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Radiation of a Vertical Dipole over Flat and Lossy Ground using the Spectral Domain Approach: Comparison of Stationary Phase Method Analytical Solution with Numerical Integration Results

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***Abstract*—In this paper we re-consider the problem of radiation from a vertical short (Hertzian) dipole above flat lossy ground, which represents the well-known in the literature 'Sommerfeld radiation problem'. Particularly, we expand on the problem's solution in the spectral domain, which ends up into simple one dimensional (1-D) integral expressions for the received EM field and represent the exact EM solution to the aforementioned problem. The advantage of the derived expressions is based on the fact that they can be analytically evaluated through the use of the Stationary Phase Method (SPM), which however is valid in the high frequency regime. To our knowledge, the literature lacks specifying the exact frequency range over which the SPM method is applicable. Hence, in this paper numerical integration on the above mentioned integral expressions is applied and the results are compared with those obtained through the SPM. These comparisons are then used as the basis of determining the frequency limits of applicability of the SPM solution. In fact it is shown that due to the specific peculiarities of the integrated expressions, which possess certain singularities, it is often preferable to use the SPM method as the best estimate for the received signal level, especially for most practical frequencies of interest in the area of wireless telecommunications. Additional practical implications that these findings suggest, as well as further research to be triggered, as a result of the overall progress that has been made by our research group so far on the specific subject, are provided as well.**

***Index Terms*—Sommerfeld radiation problem, spectral domain solution, Stationary Phase Method (SPM), high frequency approximation, numerical integration.**

I. INTRODUCTION

The so-called 'Sommerfeld radiation problem' is a well-known problem in the area of propagation of electromagnetic (EM) waves above flat lossy ground for obvious applications in the area of wireless telecommunications [1]–[5]. The classical Sommerfeld solution to this problem is provided in the physical space by using the so-called 'Hertz potentials' and it does not end–up with closed form analytical solutions. K. A. Norton [6] concentrated in subsequent years more in the engineering application of the above problem with obvious application to wireless telecommunications, and provided approximate solutions to the above problem, which are represented by rather long algebraic expressions for engineering use, in which the so-called 'attenuation coefficient' for the propagating surface wave plays an important role.

In this paper, the authors expand on their existing research work regarding the solution of Sommerfeld's radiation problem in the spectral domain. Particularly, in [7], [8] the authors demonstrated the fundamental integral representations, for the received EM field for the aforementioned problem. Then in [9]–[11], the authors suggested the use of the Stationary Phase Method (SPM method [12]), with which novel closed-form analytical expressions are derived for the calculation of the EM field in the high frequency regime. At those papers detailed comparisons between the proposed SPM method and Norton's solutions [6] showed very good agreement results.

One of the main questions that arise naturally regarding the validity of the SPM method, an inherently high frequency technique, is the frequency range where it can be securely applied. Hence, in this paper, appropriate simulations are run for various carrier frequencies. These simulations compare the estimated received EM field at an observation point above flat and lossy ground (particularly only the scattered field is calculated since the Line of Sight field is easily and analytically derivable), under two approaches: (a) SPM method [9]–[11] and (b) Numerical integration of the corresponding integral representations. The results obtained are interesting in that they favour the use of the SPM method in a wide frequency range. Particularly, most widely used frequencies in contemporary

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wireless communication systems appear to belong into this ‘SPM applicability range’ and this yields important inferences as explained in Section V below. Also note that in the rest of the paper only the expressions for the Electric field are shown. Similar expressions hold for the Magnetic fields, as well [11].

