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# Bayesian Decision Theory Application for Double-step Saccades

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## Introduction

If we wish to understand how a human being sees clearly, knows were the things are, knows whether they are stationary or moving and knows where he is in relationship to them, we have to understand how human oculomotor system works. Functionally oculomotor system can be broken down into a few subsystems, purpose of which do not seem difficult to understand. There are a few of them [1] enabling us to fixate stationary objects (fixational subsystem), to follow moving objects (smooth pursuit subsystem and oculomotor nystagmus), jump from one object to another (saccadic subsystem), to move ourselves and see clearly (vestibular ocular reflex and vestibular nystagmus), see clearly close objects with both eyes (eyes convergence subsystem). Most important in everyday life are fixational and saccadic subsystems. They enable us to scan of visual scenes, fixate interesting objects and let us acquire necessary information from the surrounding. In the latest applications eye tracking equipment is used for target pointing tasks. In this case, measuring line's of sight direction allow us to define coordinates of the target.

Many theoretical and experimental scientific tools such as neurophysiologic modeling, signal theory, control system analysis and, recently, information theory was used for the investigation of the saccadic subsystem and saccadic eye movements [2]. Recent advances in recording data from cells of the Central Nervous System (CNS) in alert animals coupled with tracking eye movement registration and analytic methods let to explain how saccadic eye movements are elicited [3].

Saccadic subsystem execute saccadic eye movements, which are accurate, high-velocity, non-ballistic used for refixation line of sight from one object of interest to another. Normally saccades take the eye 90% of the way to a target, followed by 10% corrective saccade. Therefore, normal saccades execute eye jump in to the two steps. Double-step saccade phenomena is investigated mostly discovering its' quantitative parameters [4]. It was found that double-step saccades can be either too small (hypometric) or too large (hypermetric) with respect to the real target position. Majority of the saccades are hypometric and they represent a normal strategy adopted by the saccadic system so that any subsequent corrective saccade requires computation only amplitude but not direction. In the paper [5], double-step saccades were investigated as two-stage information transfer channel. This research considers double-step saccades as Bayesian decision process and explains existence of the primary and corrective saccades by two-steps goal reaching strategy.

## Bayesian decision theory for sensorimotor control

Decision theory quantifies how people should choose in the context of a given utility function and some partial knowledge of the world. The expected utility is defined as:

$$E[Utility]=\sum_{actions} p(outcome/action)U(outcome), \quad (1)$$

where *p(outcome/action)* – the probability of an outcome given an action; *U(outcome)* – the utility associated with this outcome. According to decision theory people choose the action so as to maximize the expected value of utility.

Bayesian statistics defines how new information should be combined with prior beliefs and how information from different modalities should be integrated. Limbs movement control system aims to solve similar problems where the decision is based on notion of 'cost-to-go" from current state to a target state. The solution changes constantly according to new information coming from the sensory system and minimizes expected value of utility such as muscle energy and movement error.

Determining the appropriate motor command from the CNS could be defined as a decision process. At each point of time, we must select one or few particular commands from the set of possible actions. Decision process is activated by two components: knowledge of the initial position of the limb and knowledge of our objectives. Because signals in our sensory or motor systems are corrupted by variability of noise, which means that we always have uncertainty about our limb's true location. This uncertainty depends on the modality of sensory input: when we use proprioception to locate our limb (efferent copy from the muscles), we may have larger uncertainty about our position compared to when we have information from the visual system (afferent copy). Moreover, our acting muscles produce noisy outputs and when we quickly move to a visually not seen target location our final hand position will deviate from the intended target. This uncertainty places the problem of estimating the location of the target and the control of our motor system within the statistical framework. Bayesian statistics provides the systematic way of solving problems in the presence of action-by-action changing uncertainties [6].

**Bayesian rule (model) for double-step saccades**

In the Fig. 1, we can see experimentally recorded sequence from one to four two-dimensional eye saccades to the different target positions on the screen. Dots in this figure represent samples of the eye movement trajectories obtained every 8 msec. Smaller regions of the concentrated dots illustrate landing places of the large amplitude primary saccades. Larger regions of the dots illustrate fixational micromovements recorded when line of sight is on the target. Distance from smaller regions of the concentrated dots to the larger regions represents small amplitude corrective saccades. Most of the primary saccades undershoot target and corrective saccades put line of sight on the target. Neurophysiology of the eye movements control system explains this behavior by the smaller amount of the position receptors in the periphery of the retina and larger amount of them closer to the fovea. Therefore, eye muscles controlled by CNS first execute large amplitude primary saccade with low accuracy and later elicit small amplitude corrective saccade with high accuracy. Using information theory concepts sensory noise or target position uncertainty in the periphery of the retina is larger comparing with closer to the fovea region.

