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Electrical Field and Current Distribution in Semiconductor Plasma in the Strong Magnetic Field

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Introduction

The current interest in solid-state plasmas may be explained by the wide perspectives for the gaseous plasmas applications. An understanding of plasma behavior is crucial to the controlled release of thermonuclear energy, and to design of magneto hydrodynamic generators in which electric power is generated by jets of gas plasma traversing magnetic fields. Gaseous plasmas pervade the Universe, but it is not easy to control their properties in the laboratory. The plasmas that exist in solids are friendlier and offer an unique opportunity to observe plasma behavior under well-defined and accurately known conditions. In a solid, one can usually determine quite precisely the number of charge carriers, their masses, their random heat energy and the boundaries of the plasma. Such a degree of knowledge and control is rarely attainable in gas-plasma experiments, which often take place in transient discharges where conditions are subject to rapid change. Consequently certain aspects of plasma theory can be tested better in a solid than in a gas.

Statement of the problem

The present article is aimed at elaborating the methods of calculation of the potential and current distribution in the solid-state plasma, placed in the strong magnetic field with due regard for the effect of metal contacts. Problems of such type occur at designing and manufacturing of Hall transducers and converters. The methods proposed allow also defining the change in the resistance of the sample.

In a general case the steady magnetic field distorts the configuration of the current and potential lines in the solid-state plasma. Only for long samples at a sufficient distance from metal contacts the Hall electric field is homogeneous and fully compensates the effect of the magnetic field on the moving charge carriers, and the picture of the current lines is the same as in the absence of the magnetic field. Near the metal contacts the short-circuiting of Hall e.m.f. occurs and the configuration of the current lines becomes more complicated. Even for the areas of simple geometrical form (e.g., rectangular) the general methods of calculation of the potential distribution at present are absent [1].

Calculation of electrical field

Below the solution of the given task is provided for a rectangular sample of the finite length with two metallized ends (see Fig.1, 2), located in the strong magnetic field, when $pB >> 1$, where p – the mobility of current carriers; B, magnetic field induction. To the metallized ends the difference of potentials is applied. Since we study the rectangular sample, and the steady field B is directed along the z-axis [2], the configuration of lines of force does not depend on z (two-dimensional approximation).

For simplicity we shall limit ourselves to the case of a semiconductor sample, the electric conductivity of which is caused by electrons with isotropic effective mass (indium antimonide). Then the components of current j_x, j_y and the electric field E_x, E_y will be related by the equations of motion

$$\begin{cases} j_x - \beta j_y = \sigma E_x, \\ j_y + \beta j_x = \sigma E_y, \quad \beta = pB, \end{cases} \tag{1}$$

where σ is a specific electrical conductivity of the material at $B = 0$.

Expressing j_x, j_y through E_x, E_y we can rewrite the system of equations (1) in the form [3]

$$\begin{cases} \dfrac{\sigma}{1+\beta^2} \cdot \left(E_x + \beta E_y\right) = j_x, \\ \dfrac{\sigma}{1+\beta^2} \cdot \left(E_y - \beta E_x\right) = j_y, \end{cases} \tag{2}$$

where

$$E_x = -\frac{\partial U}{\partial x}; \quad E_y = -\frac{\partial U}{\partial y} \tag{3}$$

and function U satisfies the Laplace equation [4]

