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The Distribution of Quantum Noise in the Presence of Gaussian Noise in the Fiber

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Introduction

In this paper we determine the distribution of quantum noise in the optical IM-DD systems in the presence of variable light intensity. Let we consider the Poisson's distribution of quantum noise in the presence of variable light intensity. The light intensity is variable because of different interferences. Because of that we calculate the distribution of quantum noise by averaging the conditional Poisson's distribution. We take the case when the light intensity has Gaussian probability density function (pdf).

The quantum noise appears because of quantum natural of light [1]. It is important to determine the probability of the quant number in the some time interval. We could always make the time interval so small that no one quant of light or only one appears in that interval. It can be assumed that the probability of appearance one pulse in the time interval Δt is proportional to this interval. If it is valid, the quant number has Poisson's distribution [1]. It is important to determine the current amplitudes quantum noise probability density function. In some cases this probability density function is Gaussian. The variance of quantum noise has distribution proportional to the average value of the quantum noise. Because of that the quantum noise is the Winer's random process. When the value of the quantum noise is significant, and binary hypothesis have equal probability, the threshold is not at the medium.

In the optical IM-DD systems [1], [2] on the fiber can appear interference with Gaussian pdf. The interferences with this distribution originate from optical amplifiers along the fiber. They are 50-100km for away from each other and compensate the reducing of light. The noise, which appears because of the spontaneous emission of light has Gaussian pdf. The Gaussian noise formed in the fiber can be resulted by mixing the modes. Some new modes can be formed at one connection of the optical fibers, but at the next connection that is modal noise. The modal noise exists because of nonlinear transformations along the fiber and it is similar to the interferometer noise. The amplitudes of this noise can have Gaussian pdf [1]. Also, several interferences with uniform phase pdf can occur in the optical fiber[2]. When the number of interferences is more than 10, it can be assumed that noises have Gaussian pdf [3].

The distribution of the first order photodiode quantum noise

The intensity of the light exiting the photodiode is [2]:

$$\lambda = C(A+x)^2 . \qquad (1)$$

C is constant and it can take the value 1. The electromagnetic wave consists of two terms. First of them is constant and it depends of hypothesis from transmitter and the other one is x, and it is the interference formed in the fiber with Gaussian pdf.

The number of quants exiting the photodiode depends on intensity of light and its conditional probability in a time interval is [4]:

$$p(n/\lambda) = \frac{\lambda^n}{n!} e^{-\lambda} = \frac{C^n (A+x)^{2n}}{n!} e^{-C(A+x)^2} . \qquad (2)$$

The Gaussian noise x which appears in the fiber has the probability density function:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} , \qquad (3)$$

where σ_x^2 is the variance of the Gaussian noise.

The probability of number of quants is obtained by averaging the expression (2):

$$p(n) = \int_{\lambda} P(n/\lambda) p(\lambda) d\lambda = \int_{-\infty}^{\infty} \frac{C^n (A+x)^{2n}}{n!} e^{-C(A+x)^2} \cdot$$

$$\cdot \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} dx , \qquad (4)$$

because of $p(\lambda)d\lambda = p(x)dx$.

While we determine the statistical characteristics of signal in telecommunication systems in the presence of Gaussian noise very often we use the integral:

$$I_1(k_1,a,b,d)=\int_{-\infty}^{\infty} x^{k_1} e^{a(b+x)^2} e^{-dx^2}\,dx. \qquad (5)$$

It can be shown as:

$$I_1(k_1,a,b,d)=\int_{-\infty}^{\infty} x^{k_1} e^{ab^2+2abx+ax^2} e^{-dx^2} =$$

$$=\int_{-\infty}^{\infty} x^{k_1} e^{-x^2(d-a)+2abx+ab^2}\,dx =$$

$$=\int_{-\infty}^{\infty} x^{k_1} e^{-(d-a)\left[x^2-\frac{2ab}{d-a}x-\frac{ab^2}{d-a}\right]}\,dx\,. \qquad (6)$$

