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The Influence of Cornea to the Measurement of Three – Dimensional Rotations

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Introduction

Of late years more and more often there are used videooculographical systems to register people three – dimensional eye rotations [1,2,4,5], because they are out of contact. During the research the patient does not feel high discomfort, so the research can be continued. This system allows to do this experiment not only at labs but in a natural environment.

Using that videooculographical method, the cameras get the eye records into the computer hard disc. They are analysed using special programs which establish the pupil centre co-ordinates and by them there are calculated horizontal and vertical eye rotations. The calculation of torsion is insufficient using only pupil centre co-ordinates, for that there is used iris signature [4,5].

The research during the last decade presented the importance of registration of accurate three – dimensional components [7]. Analysing the eye rotations obtained from the videooculographical systems there are observed some inaccuracies related to qualities of using methods which have not been estimated yet.

The purpose of research

Analysing the images of videooculographical method there was observed that when the eye rotates the pupil and iris form changes. The cornea and anterior chamber can give influence forming the image of iris and pupil. These mentioned eye elements form an optical system which can distort the image of iris and pupil (Fig. 1).

The purpose of this research is to estimate the influence of cornea to the formation of the image of iris and measurement of three – dimensional rotations.

Method

The light rays reach the cornea through air and break in it. A part of light comes into the lens through the pupil and the other part is reflected of iris and comes back. The corneal refractive power reaches 43dpt. The refractive power of anterior chamber is 3,5dpt.

The influence of corneal optics to the iris and pupil picture we will reach using mathematical model of corneal optics. The mathematical model of corneal contour we will describe using the equations:

$$\begin{cases} x^2 + y^2 = R_{out}^2, \\ (x - x_0)^2 + (y - y_0)^2 = R_{in}^2; \end{cases} \quad (1)$$

where x_0, y_0, – the co-ordinates of circle centre; R_{out} – external corneal radius, R_{out} – internal corneal radius.

The mathematical model of iris contour we will describe using the equations:

$$\begin{cases} \mathrm{x} = const, \\ y = 0 \ldots R_{iris}; \end{cases} \quad (2)$$

where R_{iris} – iris radius.

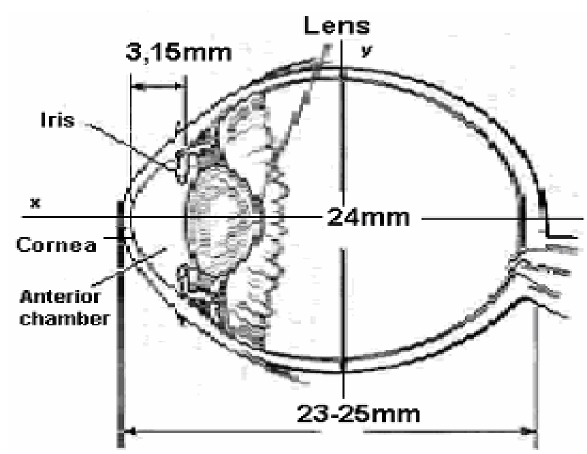

Fig. 1. Cut of eyeball

To make the model there are necessary parameters chosen from the reference books. The external curvature radius of the cornea is 7,8mm, and the internal is – 6,5mm. The standard iris diameter is 11mm [2,3]. The iris is flat, disc shaped [3,4,5]. Its base is consisted of muscle, which can contract or stretch. In that way there is obtained a pupil of less or bigger diameter. The model is accomplished using MATLAB™ program and presented in Fig. 2.

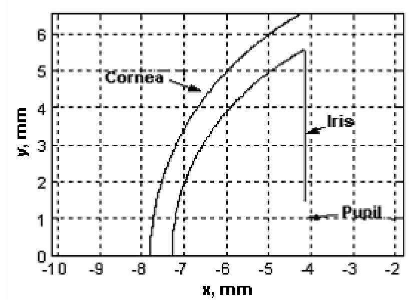

Fig.2. The simulated contours of cornea and iris

In the monotonous environment, the light spreads directly. For that reasons modelling of the rays there are used lines, going in parallel fibre, in equal distance (step 0,1mm) get into the external corneal surface. In the simulation were used 59 rays.

