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Simply Construction Method for Exponential Source's Scalar Quantizers

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Introduction

Considerable attention has been focused on the design of optimal quantizers for sources encountered in image, speech, and other compression applications. Sources having exponential and Laplacian probability density function are commonly in use [1] and the methods for designing quantizers for these sources are very similar. The problem of determining maximal amplitude of the input signal e.g. problem of determining granular region, which is very important and was considered in [2] and [3], is obviated using the logic presented in this paper. The method that is the most commonly in use for construction of scalar quantizers is Lloyd-Max's method. The problem of finding the sets of optimum parameters in [4] is settled by introducing the Lambert W function and some approximations. Approximation method suggested in this paper is simpler than well known Lloyd-Max's method [5]. Therefore, in this paper the goal is to evaluate the necessary parameters for construction of scalar quantizers for exponential sources on the most easily way. First of all we consider scalar quantization, in which each input random variable is separately mapped to its output approximation. After that we focalize on scalar quantizers for exponential sources. We suggest one very fast and simple approximation method for solving transcendental equations. This method enables obtaining nearly accurate parameter's values which are necessary for designing of scalar quantizers.

Scalar quantization

Consider an input random variable x having a probability density function (pdf) $f(x)$ which is greater than zero over (0, ∞) and zero elsewhere. First, we consider only a one-sided pdf for convenience, without loss of generality to the similar problem posed for other regions of support. Let an n-level quantizer $Q^{(n)}(\cdot)$ be defined in terms of a set of $n-1$ positive step sizes $\{\alpha_i\}_{i=1}^{n-1}$ (defining $\alpha_0 = \infty$) and a set of n nonnegative distances from the representative levels to the nether decision thresholds $\{\delta_i\}_{i=0}^{n-1}$ as shown in Fig. 1. and Fig. 2.

Let $n+1$ decision thresholds of the quantizer $\{t_i^{(n)}\}_{i=0}^{n}$ are given by [4]:

$$t_i^{(n)} = \sum_{j=i}^{n-1} \alpha_j , \quad i = 0,...,n \tag{1}$$

and we define $t_{i=n}^{(n)} = 0$. Let n output values of the quantizer (representative levels) $\{y_i^{(n)}\}_{i=0}^{n-1}$ are given by

$$y_i^{(n)} = t_{i+1}^{(n)} + \delta_i . \tag{2}$$

The n-level scalar quantizer $Q^{(n)}(\cdot)$ is defined as a functional mapping of an input value $x > 0$ onto an output representation $Q^{(n)}(x)$, such that

$$Q^{(n)}(x) = y_i^{(n)}, \text{ for } t_{i+1}^{(n)} < x \le t_i^{(n)} . \tag{3}$$

Note that the i index subscript decreases to the right of the zero-input location, and that the quantizer is defined in terms of its step sizes $\{\alpha_i\}_{i=1}^{n-1}$ and the distances from the representative levels to the nether decision thresholds $\{\delta_i\}_{i=0}^{n-1}$ rather then the most common convention using the threshold $\{t_i^{(n)}\}_{i=0}^{n}$ and output values $\{y_i^{(n)}\}_{i=0}^{n-1}$.

The quantizer distortion is given by

$$D_f^{(n)} = \sum_{i=0}^{n-1} \int_{t_{i+1}^{(n)}}^{t_i^{(n)}} d\left(x - y_i^{(n)}\right) f(x) dx . \tag{4}$$

The most commonly used distortion measure is mean-square error (MSE), denoted here by $d_{mse}(\Delta)$ and given by:

$$d_{mse}(\Delta) = |\Delta|^2 \tag{5}$$

and in that case [4] optimal values of the distances from the representative levels to the nether decision thresholds are given by:

$$\delta_i^* = \frac{\int_0^{\alpha_i} x f\left(x + t_{i+1}^{(n)}\right) dx}{\int_0^{\alpha_i} f\left(x + t_{i+1}^{(n)}\right) dx}. \quad (6)$$