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0. \tag{4}$$

The boundary conditions have the form

$$E_x = 0, \quad y = 0 \quad \text{or} \quad y = b, \tag{5}$$

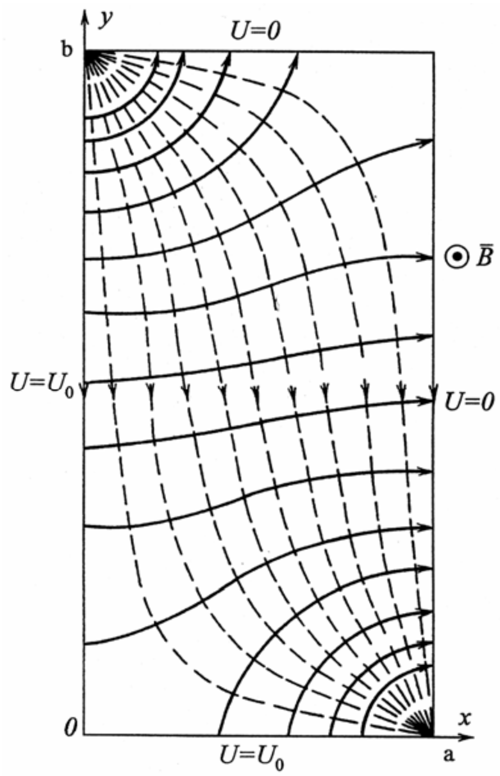

Fig. 1. The picture of equipotential (——) and electric field (– – –) lines calculated from the formula (13) for the case b=2a. Equipotentials at the same time are the current lines. The directions of the current and the field at every point are interpendicular and denoted by arrows

$$j_x = 0, \quad x = 0 \text{ or } x = a\,. \tag{6}$$

Taking into account (2) we have instead of (6)

$$E_x + \beta E_y = 0, \quad x = 0 \quad \text{or } x = a. \tag{7}$$

In the case of the strong magnetic field $\beta >> 1$ (for example, for indium antimonide in the liquid nitrogen $\beta >> 1$ if $B > 1T$) it follows that at the boundaries $x = 0$ and $x = a$ the ratio

$$\left|\frac{E_x}{E_y}\right| = \beta >> 1 \tag{8}$$

and the normal component of the electrical field is considerably greater than tangential one, i.e. boundaries $x = 0$ and $x = a$ are practically equipotential. Hence, all four boundaries of the rectangle $y = b$, $y = 0$, $x = 0$, $x = a$ are equipotential lines.

It follows from (6) directly that boundaries $x = 0$ and $x = a$ are the current lines. On the basis of (1) instead (5) we shall have

$$j_x - \beta j_y = 0, \quad y = 0 \quad or \quad y = b. \tag{9}$$

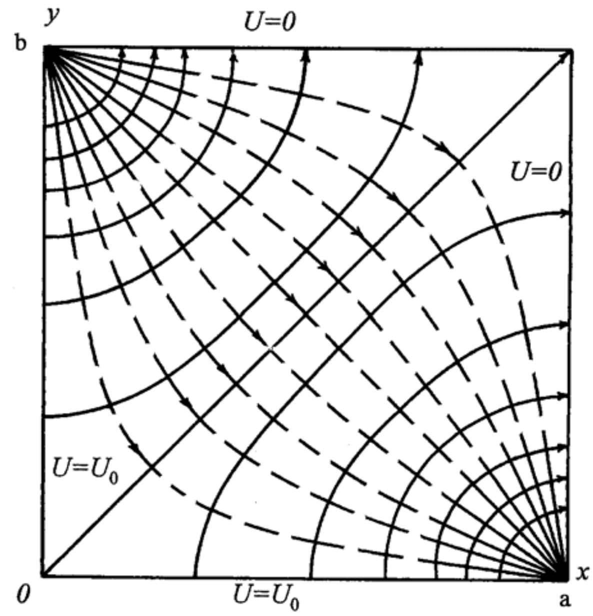

Fig. 2. The picture of equipotential (——) and electric field (– – –) lines calculated from the formula (13) for the case b=a. Equipotentials at the same time are the current lines. The directions of the current and the field at every point are interperpendicular and denoted by arrows *(b* = 2*a)*

From (9) it follows that at the boundaries $y = 0$ and $y = b$ the ratio

$$\left|\frac{j_x}{j_y}\right| = \beta >> 1 \tag{10}$$

and the tangential component of the current exceeds considerably the normal one, i.e. the boundaries $y = \mathbf{0}$ and $y = b$ coincide practically with the current lines. Hence, all four boundaries $y = b$, $y = 0$, $x = 0$, $x = a$ of the rectangle coincide with the current lines.