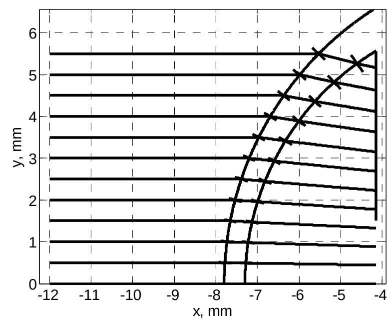

Fig. 3. The propagation of light rays

The eye is made of mediums having different refraction indices. The rays refract getting over the boundary of mediums with different refraction indices. The angle between the perpendicular to the boundary of two environments and the light ray is marked *α* and called as an incidence angle. The angle between the perpendicular and the broken light ray is marked *β* and called as a refraction angle. The auxiliary angles *τ* and *γ* we will use to calculate the angle α. The Refraction angle *β* is calculated from this equation:

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}. \qquad (2)$$

Here n_1 and n_2 – the indices of medium refraction; *α* – ray incidence angle; *β* – ray refraction angle

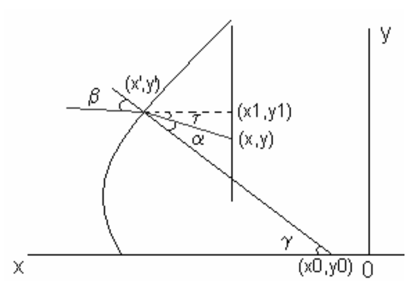

Fig. 4. The angles of light ray incidence α and refraction *β*

The incidence angle α is calculated expressing these equations:

$$\begin{cases} \gamma = arctg\left(\dfrac{y' - y_0}{x' - x_0}\right), \\ \tau = arctg\left(\dfrac{y' - y_1}{x' - x_1}\right), \\ \alpha = \gamma - \tau; \end{cases} \qquad (3)$$

here *τ* and *γ* – auxiliary angles (Fig. 4.); *α* – the ray incidence angle; x_0 and y_0 – the junction of corneal perpendicular and axis of *x* co-ordinates; *x*‘ and *y*‘ – the junction of the ray and corneal perpendicular; x_1 and y_1 – the junction of iris and it's perpendicular.

In the diagrams there are presented changes of angles *α* and *β*, getting over the environment of different indices of refraction. *β*.

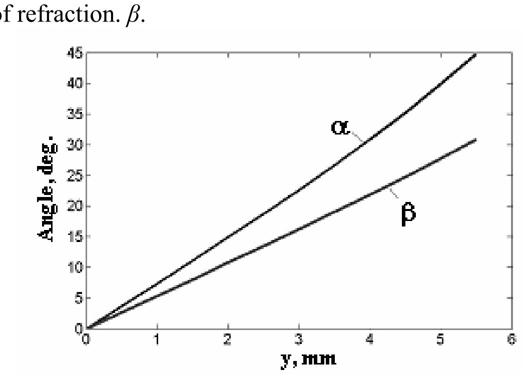

Fig. 5. α and β angle changes in boundary air – cornea

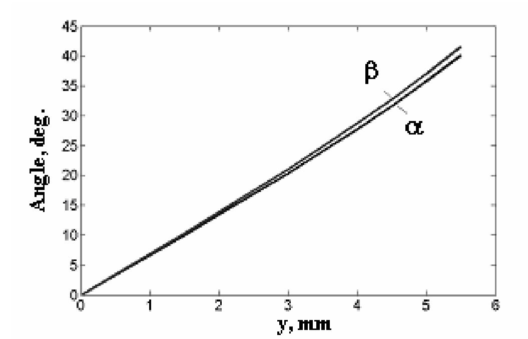

Fig. 6. *α* and *β* angle changes in boundary cornea–anterior chamber

The way of light ray propagation inside anterior chamber is described by coordinates x_2 and y_2, which can be found estimating the equation system of the line and circle arc:

$$\begin{cases} y_2 + y' = tg(\beta)\cdot(\sqrt{R^2 - y_2^2} + x'), \\ x_2^2 + (tg(\beta)\cdot(x_2 + x') - y')^2 = R^2. \end{cases} \qquad (4)$$

The obtained light ray propagation path is shown Fig.3.

After the ray distribution analysis, we obtain that the boundary between the cornea and anterior chamber has little influence to ray propagation direction. Bigger changes are when the rays go from the air into the cornea.

We can conclude the purpose of cornea is to turn the light rays from air to the iris plane and to the pupil. The light ray stream passing the cornea and the anterior chamber lights up the iris plane. The iris image will be seen when the reflective rays of the iris will come back through the optical environments. The light images situated on the iris in not of equal distance (though the light was turned in parallels each 0,1 mm) that proves the iris image has nonlinear transformation (Fig. 7).