A second necessary condition for optimality [4] can be given by

$$d\left(\alpha_{i+1}^* - \delta_{i+1}\right) - \lambda \log_2\left(p_{i+1}^{(n)}\right) = d\left(-\delta_i\right) - \lambda \log_2\left(p_i^{(n)}\right). \quad (7)$$

In this paper we consider case $\lambda = 0$ thereby (7) becomes:

$$d\left(\alpha_{i+1}^* - \delta_{i+1}\right) = d\left(-\delta_i\right). \quad (8)$$

Exponential source

Let we consider the exponential source with memoryless property. Namely, x is an exponentially distributed random variable and has a value exceeding some fixed nonnegative threshold t. Regarding to the memoryless property of this exponential source the conditional pdf of $x - t$ is the same as the pdf of the original random variable x and can be expressed by

$$f_e(x) = \mu^{-1} e^{-x/\mu}, \quad x > 0, \mu > 0. \quad (9)$$

Without loss of generality we can assume that μ=1 and (9) becomes:

$$f_e(x) = e^{-x}. \quad (10)$$

Using (10) the memoryless property of the exponential pdf allows the substitution:

$$f_e\left(x + t_{i+1}^{(n)}\right) = e^{-t_{i+1}^{(n)}} f_e(x). \quad (11)$$

By substituting (11) in (6), the expression for determining optimal values of the distances from the representative levels to the nether decision thresholds as a function of the step sizes α_i is derived as

$$\delta_i^*(\alpha_i) = 1 - \frac{\alpha_i e^{-\alpha_i}}{1 - e^{-\alpha_i}}. \quad (12)$$

The optimal values of the distances from the representative levels to the nether decision thresholds are obtained for the optimal values of the step sizes α_i^*, thus from (12) we have:

$$\delta\left(\alpha_i^*\right) = 1 - \frac{\alpha_i^* e^{-\alpha_i^*}}{1 - e^{-\alpha_i^*}} \quad (13)$$

and from (8)

$$d\left(\alpha_{i+1}^* - \delta\left(\alpha_{i+1}^*\right)\right) = d\left(-\delta\left(\alpha_i^*\right)\right). \quad (14)$$

If we use mean-square error distortion measure then from (14) and (5) follows

$$\left(\alpha_{i+1}^* - \delta\left(\alpha_{i+1}^*\right)\right)^2 = \left(-\delta\left(\alpha_i^*\right)\right)^2. \quad (15)$$

Substituting (13) in (15) and using a simple set of mathematical operations the transcendental equation is obtained (16):

$$\alpha_{i+1}^* + e^{-\alpha_{i+1}^*}\left(1 + \sqrt{\left(\frac{\alpha_i^* e^{-\alpha_i^*}}{1 - e^{-\alpha_i^*}} - 1\right)^2}\right) - \left(1 + \sqrt{\left(\frac{\alpha_i^* e^{-\alpha_i^*}}{1 - e^{-\alpha_i^*}} - 1\right)^2}\right) = 0. \quad (16)$$

By solving equation (16) it is possible to determine optimal values of the step sizes α_{i+1}^* as a function of optimal values of the previous step sizes α_i^*. Thus, knowing the α_i^* may be sufficient to determine α_{i+1}^*.

Numerical results

As in [4], in this paper is also considered transcendental equations solving problem for $\lambda = 0$ and is settled by introducing the Lambert W function and some approximations.

We suggested more efficiently solution of this problem. The solutions of transcendental equations which are iteratively found are the exact solutions. In the way to easily find solutions which are nearly to the exact solutions than it was done in [4], we suggest approximation method for solving transcendental equations and therefore we introduce the following approximation:

$$e^{-x} \approx e^{-x_0} - (x - x_0) e^{-x_0}. \quad (17)$$

By introducing this approximation the solving of transcendental equations are replaced with solving of linear equations.