If we use

$$a_1x^2+b_1x+c_1=a_1\left(x+\frac{b_1}{2a_1}\right)^2+\frac{4a_1c_1-b_1^2}{4a_1}\,, \qquad (7)$$

from (6) we obtain:

$$I_1(k_1,a,b,d)=\int_{-\infty}^{\infty} x^{k_1} e^{-a_2(x+b_2)^2+c_2}\,dx, \qquad (8)$$

where are:

$$\begin{cases} a_2=d-a, \\ b_2=\dfrac{b_1}{2a_1}=-\dfrac{ab}{d-a}, \\ c_2=\dfrac{4a_1c_1-b_1^2}{4a_1}. \end{cases} \qquad (9)$$

Integral (8) can be given by:

$$I_1(k_1,a,b,d)=e^{c_2}\int_{-\infty}^{\infty} x^{k_1} e^{-a_2(x+b_2)^2}\,dx. \qquad (10)$$

After the substitution:

$$\begin{cases} (x+b_2)\sqrt{a_2}=t, \\ x=\dfrac{t}{\sqrt{a_2}}-b_2 \end{cases} \qquad (11)$$

and using binomial expansion we get

$$I_1(k_1,a,b,d)=e^{c_2}\int_{-\infty}^{\infty}\left(\frac{t}{\sqrt{a_2}}-b_2\right)^{k_1} e^{-t^2}\,\frac{dt}{\sqrt{a_2}}=$$

$$=e^{c_2}\int_{-\infty}^{\infty}\sum_{n=0}^{k_1}\binom{k_1}{n}\frac{t^{k_1-n}b_2^{k_1}}{a_2^{\frac{k_1-n}{2}}}e^{-t^2}\,\frac{dt}{\sqrt{a_2}}=$$

$$=e^{c_2}a_2^{-\frac{k_1-n+1}{2}}b_2^{k_1}\sum_{n=0}^{k_1}\binom{k_1}{n}\int_{-\infty}^{\infty}t^{k_1-n}e^{-t^2}\,dt=$$

$$=e^{c_2}a_2^{-\frac{k_1-n+1}{2}}b_2^{k_1}\sum_{n=0}^{k_1}\binom{k_1}{n}I_2(k_1-n), \qquad (12)$$

where the integral $I_2(n)$ is defined by

$$I_2(n)=\int_{-\infty}^{\infty} x^n e^{-x^2}\,dx. \qquad (13)$$

The quant number probability exiting the photodiode in the presence of Gaussian noise in the fiber for the digital optical telecommunication IM-DD systems is:

$$P(n)=\frac{C^n}{n!}\frac{1}{\sqrt{2\pi}\sigma_x}\int_{-\infty}^{\infty}\sum_{i=0}^{2n}\binom{2n}{i}A^{2n-i}x^i e^{-C(A+x)^2}e^{-\frac{x^2}{2\sigma_x^2}}\,dx=$$

$$=\frac{C^n}{n!}\frac{1}{\sqrt{2\pi}\sigma_x}\sum_{i=0}^{2n}\binom{2n}{i}A^{2n-i}\int_{-\infty}^{\infty}x^i e^{-C(A+x)^2}e^{-\frac{x^2}{2\sigma_x^2}}\,dx=$$

$$=\frac{C^n}{n!}\frac{1}{\sqrt{2\pi}\sigma_x}\sum_{i=0}^{2n}\binom{2n}{i}A^{2n-i}I\left(i,C,A,\frac{1}{2\sigma_x^2}\right). \qquad (14)$$

