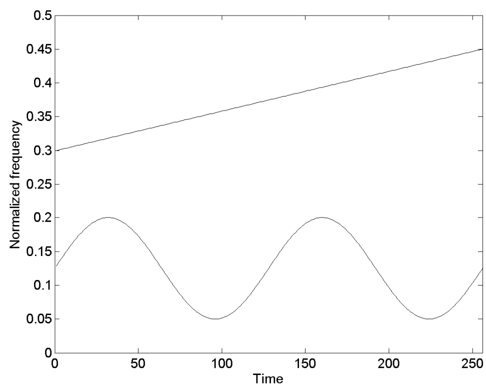

Fig. 1. Frequencies "trace" of test-signal

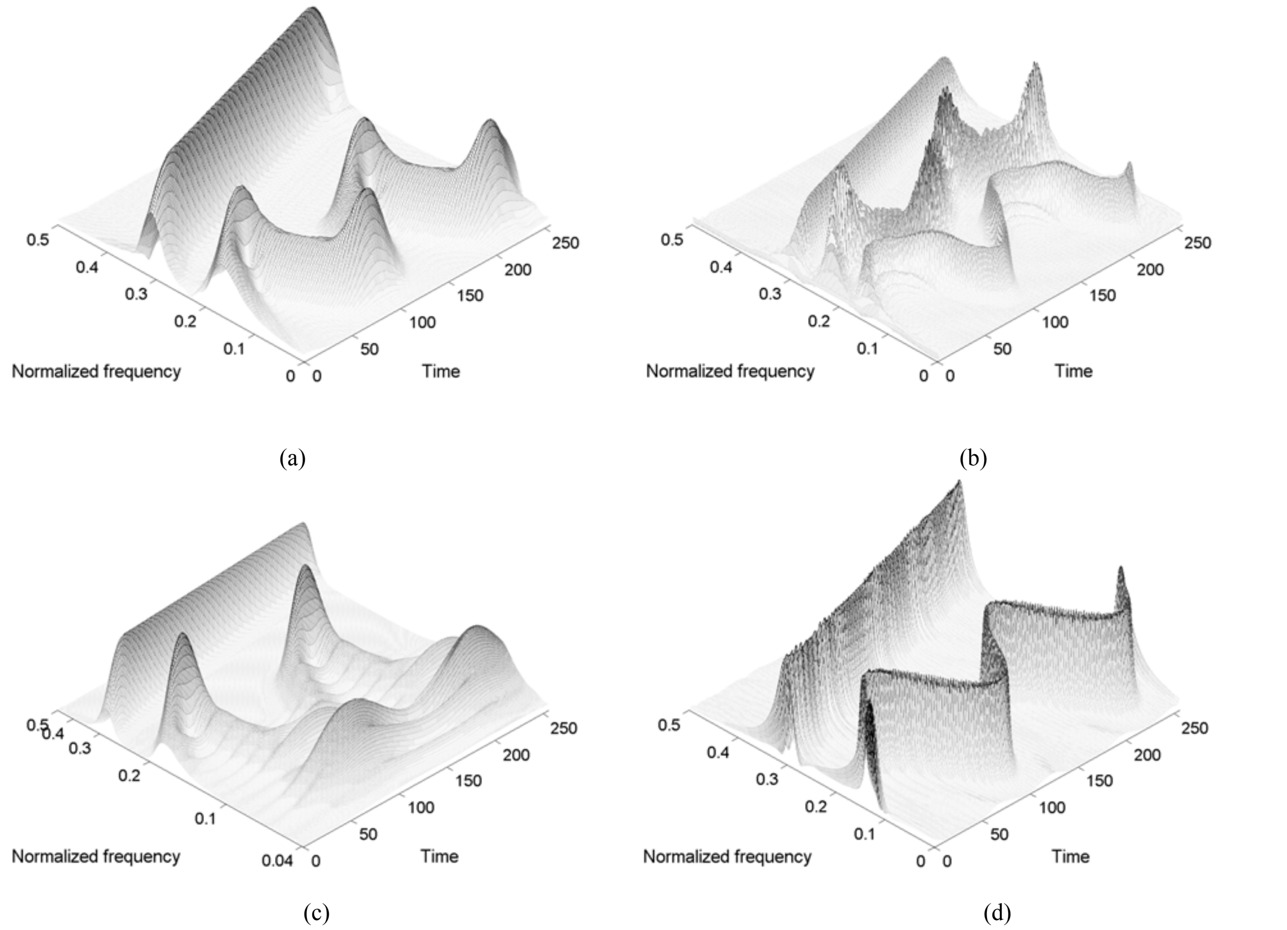

Fig. 2. Time-frequency representations of test signal: (a) – STFT, (b) – Wigner distribution, (c) – Wavelet analysis, (d) – suggested signal dependent transformation

The time-frequency representations obtained by the proposed and three "classical" approaches are demonstrated in Fig.2. For STFT analysis a Hamming window with length of 81 samples is used. STFT (Fig.2a) clearly identifies both subcomponents of test-signal, but with low resolution. The pseudo (frequency smoothing Hamming window is used) Wigner distribution provides good resolution (Fig.2b), but significant cross-term appears in time-frequency representation additionally to the true test-signal components. The Fig.2c illustrates the limited temporal resolution of wavelet transform at low frequencies and limited frequency resolution at high frequencies. The frequency axis in Fig.2c is not linear due to multiresolution nature of WT.

The time-frequency representation obtained by suggested approach is shown in Fig.2d. It demonstrates high temporal and frequency resolution without cross-terms. The used algorithm assumes the a priori knowledge of signal autocorrelation nature. In practice there is a possible case when some information about the signal autocorrelation characteristics is known, and a case when only signal samples are known. In the second case the obtaining of transformation functions can be managed by the iterative update of local autocorrelation matrix values. A procedure similar to the one which is described in literature [10, 11] can be exploited. The time-frequency representation obtained in such an iterative way is shown in Fig.3.