The analytical expression for the function U may be represented by the infinite series of Eigen functions [5]:

$$U = \sum_{m=1}^{\infty} (c_m \sin mx + d_m \cos mx)(g_m shmy + h_m chmy), \tag{11}$$

where c_m, d_m, g_m, h_m are coefficients, defined by boundary conditions

$$U = U_0, \quad y = 0, \tag{11a}$$

$$U = U_0, \quad x = 0, \tag{11b}$$

$$U = 0, \quad y = b, \tag{12a}$$

$$U = 0, \quad y = 0. \tag{12b}$$

By the help of the standard procedure we shall get

$$U = \frac{4U_0}{\pi}\sum_{k=1}^{\infty} \frac{1}{2k-1} sin\left[\frac{(2k-1)\pi x}{a}\right]\times \times\frac{sh[(2k-1)\pi(b-y)/a]}{sh[(2k-1)\pi b/a]} + \frac{4U_0}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\times \times sin\left[\frac{(2k-1)\pi y}{b}\right]\times\frac{sh[(2k-1)\pi(a-x)b]}{sh[(2k-1)\pi a/b]}, \tag{13}$$

$k = 1, 2, 3,...$

The pictures of the equipotential lines (they also are the current lines) and electrical field lines, obtained on the basis (13) for the cases $b = 2a$ and $b = a$ are shown in the Figs. 1, 2.

Near the high field corner the carrier velocity is nearly radial. In particular, near the contact the velocity is in a direction nearly parallel to the contact rather than perpendicular to t as it is in zero magnetic field.

In the electrodynamics of elementary charges (electrons and holes), the finite velocity of propagation of electromagnetic disturbances is of fundamental significance. If the action of a field affects the energy or momentum of a charged particle, the change can be directly transmitted only to the surrounding electromagnetic field, because, for the energy and momentum of other particles to change, a finite time interval is required before the electromagnetic disturbance excited by the charge reaches them. This means that the electromagnetic field itself possesses energy and momentum; otherwise there two important mechanical quantities would not always be conserved, vanishing at the intent when it is received. Since an electromagnetic field possesses momentum and energy, it can be treated as an independent physical entity in exactly the same way as charged particles. The equations of motion must directly describe the propagation of electromagnetic disturbances in space and the interactions of the charges with the field.

It is important also to note that the magnitude of magnetic flux does not depend on the specific from of the surface stretched on the circuit. This can be visually explained by the fact that the magnetic field lines cannot originate or terminate in an empty space devoid of magnets.

It should be remembered, however, that this treatment of the carrier velocity is valid only over distances large compared to a mean – free path and hence only describes carrier motion over distances of many microns.

In publication [6] the solution of the similar problem for the semi-infinite slab was obtained by the help of Schwartz–Christoffel conformed transformation. This approximation is quite good for all cases where the length b is greater than twice of the width a. Our solution is valid for all values a and b.

Conclusions

Let us define the order of error, introduced into calculations due to the finiteness off the parameter β. Let us go back to the boundary conditions (11–15) for the potential functions *U*. If conditions (12) and (14) are precise, the conditions (13) and (15) follows from (8) only approximately, i.e., by matching the boundaries $x = 0$ and $x = 0$ and $x = a$ with equipotentials, in accordance with (8) we neglect on them the tangential component of the field E_y which is small in comparison with the normal component E_x. Actual equipotential may constitute with the calculated according to (16) the angle approximately equal to 1/β .

The analogous situation also exists at the determining the configuration of the current lines. Approximation is related with the matching of the boundaries y = 0 and y= b with the current lines and the neglecting on them according to (10) the normal component j_y, which is small as compared to the tangential component j_x. Thus the actual current lines may constitute with the calculated the angle of the order 1/β.

Experimental determination of equipotential lines with the help of probe, carried out for rectangular samples of indium antimonide at β>5, confirmed the pictures, given in the Figs, the precision of coincidence increasing with the augmentation of β. The greatest deviation (as to the voltage) did not exceed 6 %.

The demonstration that properties of plasma can be duplicated in semiconductor and superconductors has been a spurce of satisfaction to the experimenter which is always looking for new ways to study complex phenomena.

The given methods make it possible to evaluate the configuration of the current and electrical field lines in the Hall elements with the consideration of the effect of metal contacts in strong magnetic fields.