II. Geometry of the Problem

The geometry of the problem is given in Fig. 1. Here a Hertzian (small) dipole with dipole moment *p* directed to positive *x* axis, at altitude x_0 above the infinite, flat and lossy ground, radiates time-harmonic electromagnetic (EM) waves at angular frequency ω = 2π*f*. Here the relative complex permittivity of the ground is $£'_r = £'/£_0 = £_r + i± / \mu£_0$, σ being the ground conductivity, *f* the frequency of radiation and $\varepsilon_0 = 8{,}854 \times 10^{-12}$ F/m is the absolute permittivity in vacuum or air. Then the wavenumbers of propagation in the air and lossy ground, respectively, are given by the following equations:

$$k_{01} = \omega/c_1 = \omega\sqrt{\varepsilon_1\mu_1} = \omega\sqrt{\varepsilon_0\mu_0}, \quad (1)$$

$$k_{02} = \omega/c_2 = \omega\sqrt{\varepsilon_2\mu_2} = k_{01}\sqrt{\varepsilon_r + \mathrm{i}(\sigma/\omega\varepsilon_0)}. \quad (2)$$

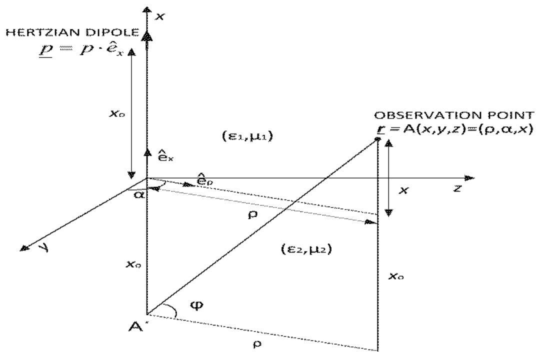

Fig. 1. Geometry of the problem.

III. Integral Representation for the Received Electric Field in the Spectral Domain

Following [7], [8] the following integral representation for the space wave regarding the electric field above the ground level (*x* > 0) is derived:

$$\underline{E}(\underline{r}) = \underline{E}^{\mathrm{LOS}}(\underline{r}) - \frac{ip}{8\pi\varepsilon_{r1}\varepsilon_0}\times$$
$$\times\left\{\hat{e}_\rho \int_{-\infty}^{\infty} k_\rho^2 \frac{\varepsilon_{r2}\kappa_1 - \varepsilon_{r1}\kappa_2}{\varepsilon_{r2}\kappa_1 + \varepsilon_{r1}\kappa_2} e^{i\kappa_1(x+x_0)} \times \mathrm{H}_0^{(1)}(k_\rho\rho)dk_\rho -\right.$$
$$\left.-\hat{e}_x \int_{-\infty}^{\infty} k_\rho^3 \frac{\varepsilon_{r2}\kappa_1 - \varepsilon_{r1}\kappa_2}{\kappa_1(\varepsilon_{r2}\kappa_1 + \varepsilon_{r1}\kappa_2)} e^{i\kappa_1(x+x_0)} \times \mathrm{H}_0^{(1)}(k_\rho\rho)dk_\rho\right\}, \quad (3)$$

where:

$$\begin{cases} \kappa_1 = \sqrt{k_{01}^2 - k_\rho^2}, \\ \kappa_2 = \sqrt{k_{02}^2 - k_\rho^2}, \end{cases} \quad (4)$$

and $\mathrm{H}_0^{(1)}$ is the Hankel function of first kind and zero order. Moreover, $\underline{E}^{\mathrm{LOS}}$ is the Line of Sight (LOS) electric field vector, which as mentioned above, is not shown in our simulations, since it can be directly evaluated with analytic expressions [10].

IV. Analytical Closed-Form Expressions for the Scattered Electric Field Obtained through the Application of the Stationary Phase Method (SPM)

Application of the SPM method on (3) and (4) above leads to the following analytic expressions for the EM field in the higher half space (*x* > 0) [10], [11]