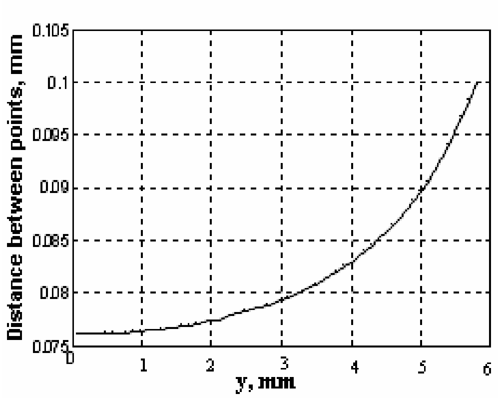

Fig. 7. The distance change among the light images formed in the iris versus vertical coordinate

We try to estimate this iris transformation. The distance among the light images on the iris plane changes from A_1 =0,0761mm to A_{58} = 0,1mm (Fig. 8), while incident parallel light rays has equal distance. So, we try to estimate iris spherical surface, where distance between the light images is equal to 0,1mm (B_i=0,1mm, here i=1…58).

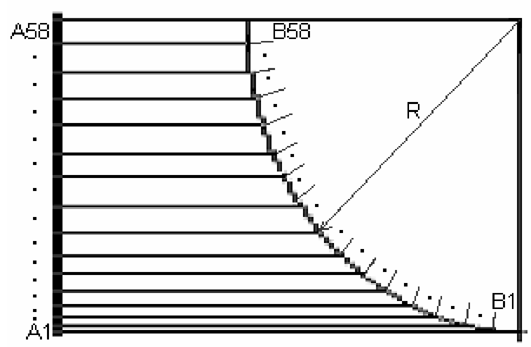

Fig. 8. Corneal image points establishment

The corneal image points we will establish using the equation system:

$$\begin{cases} a^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2, \\ y_2 - y_1 = (x_2 - x_1) \cdot tg\lambda. \end{cases} \qquad (5)$$

Here a – constant, equal to 0,1mm; x_1 and y_1 – the primary co-ordinate; x_2 and y_2 – co-ordinate of the iris image; λ – the propagation angle of the light ray going to the iris plane. To calculate the iris image co-ordinates we used MATLAB™ program.

Results

After the calculation, there are got the iris image points. They are presented in the Fig. 9. It is tested and there is established that the distance between the points is equal 0,1mm. So, the iris image points are calculated well.

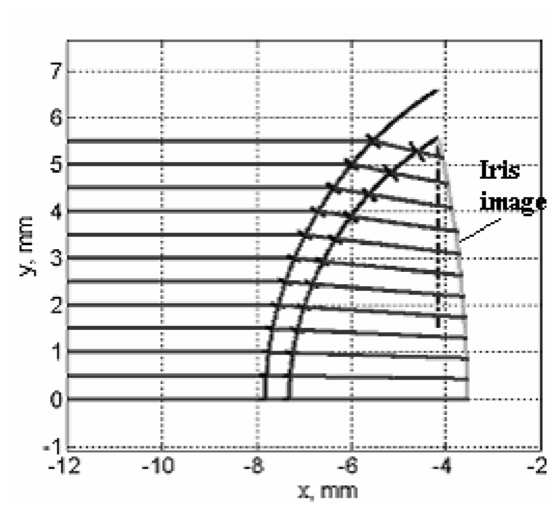

Fig. 9. Spherical iris image

As we see at the diagram, the iris image is distorted. The iris image points were approximated in the circle arc (Fig. 9) and its curvature radius is 26,2mm. So, if we want to estimate corneal optics influence to form the iris and pupil image on the plane, the image projection of iris and pupil on the plane we must get from spherical iris (not plane) [6], which radius is 26,2mm.

Now, we shall establish what influence has cornea to the eye three – dimensional measurements. Doing that, we will use mathematical model of eye iris image at the plane [6]. In that model there were not estimated corneal optics influence, so we will correct the model using the results of this research. We will repeat simulations done in past work [6], using estimated influence of corneal optics forming the iris and pupil image. First of all there was investigated the changing form of pupil and iris. The change tendencies of the iris and pupil are similar being on model of iris [6] without estimated corneal optics influence.

Further we will examine, how do the geometrical parameters of iris beams change during eye rotations [6]. Doing horizontal, vertical, oblique and 3D rotations with the obtained model there is noticed that change not only iris beams length and declination angle but also the iris beams bend. During the experiment the diapason of vertical and horizontal rotations was ±30 degrees, and the torsion – ±4 degree. When the head is not moving the torsion does not exceed 4 degrees. The parameters of iris beams carrying out 3D rotations are presented on Fig. 10, Fig. 11, Fig. 12.