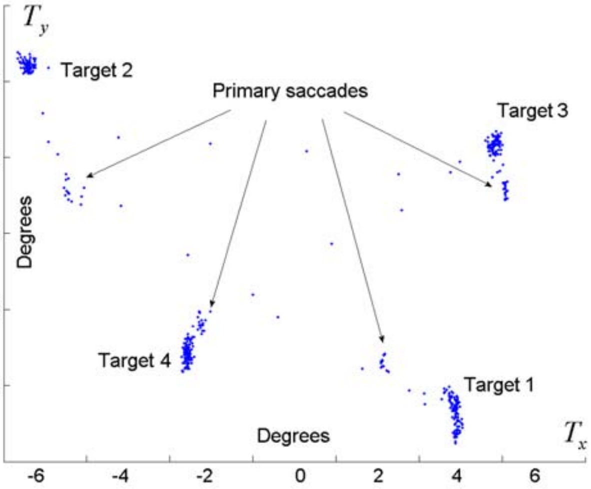

**Fig. 1.** Two-dimensional eye saccades (dots represent samples of the eye movement trajectories obtained every 8 msec). Smaller regions of the dots illustrate landing places of the primary saccades and larger regions – fixational micromovements at the end of saccades

If we assume two-dimensional target position on the screen prior probability distribution as $p_t(T_x,T_y)$ and visual estimation of the real target coordinates $T_{xo}$, $T_{yo}$ in the periphery of the retina leads likelihood $p_v(v_x,v_y/T_x,T_y)$, then posterior probability distribution according Bayesian rule would be:

$$p_p(T_x,T_y/v_x,v_y)=p_v(v_x,v_y/T_x,T_y)\frac{p_t(T_x,T_y)}{p_f(v_x,v_y)}. \qquad (2)$$

In this equation $p(v_x\ ,v_y)$ is probability distribution of the vision in the visual field and $v_x$, $v_y$ are visual field coordinates. If there are no defects in the visual field, $p_f(v_x,v_y)$ could be assumed as constant value. Fig. 2 shows example of the above-mentioned distributions in the horizontal direction. Because of the bigger target probability in the centre of the screen and zero probability outside the screen, the possible target positions distribution is defined as abrupt Gaussian probability distribution $p_t(T_x)$. Noisy visual estimation of the observed target $T_{xo}$ is assumed as a symmetrical (regard target) Gaussian likelihood $p_v(v_x/T_x)$ with standard deviation $\sigma_t$ and mean $\mu_t$. Analytically computed posterior probability distribution $p_p(T_x,v_x)$ is depicted in the same Fig. 2c. The main conclusion from the Fig. 2 is that estimated target position $\mu_p$ comparing with $\mu_t$ is shifted to the centre of the screen. It explains why majority centrifugal saccades undershot target. If saccadic eye movements would be centripetal, estimated target position would not be shifted. This Bayesian statistics approach to the double-step saccades mach experimental findings presented by D. A. Robinson and Z. Kapoula.

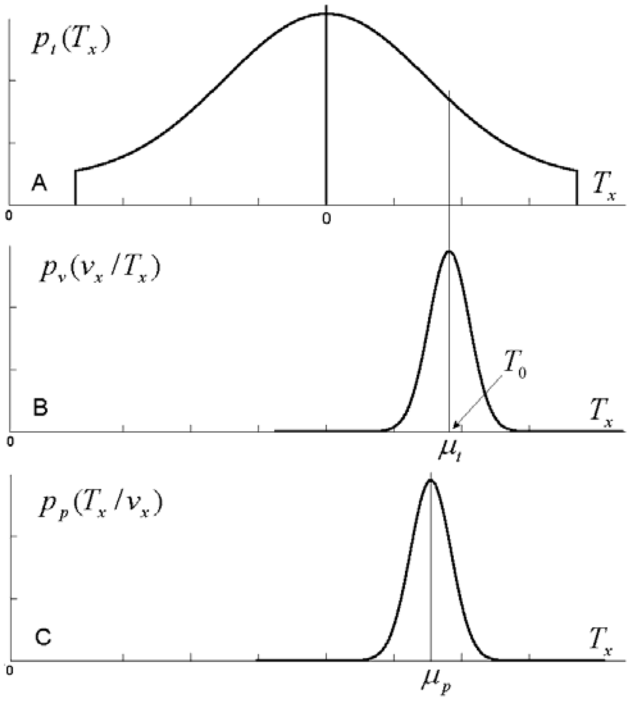

**Fig. 2.** (A) Prior $p_t(T_x)$, (B) likelihood $p_v(v_x/T_x)$ and (C) posterior $p(T_x/v_x)$ probability distributions

Corrective saccades have small amplitudes not bigger than 1-2 degrees and therefore after primary saccade target is close to fovea and accurately estimated. If undershoot is large and estimation of the target position after primary saccade remains poor two corrective saccades could be executed. This behavior match Bayesian rule adopting the first corrective saccade as a separate saccadic eye movement.