$$\underline{E}_{x>0}^{SC} = \frac{p}{4\pi\varepsilon_0\varepsilon_{r1}\rho^{1/2}(x+x_0)^{1/2}} \times \frac{\kappa_{1s}^{1/2}k_{\rho s}^{3/2}}{k_{01}}$$
$$\times\frac{\varepsilon_2\kappa_{1s} - \varepsilon_1\kappa_{2s}}{\varepsilon_2\kappa_{1s} + \varepsilon_1\kappa_{2s}} \times e^{ik_{\rho s}\rho} \times e^{i\kappa_{1s}(x+x_0)} \times$$
$$\times(\kappa_{1s}\hat{e}_\rho + k_{\rho s}\hat{e}_x) = \frac{pk_{01}\cos\varphi}{4\pi\varepsilon_0\varepsilon_{r1}(\mathrm{A'A})}\times$$
$$\times\frac{\varepsilon_2\kappa_{1s} - \varepsilon_1\kappa_{2s}}{\varepsilon_2\kappa_{1s} + \varepsilon_1\kappa_{2s}} \times e^{ik_{\rho s}\rho} \times e^{i\kappa_{1s}(x+x_0)} \times$$
$$\times(\kappa_{1s}\hat{e}_\rho + k_{\rho s}\hat{e}_x), \quad (5)$$

where (A´A) is the distance between the image point and the observation point, shown in Fig. 1. Moreover, in (5) above, the following expressions hold:

$$k_{\rho s} = \frac{k_{01}\,\rho}{\sqrt{(x+x_0)^2 + \rho^2}} = \frac{k_{01}}{\sqrt{1 + \left(\frac{x+x_0}{\rho}\right)^2}} = k_{01}\cos\varphi, \quad (6)$$

$$\begin{cases} \kappa_{1s} = \sqrt{k_{01}^2 - k_{\rho s}^2} = k_{01}\sin\varphi, \\ \kappa_{2s} = \sqrt{k_{02}^2 - k_{\rho s}^2}, \end{cases} \quad (7)$$

with $k_{\rho s}$ being the stationary point obtained from the SPM method and φ the angle shown in Fig. 1, also known in the literature as the ‘grazing angle’ [13].

V. Numerical Results: Comparison of SPM with Numerical Integration Techniques

In this Section the results obtained using the analytical, closed-form formulas of the SPM method, described in Section IV above, i.e. (5), are compared with the corresponding results obtained by numerically evaluating the corresponding integral expression of (3), given in Section III. The whole set of simulation parameters, used in this problem, are summarized in Table I.

TABLE I. SIMULATION PARAMETERS.

Symbol	Description	Value
f_{min}	Minimum Frequency	10 KHz
f_{max}	Maximum Frequency	1 GHz
x_0	Height of transmitting dipole	60 m
x	Height of observation point (receiver)	15 m
I	Current of the radiating Hertzian dipole[1]	1 A
$2h$	Length of the Hertzian dipole[2]	0.1 m
ε_r	Relative dielectric constant of ground	20
σ	ground conductivity	0.01 S/m

Notes:
[1] Relation between current I and dipole moment *p*: $I(2h) = \mathrm{i}\omega p$, where $\omega = 2\pi f$ and i is the unit imaginary number
[2] much smaller than the wavelength $\lambda = c/f$ in both cases