Estimating the results we can affirm, that there is the corneal influence to the iris and pupil image. The length changes of the beam chord of the iris and declination angles is asymmetric.

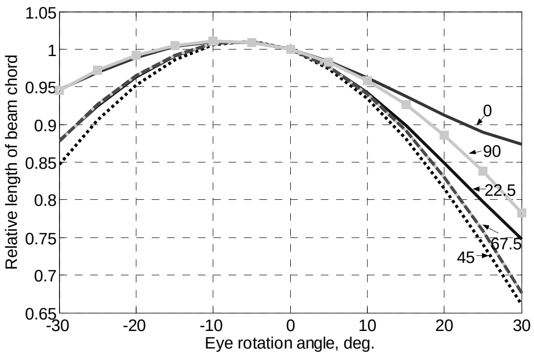

Fig. 10. The length changes of the iris beams chords

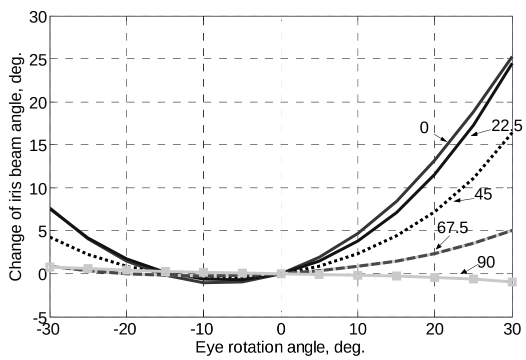

Fig. 11. The angle changes of the iris beams chords

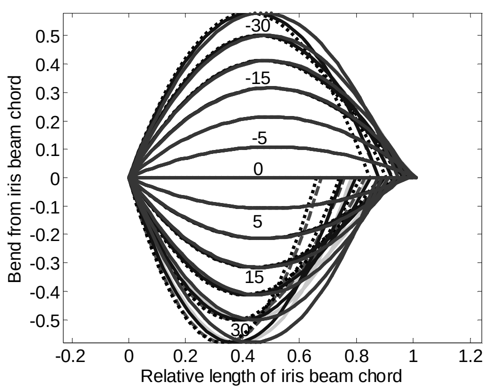

Fig. 12. Bends of the iris beams

The changes depend on the beam position in the iris. If the beam is in the right side of the iris and the eye is rotated to right, thus the beam will not change its length for a while and it has lower variation. If the eye is turned to the left, the beam has higher variation. Doing the 3D rotations maximum changes of the length at 45 degree, at least – 0 degrees. The biggest angle alteration is 0 degrees. The bends of all the angle beams are almost equal and at the biggest angles are 0,6 degree.

Conclusions

The mathematical model for simulation of propagation of the ligth ray fiber through the eye cornea was developed. The model was implemented by programming in MATLAB. This model is used for analysis of the corneal influence to form the iris and pupil image.

In the work it was established that because of corneal influence the iris image has nonlinear transformation and it's distorted. If we want to estimate corneal optics influence to formation of the iris and pupil image on the plane, we must get from spherical iris (not plane) [6], which radius is 26,2mm.

There are bends in iris beams, when corrected iris model on the plane [6] is used to simulate rotations in ±30 degrees diapason. In the rotation diapason they have maximal value of 0,6 degree. The change tendencies of the iris and pupil are similar being on iris model without corneal influence. [6]. The diagrams of horizontal, vertical and oblique rotations are similar by their form and the change regularity to the diagrams of model without corneal influence [6]. The length and the angle changes by the parabola law. But exact examination revealed differences in dependencies courses, because changes is asymmetric, when rotations are in the same angles, but in the opposite directions.

When eye is making combined 3D rotation the form of pupil and iris image become ellipse. We must take into account this, when we calculate the centre co-ordinates of the pupil. If we don't estimate bends, length and angle changes of the iris beams during 3D rotations, the big errors during estimation of torsonal eye co-ordinates are possible, when we use iris signature correlation method.

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E. Paliulis. Akies ragenos įtaka erdvinių judesių matavimui // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 2(58). – P. 78 – 82.