The distribution of the *n*-th order photodiode quantum noise

The number of quant $n_1,n_2,\ldots,n_n$ exiting the photodiode at non coincide time intervals τ_1, $\tau_2,\ldots,\tau_n$. The light intensities in that intervals are $\lambda_1,\lambda_2,\ldots,\lambda_n$. If the light intensities $\lambda_1,\lambda_2,\ldots,\lambda_n$ are constant, the probabilities of number of quant $n_1,n_2,\ldots,n_n$ are:

$$\begin{cases} P(n_1/\lambda_1)=\dfrac{\lambda_1^{n_1}}{n_1!}e^{-\lambda_1}, \\ P(n_2/\lambda_2)=\dfrac{\lambda_2^{n_2}}{n_2!}e^{-\lambda_2}, \\ \quad\ldots \\ P(n_R/\lambda_R)=\dfrac{\lambda_R^{n_R}}{n_R!}e^{-\lambda_R}, \\ P(n_n/\lambda_n)=\dfrac{\lambda_n^{n_n}}{n_n!}e^{-\lambda_n}. \end{cases} \qquad (15)$$

If the time intervals τ_1, $\tau_2,\ldots,\tau_n$ do not coincide, then the conditional joint quant number probability is equal to product of probabilities:

$$p(n_1,n_2,\ldots,n_n/\lambda_1,\lambda_2,\ldots,\lambda_n)=$$

$$=\frac{\lambda_1^{n_1}}{n_1!}e^{-\lambda_1}\,\frac{\lambda_2^{n_2}}{n_2!}e^{-\lambda_2}\ldots\frac{\lambda_n^{n_n}}{n_n!}e^{-\lambda_n}=\prod_{k=1}^{n}\frac{\lambda_k^{n_k}}{n_k!}e^{-\lambda_k}\,. \qquad (16)$$

If the interference with Gaussian pdf exists in the fiber, the light intensities $\lambda_1,\lambda_2,\ldots,\lambda_n$ are:

$$\begin{cases} \lambda_1=C(A_1+x_1)^2\lambda_2=C(A_2+x_2),\lambda_2=C(A_2+x_2)^2,\ldots, \\ \lambda_n=C(A_n+x_n)^2 \end{cases} \qquad (17)$$

The conditional joint probability of number of quant $n_1, n_2, \ldots n_n$ in time intervals $\tau_1, \tau_2, \ldots, \tau_n$ is then

$$p(n_1, n_2, \ldots, n_n / x_1, x_2, \ldots, x_n) = \\ = \prod_{k=1}^{n} \frac{C(A_k + x_k)^{2n_k}}{n_k} e^{-C(A_k + x_k)}. \quad (18)$$

The joint probability density of Gaussian variables x_1, x_2,..., x_n is:

$$p(x_1, x_2, \ldots, x_n) = \frac{1}{(2\pi)^{n/2}\sqrt{D}} e^{-\frac{1}{D}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i V_{ij} x_j}, \quad (19)$$

where D is determinant of matrix covariance, and Vij cofactor of matrix covariance. The joint probability of number of quant $n_1, n_2, \ldots, n_n$ in time intervals $\tau_1, \tau_2, \ldots, \tau_n$ is finally:

$$p(n_1, n_2, \ldots, n_n) = \\ = \prod_{k=1}^{n} \int_{x_1} \underbrace{dx_1 \ldots}_{n} \int_{x_n} dx_n \frac{C(A_k + x_k)^{2n_k}}{n_k} e^{-C(A_k + x_k)^2} \cdot \\ \cdot \frac{1}{(2\pi)^{n/2}\sqrt{D}} e^{-\frac{1}{\sqrt{D}}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i V_{ij} x_j}. \quad (20)$$

Conclusion

In this paper we determined the distribution of quantum noise in the presence of variable light intensity. The distribution of quantum noise in the presence of variable light intensity is Poisson's. The light intensity is variable because of different interferences. Because of that the distribution of quantum noise is calculated by averaging the conditional Poisson's distribution. We considered the case when the light intensity has Gaussian pdf. If the amplifier noise appears in the fiber, electromagnetic field in optical fiber has Gaussian pdf. Also, when more interferences exists, it can be assumed that they have Gaussian pdf.

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Nagrinėjamas kvantinio triukšmo pasiskirstymas esant nevienodam šviesos intensyvumui. Tai yra Puasono dėsnis. Šviesos intensyvumas yra skirtingas dėl įvairių trukdžių. Čia išnagrinėjome atvejį, kai šviesa turi Gauso pdf. Bibl. 4 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

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