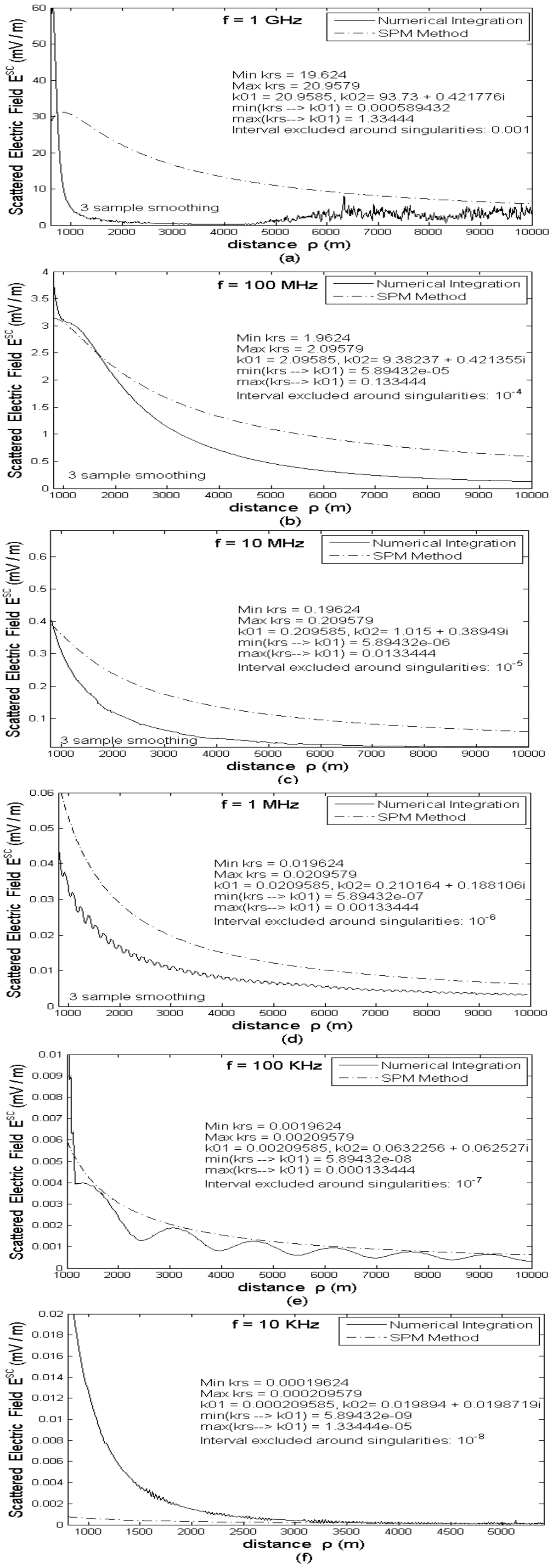

Fig. 2. Scattered electric field values as a function of horizontal distance ρ between transmitter and receiver calculated by the SPM method and numerical integration techniques.

As mentioned in Section I, the objective of this comparison is to identify the frequency ranges where the SPM technique, which is an inherently high frequency method [12], can be effectively applied. For that purpose, Fig. 2, below, provides indicative results for the scattered electric field of (3) and (5), as explained above, for a wide range of frequencies of interest (corresponding results for the magnetic field are very similar).

In order to evaluate the results of Fig. 2, the following remarks, which essentially summarize the simulation procedure, must be made:

– The estimation of the integral expression of (3) was performed through the adaptive Simpson's algorithm [14]. The error tolerance for convergence was set to 10^{-6}. Moreover, to mitigate the 'small scale' oscillating behavior of the resulting graphs, which is an outcome of the fact that the integrated expressions consist of complex numbers of fast-varying phases, a typical three sample smoothing (averaging) was applied, where appropriate.

– Careful examination of (3) reveals the fact that there exists a *singular point* at $\kappa_1 = 0$, that is at $k_\rho = k_{01}$ and hence it must be excluded by a sufficient range around k_{01}, for the numerical integration algorithm to converge. Our tests, showed that this range needs to be no less than 5×10^{-5} times the upper value of $k_{\rho s}$, which is k_{01} [according to (6) $k_{\rho s} < k_{01}$]. However, this indicates that a segment around the stationary point is also excluded from the calculation. The inferences for this are discussed below.

– Note that the excluded interval of integration around the singular point, k_{01}, is shown in Fig. 2, where min(krs-- > k01) and max(krs-- > k01) denote the minimum and maximum distance between $k_{\rho s}$ and k_{01} respectively (see labels in respective diagrams). In the same figure, the corresponding $k_{\rho s}$ values, which depend on horizontal distance ρ, as evident from (6), are shown as well. This is shown in order to give an indication of the range that was excluded, around the stationary point $k_{\rho s}$, regarding the calculation of the integral expression.