Sukurtas ragenos optikos matematinis modelis, kuriuo galima modeliuoti šviesos spindulių srauto perėjimą per rageną. Matematinis modelis įgyvendintas MATLAB programų paketu panaudotas ragenos įtakai rainelės ir vyzdžio atvaizdui tirti. Dėl ragenos įtakos rainelės atvaizdas netiesiškai transformuojasi ir yra iškraipomas. Norėdami įvertinti ragenos optikos įtaką rainelės atvaizdui plokštumoje, rainelės atvaizdo projekcijos į plokštumą skaičiavimams turime naudoti sferinę rainelę (ne plokščią) [6], kurios kreivumo spindulys 26,2mm. Modeliuojant patikslintu modeliu akies posūkius ±30 laipsnių diapazone, rainelėje pasireiškia spindulių išlinkimai. Sukimo diapazonų ribose jie siekia 0,6 laipsnio. Rainelės ir vyzdžio kitimo tendencijos panašios kaip ir modelyje, kuriame nebuvo įvertinta ragenos įtaka [6]. Horizontaliųjų, vertikaliųjų bei įstrižųjų posūkių grafikai panašūs savo forma ir kitimo dėsningumais į grafikus, gautus neįvertinus ragenos įtakos [6] . Ilgis ir kampas kinta pagal parabolės dėsnį. Skirtumas tas, kad nebėra simetrijos, sukant tais pačiais kampais į priešingas puses. Akiai atliekant erdvinius judesius vyzdžio ir rainelės atvaizdo forma virsta elipse. Skaičiuodami vyzdžio centro koordinates, turime tai įvertinti. Jeigu neįvertinsime rainelės spindulių išlinkimų, ilgio ir kampo kitimų, atliekant erdvinius judesius, galima gauti dideles sukties judesio paklaidas, kada taikomas rainelės parašo koreliacijos metodas. Il. 12, bibl. 7 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

E. Paliulis. The Influence of Cornea to the Measurement of Three – Dimensional Rotations // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 2(58). – P. 78 – 82.

The mathematical model of corneal optics was developed, which is used for simulation of propagation of the light rays fiber through the cornea. The model was implemented by programming in MATLAB. This model is used for analysis of the corneal influence to form the iris and pupil image. Because of the corneal influence the iris image has nonlinear transformation and it’s distorted. If we want to estimate corneal optics influence to form the iris and pupil image on the plane, the image projection of iris and pupil on the plane we must get from spherical iris (not plane) [6], which radius is 26,2mm. There are bends in iris beams, when corrected iris model is used for simulation of rotations in ±30 degrees diapason. In the limits of the rotation diapason they are 0,6 degree. The change tendencies of the iris and pupil are similar being on iris model without corneal influence [6]. The diagrams of horizontal, vertical and oblique rotations are similar by their form and the change regularity to the diagrams of model without corneal influence [6]. The length and the angle changes by the parabola law. But it makes difference, because changes is asymmetric, when rotations are in the same angles, but in the opposite directions. When eye is making 3D rotation, the form of pupil and iris image become ellipse. We must take into account it, when we calculate the centre co-ordinates of the pupil. If we don’t estimate bends, length and angle changes of the iris beams during 3D rotations, the big errors are possible for estimatation of torsional eye rotations, when we use iris signature correlation method. Ill. 12, bibl. 7 (in English; summaries in Lithuanian, English, Russian).

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Создана математическая модель оптики роговицы, при помощи которой возможно моделирование прохода потока световых лучей через роговицу. Математическая модель реализована в пакете программ MATLAB. Упомянутая модель применяется для исследования воздействия роговицы на изображение радужной оболочки и зрачка. Вследствие воздействия роговицы изображение радужной оболочки имеет нелинейное преобразование и является искривленной. Чтобы оценить воздействие роговицы на изображение радужной оболочки и зрачка на плоскоть, проекция изображения радужной оболочки и зрачка на плоскоть должны быть получены от сферической роговицы (не от плоской), кривизна которой составляет 26,2 мм. При моделировании поворот с уточнённой моделью глаза [6] в диапазоне ±30 градусов на радужной оболочке наблюдаются изгибы лучей. В границах диапазонов поворотов они достигают 0,6 градуса. Тенденции изменения зрачка и радужной оболочки схожи, как и при использовании модели без оценки воздействия роговицы [6]. Графики горизонтальных, вертикальных и диагональных поворотов похожи по своей форме и закономерностям изменения на графики, полученные без оценки воздействия роговицы [6] . Длина и угол изменяются по закону параболы. Разница заключается в том, что исчезает симметрия при поворотах под теми самыми углами в противоположные стороны. Когда глаз делает пространственные вращения, форма зрачка и радужной оболочки становится овалом. Это нужно оценить при вычислении координат зрачкового центра. Если не оценить измены поворотов, длины и углов в лучах радужной оболочки, когда глаз делает пространственное вращение, можно получить большие ошибки при вычислении скручивых вращений глаза, когда пользуемся методом соотношения подписей радужной оболочки. Ил. 12, библ. 7 (на английском языке; резюме на литовском, английском и русском языках).