References

1. **Goedbloed J. P. H., Poedts S.** Principles of Magnetohydrodynamics. – Cambridge Univ. Press, 2005. – 632 p.
2. **Jankauskas Z., Kvedaras V., Laurinavičius L.** Experimental realization of Helicon Maser in magnetized solid-state plasma // *Physica B*. – 2004. – V. 346–347. – P. 539–542.
3. **Jankauskas Z., Kvedaras V.** Investigation of helicon Maser // Electronics and Electrical Engineering, 2005. – No. 3(59). – P. 70–72.
4. **Strauss W.** Partial Differential Equations. – Wiley, 1992. – 438 p.
5. **Binns K. J., Lawrenson P. J.** Analysis and computation of electric and magnetic field problems. – Pergamon Press, Oxford, 1963. – 583 p.
6. Tompson A. H., Kino G. S. Noise emission from IN // Sb. Journal of Applied Physics, 1970. – V. 41, No 7. – P. 3064–3075.

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Z. Jankauskas, V. Kvedaras. Electrical Field and Current Distribution in Semiconductor Plasma in the Strong Magnetic Field // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. –No. 2(74). – P. 41–44.

In general case the steady magnetic field distorts the configuration of the current and potential lines in the solid-state plasma. Only for the long samples at a sufficient distance from the metal contacts the Hall electric field is homogeneous and fully compensates the effect of the magnetic field on the moving charge carriers, and the picture of the current lines is the same as in the absence of the magnetic field. Near the metal contacts the short-circuiting of Hall e.m.f. occurs and the configuration of the current lines becomes more complicated. Even for the samples of simple geometrical from (e.g. rectangular) the general methods of calculation of the potential distribution are absent. The new method of calculation of the potential and current distribution in the magnetized solid-state plasma is proposed regarding the role of metal contacts. Problems of this type occur at designing and manufacturing of Hall transducers, magnetohydrodynamic generators and converters. The pictures of electrical field and current lines in magnetized semiconductor plasma with regarding the role of metal contacts were obtained. Ill. 2, bibl. 6 (in English; summaries in English, Russian and Lithuanian).

З. Янкаускас, В. Квядарас. Распределение электрического поля и тока в полупроводниковой плазме в сильном магнитном поле // Электроника и электротехника.- Каунас: Технология, 2007. – № 2(74) . – С. 41–44.

В общем случае постоянное магнитное поле сильно искажает конфигурацию линий тока и потенциала в плазме твердого тела. Лишь для длинных образцов при достаточном удалении от металлических контактов электрическое поле Холла однородно и полностью компенсирует воздействие магнитного поля на движущиеся носители заряда, а картина линии тока такая же, как и в отсутствие магнитного поля. Вблизи же металлических контактов происходит закорачивание э.д.с. Холла и конфигурация линии тока усложняется. Даже для областей простой геометрической формы (например, прямоугольной) общая методика расчета распределения потенциала в настоящее время отсутствует. Предложена новая методика расчета распределения потенциала и тока в плазме твердого тела, находящегося и сильном магнитном поле, с учетом влияния металлических контактов. Задачи такого типа возникают при проектировании и производстве датчиков Холла, магнитогидро-динамических генераторов и конвертеров. Ил. 2, библ. 6 (на английском языке; рефераты на английском, русском и литовском яз.).

Z. Jankauskas, V. Kvedaras. Elektrinio lauko ir srovės pasiskirstymas puslaidininkinėje plazmoje stipriame magnetiniame lauke // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 2(74) – P. 41–44.

Bendruoju atveju nuolatinis magnetinis laukas iškraipo srovės ir elektrinio lauko linijų konfigūraciją kietojo kūno plazmoje. Tik ilguose bandiniuose pakankamu atstumu nuo metalinių kontaktų Holo elektrovaros laukas yra homogeninis ir visiškai kompensuoja magnetinio lauko poveikį judantiems krūvininkams, o srovės bei elektrinio lauko linijų konfigūracija yra tokia pat, kaip ir nesant magnetinio lauko. Šalia metalinių kontaktų Holo elektrovara yra trumpai sujungta ir srovės linijų konfigūracija tampa sudėtingesnė. Netgi paprastos geometrinės formos (pvz., stačiakampės) bandiniams bendrų potencialo pasiskirstymo skaičiavimo metodų kol kas nėra. Pateikiamas naujas metodas elektrinio lauko ir srovės pasiskirstymui puslaidininkinėje plazmoje stipriame magnetiniame lauke skaičiuoti, atsižvelgiant į metalinių kontaktų įtaką. Tokio tipo uždaviniai iškyla projektuojant ir gaminant Holo jutiklius, magneto-hidrodinaminius generatorius ir keitiklius. Il. 2, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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