– Also note that although point $k_\rho = 0$ appears to be another possible singular point, in the integral expression of (3), being the zero argument point of the Hankel function, in fact this is not the case due to the presence of the k_ρ^2 and k_ρ^3 factors in the integrands. Indeed, one can easily show that: $k_\rho^2 \cdot H_0^{(1)}(k_\rho \cdot \rho) \rightarrow 0$ *as* $k_\rho \rightarrow 0$

Examining the curves of Fig. 2, it is evident that for frequencies of about 100 KHz and above the results taken through the numerical integration approach underestimate the received signal level compared with the SPM method. This is related with the nature of the SPM method, which indicates that for large frequencies the integral expression can be asymptotically approximated by taking into consideration only the contributions of the area around the stationary point [12]. However, as mentioned above, when numerically evaluating the integral expression of (3), a sufficiently large interval around k_{01} has to be excluded. In most cases, this interval overlaps with the stationary point, which means that a significantly contributing part in the integral calculation is missed.

On the contrary, regarding the last curve of Fig. 2 (f = 10 KHz), it is, in this case, the SPM method which seems to underestimate the EM field values (we also reach the same findings for f < 30 KHz). Indeed, for such low frequencies, the large argument approximation of the SPM method cannot be invoked, in other words, (5) through (7) are not accurate at all. It is still necessary to exclude a range around point $k_\rho = k_{01,}$ for the numerical integration algorithm to converge, however this time this range is not a major contributor to the overall outcome.

The results shown in Fig. 2, above, have direct practical implications when considering the design of efficient radio network planning simulation tools, in which case the appropriate computation method for estimating the signal level must be used. These *conclusions* are summarized in Table II.

TABLE II. SELECTION OF EM FIELD CALCULATION METHOD.

Method Suitability SPM vs. Numerical Integration(NI)		
Frequency Range ([13])	**Suitable Method**	**Observation**
Medium Frequency (MF) range and above (>300 KHz)	SPM	SPM provides more accurate results for the received signal level and hence should be the selection of choice for prediction purposes.
Low Frequency (LF) range (30 – 300 KHz)	SPM and NI	The results given by the two methods seem comparable and our research group proposes that a closer examination and potential fine-tuning of the numerical integration algorithm is necessary, before reaching ‘safe’ inferences (see Section VI below).
Very Low Frequency (VLF) range (<30 KHz)	NI	Numerical Integration is more suitable. SPM fails and the estimation must be based on numerical integration techniques.

VI. CONCLUSIONS

In this paper we re-examined the solution of Sommerfeld’s radiation problem in the spectral domain and we studied the practical aspects that this novel exact electromagnetic method gives in the determination of the received signal level. As mentioned above, the resulting integral expressions expedite the simulation process and allow direct comparisons between numerical integration techniques and the analytic expressions obtained through application of the SPM.

Particularly, in this paper, we explained why the proposed SPM method is a robust technique, since it provides more accurate results than common numerical integration methods for most frequencies of interest (at least above 300 KHz) in the area of wireless telecommunications and hence can be the basis for an efficient simulation tool for radio signal propagation.

Corresponding research in the near future by our research group will concentrate on the solution of Sommerfeld’s radiation problem, but this time for a *horizontal* dipole. Furthermore, we intend to calculate the received EM field, above or below the ground, for *any frequency of the radiating dipole*, in an exact and analytical manner [15]. In this context, the behavior of surface waves will become evident through the use of the residue theorem, when applied to (3) above, in a way similar to [5].

Finally, in the near future our research group will focus on the design of a *software product* for accurate prediction of pass loss in different types of environments. The above software tool will be based on the exact electromagnetic (EM) method proposed in this paper, as well as in [9]–[11], and therefore it is expected that it will exhibit important advantages over previously developed corresponding software tools. Some of these advantages might include accuracy, speed, efficiency and low complexity, since the various calculations will be based on closed form analytical expressions, instead of resource starving and time consuming numerical methods. We also intend to fine-tune the numerical method presented in this paper (e.g. experiment with convergence tolerances), as well as to test alternative numerical integration algorithms (e.g. the ‘adaptive Lobatto’ algorithm [14] could be examined) and to use the most appropriate as the back-up method in situations where the SPM is not sufficient. In this framework, comparisons with existing commercial software tools, implementing various empirical models, can also be performed [16].

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