Analyzing an Eq. (13) we can make two predictions $\delta\left(\alpha_1^*\right) < 1$ and $\alpha_1^* - \delta\left(\alpha_1^*\right)$ which is a little bit greater than $\delta\left(\alpha_1^*\right)$. Therefore, if initial value is $\alpha_1^* \approx 2$ the mistake won't be large.

Choosing the better initial value causes faster and more accurate getting of the final solutions. It is very important to notice that the representative levels $y_i^{(n)}$ are not obtained as an arithmetical mean of the decision thresholds which determine the quantization levels.

Assuming the $\alpha_0^* = \infty$, $\delta\left(\alpha_0^*\right) = 1$, $\alpha_{1,0}^* \approx 2$ and using

$$e^{-\alpha_1^*} \approx e^{-\alpha_{1,0}^*} - (\alpha_1^* - \alpha_{1,0}^*) e^{-\alpha_{1,0}^*} \quad (18)$$

from an Eq.(16) it is easy to calculate α_1^* and that value we mark as $\alpha_1^{*(1)}$. After the substitution $\alpha_{1,0}^*$ with $\alpha_1^{*(1)}$ in (18) and also in (16) the new value for α_1^* is obtained and that value is a final value. Procedure of determining subsequent values of the step sizes α_i^* $i = 1,..., n$ identically repeats as it is shown for α_1^*.

Table 1. Parallel comparison of the accurate optimal values of the step sizes, for scalar quantizers with n=4 levels, with appropriate optimal values of the step sizes obtained by using the approximation method

Accurate optimal values of the step sizes	**Optimal values of the step sizes obtained by using the approximation method**
$\alpha_0^*=\infty$	$\alpha_0^*=\infty$
$\alpha_1^*=1.5936$	$\alpha_1^*=1.5940$
$\alpha_2^*=1.0175$	$\alpha_2^*=1.0230$
$\alpha_3^*=0.7539$	$\alpha_3^*=0.7583$

Behaviors of the four levels MSE-optimal scalar quantizer for exponential source are shown in Fig.1. and Fig.2. The values of the step sizes α_i^* $i = 1,..., n$ on Fig.1. are obtained iteratively, by solving the transcedental equations, while the values of the step sizes α_i^* $i = 1,..., n$ on Fig.2. are obtained using suggested approximation method. From Fig.1., Fig.2. and Table.1. and Table.2. it is possible to notice that the appropriate values of α_i^* $i = 1,..., n$, and also the appropriate values of δ_i^* $i = 1,..., n$ are almost identical.

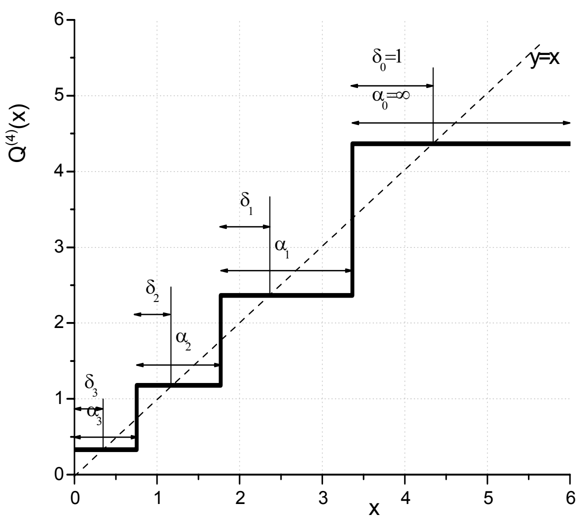

Fig.1. Behavior of the four levels MSE-optimal scalar quantizer for exponential source