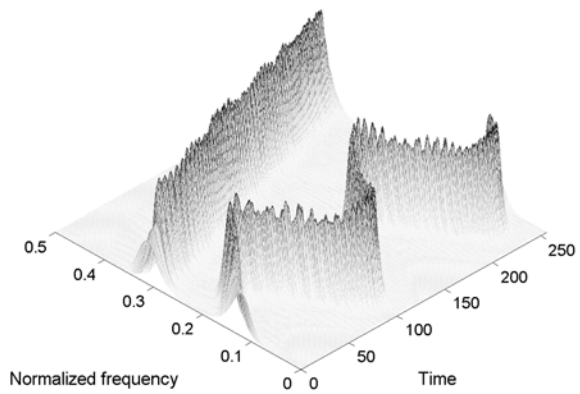

Fig. 3. Time-frequency representation obtained by suggested approach with iterative updating of signal autocorrelation

The beneficial features of the proposed method are kept.

Discussion

The main advantage of the proposed approach is the increased resolution in comparison with STFT. It is achieved by making the transformation kernel dependent on the instantaneous spectral characteristics of the signal. The developed analysis method has no problems with side-loops and cross-terms. Simulation results have shown that the proposed method provides narrow frequency peaks, permitting more precise frequency identification enhancing the ability to determine frequency changes at any time instant. The proposed method preserves the relative amplitudes of multicomponent signals thereby overcoming drawback of autoregressive model based methods [5].

The proposed time-frequency algorithm is developed for application to arbitrarily distributed signal samples. The benefit gained from that feature is the possibility to use sampling point flows with mean rate considerably below Nyquist. The Fig.4 illustrates the time-frequency representation of test-signal in the case when only 64 samples (one fourth of 256 samples used in Fig.2 is left in random way) are used for processing. The analysis is done on the same grid as in Fig.2. The shortage of samples influences the magnitudes of frequency peaks, while the resolution of representation and ability to determine precise frequency tracking remain.

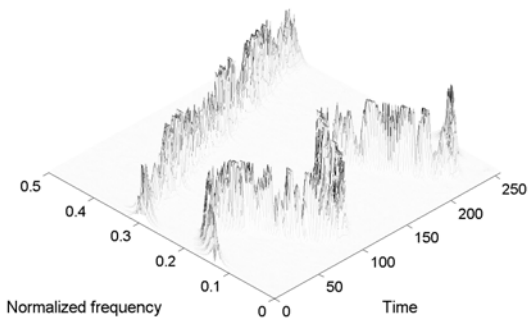

Fig. 4. Time-frequency representation obtained in the case of nonuniform sampling with density ¼ of Nyquist rate

The similar nature of the result of the proposed transformation with the short time Fourier transform provides the simplicity of signal reconstruction – the inverse STFT can be used for that.

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M. Greitāns. Nestacionarių signalų apdorojimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 3(59). – P.42–45.

Signalas yra stacionarus, jeigu jo statistinės charakteristikos nekinta laike. Signalai naudojami praktiniams tikslams dažnai netenkina šio reikalavimo. Trumpalaikė Furje transformacija, laikinis-dažninis pasiskirstymas ir banginė transformacija – tai klasikiniai būdai naudojami nestacionarių signalų analizei. Tačiau jie turi ribotas pritaikymo galimybes, jei signalo diskretizavimo tankis yra mažesnis nei Naikvisto kriterijus. Laikinė-dažninė analizė paprastai tinka tiem siganlams, kurių momentinis juostos plotis yra gerokai mažesnis už analizuojamą juostos plotį. Siūlomas nestacionarių signalų apdorojimo pagerinimo būdas, kuris pagrįstas transformacijos funkcijų adaptacija momentiniam spektrui. Pagrindiniai šio metodo privalumai yra padidėjusi skiriamoji geba ir kintančio dikretizavimo panaudijimas, kai diskretizavimo tankis mažesnis už Naikvisto kriterijų. Il. 4, bibl. 11 (anglų kalba, santraukos lietuvių, anglų ir rusų k.).

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A signal is stationary if its statistical characteristics do not change with time. Signals of practical interest often do not comply with this requirement. Short time Fourier transform, time-frequency distribution and wavelet transform are the classical approaches used to analyze nonstationary signals. However they have limited applicability if the signal sampling density is below Nyquist. Time-frequency analysis typically deals with signals, where the instantaneous bandwidth is considerably narrower than the bandwidth of analysis. The paper proposes an enhancement of non-stationary signal processing, which is based on the adaptation of transformation functions to instantaneous spectrum. The main advantages of the proposed approach are increased resolution, suppressed side-loops and cross-terms and applicability to nonuniform sampling with a sampling density less than the Nyquist rate. Ill. 4, bibl. 11 (in English, summaries in Lithuanian, English, Russian).

М. Грейтанс. Усовершенствованная обработка нестационарных сигналов // Электроника и электротехника. – Каунас: Технология, 2005. – №3(59). – С.42–45.

Для нестационарных сигналов полоса моментного спектра часто значительно уже, чем полоса анализа. Традиционно для анализа таких сигналов используют локальное преобразование Фурье, временное-частотное распределение и Вейвлет преобразование, но их возможности ограничены когда частота дискретизации сигнала меньше критерия Найквиста. Предложен подход улучшения обработки нестационарных сигналов, основанный на адаптации функции трансформации к моментному спектру. Ил. 4, библ. 11 (на английском языке; рефераты на литовском, английском и русском яз.).