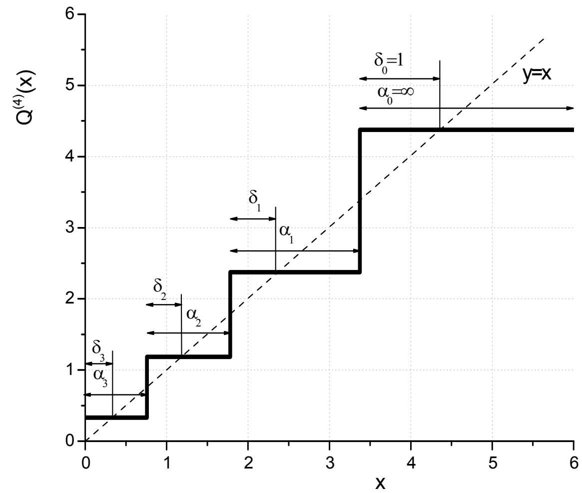

Fig.2. Behavior of the four levels MSE-optimal scalar quantizer for exponential source and using the approximation method

Table 2. Parallel comparison of the accurate optimal values of the distances from the representative levels to the nether decision thresholds, for scalar quantizers with n=4 levels, with appropriate optimal values of the distances from the representative levels to the nether decision thresholds obtained by using the approximation method

Accurate optimal values of distances from the representative levels to the nether decision thresholds	**Optimal values of the distances from the representative levels to the nether decision thresholds obtained by using the approximation method**
$\delta_0^*=1$	$\delta_0^*=1$
$\delta_1^*=0.5936$	$\delta_1^*=0.5937$
$\delta_2^*=0.4239$	$\delta_2^*=0.4257$
$\delta_3^*=0.33$	$\delta_3^*=0.3316$

Conclusion

Logic, presented in this paper enables obviating the problem of determining maximal amplitude of the input signal e.g. problem of determining granular region, which is very important and was considered in [3] and [4]. The exact solution of the transcendental equations are obtained using the iteratively method but using the approximation method for finding for solutions we could, on very fast and simply way, get the solutions very close to the exact solutions. In this paper is also shown equation for finding distances from the representative levels to the nether decision thresholds as a function of the step sizes.

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J. Nikolic, D. Antic, Z. Peric. Skaliarinio kvantizatoriaus eksponentinių šaltinių paprastasis konstrukcinis metodas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 2(58). – P. 18–21.

Pateikiami metodai, padedantys nustatyti reikalingus skaliarinių kvantizatorių konstrukcinius parametrus. Siūlomi du optimalių parametrų parinkimo metodai. Vienas paremtas iteracine procedūra, o kitas, mūsų siūlomas, yra aproksimacijos metodas, skirtas transcendentinėms lygtims spręsti. Šis metodas gali būti taikomas, kai reikalingas greitas ir paprastas sprendimas, artimas tiksliam sprendimui. Il. 2, bibl. 5 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

J. Nikolic, D. Antic, Z. Peric. Simply Construction Method for Exponential Source's Scalar Quantizers // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 2(58). – P. 18–21.

In this paper we consider methods for determining necessary parameters for constructions of the scalar quantizers for exponential source. There are two approaches to the problem of finding the sets of optimum parameters. One is an exact solution based on an iterative procedure, and another, that we suggested, is an approximation method for solving transcendental equations. This method can be used for simply and fast finding for solutions which are very close to the exact solutions. Ill. 2, bibl. 5 (in English; summaries in Lithuanian, English and Russian).

Е. Николич, Д. Антич, З. Перич. Простой конструктивный метод для экспонентных источников скалярного квантизатора // Электроника и электротехника. – Каунас: Технология, 2005. – № 2(58). – С. 18–21.

Представлены методы, помогающие определить нужные конструктивные параметры для скалярных квантизаторов. Предлагаются два метода для выбора оптимальных параметров. Один основан на итеративной процедуре, другой, предлагаемый нами, является методом аппроксимации и используется для решения трансцедентальных уравнений. Этот метод может быть использован для быстрого и простого решения, близкого к точному решению. Ил. 2, библ. 5 (на английском языке; рефераты на литовском, английском и русском яз.).

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