Appending Bayesian rule for double-step saccades it is necessary to pay attention not only to the visual noise estimating target position but also to the eye globe muscles which make mistakes performing eye jump. Errors in the motor system explain why every single primary saccade is more scattered and why not every centrifugal saccade undershoots and not every centripetal saccade overshoots target position. Fig. 3 demonstrates integration of the visual (sensor) noise and motor errors covering eye jump.

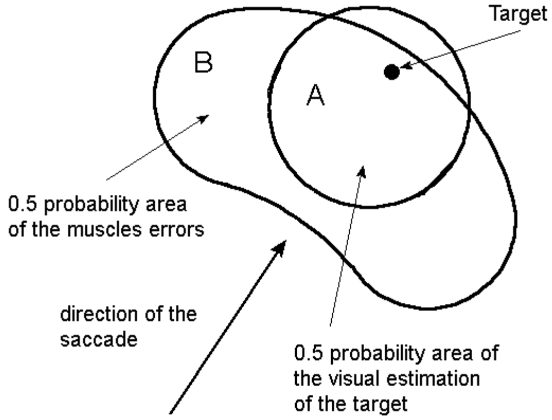

**Fig. 3.** Bayesian visual (sensor) and motor integration. (A) 0.5 probability area of the visually estimated target position. (B) 0.5 probability area of the primary saccades landing region caused by the muscles errors

There are presented possible 0.5 probability area (A) of the visually estimated target position and 0.5 probability area (B) of the line of sight landing zone caused by muscles errors. Area A is shifted to the initial eye position as explained in the figure 2. Final position of the area of the landing region of the primary saccades could be defined as second step of the Bayesian estimation or sensorimotor integration. Now second step prior probability distribution could be defined as posterior probability distribution obtained after visual target estimation $p_n(L_x,L_y) = p_p(T_x,T_y/v_x,v_y)$ and likelihood $p_m(m_x,m_y/L_x,L_y)$ as eye jump scatter influenced by muscles errors. Finally, two-dimensional probability distribution of the landing positions of the primary saccades could be defined as new posterior probability distribution:

$$p_s(L_x,L_y/m_x,m_y)=p_m(m_x,m_y/L_x,L_y)\frac{p_n(L_x,L_y)}{p(m_x,m_y)}. \quad (3)$$

In this equation $p(m_x,m_y)$ is probability distribution of the eye muscles ability to bring eye to the different positions in the eye movements range and could be assumed as constant value.

## Experimental results

Five subjects took part in the saccadic eye movement experiments. Control of target displacement, line of sight direction registration and data processing were carried out using eye tracker *EyeGaze System*. Fig. 4 depicts averaged final positions of ten primary saccades elicited to the four different target displacements. Arrows in this figure illustrate directions from the initial positions of the jumps and different amplitudes in degrees. Two-dimensional scatter of the final points of the primary saccades (dots in the Fig. 4) shows that landing places are distributed in the polar coordinates with density maximum shifted to the initial eye direction.

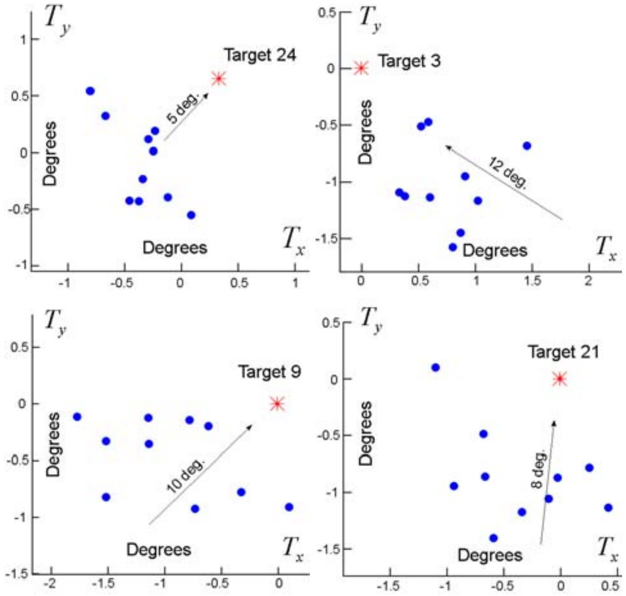

**Fig. 4.** Averaged final positions (dots) of ten primary saccades elicited to the same targets. Arrows illustrate directions from the initial positions of the jumps and different amplitudes in degrees

Further investigation was carried out trying to define characteristics of the probability distribution of the landing places of the primary saccades. This was done increasing number of trials to 150 of the saccadic eye movements and defining statistical parameters such as means and standard deviations of the two-dimensional distributions for all five subjects. It was defined that two-dimensional probability distribution is shaped in the polar coordinates which size of scatter depends on the amplitude $A$ and on the direction $\varphi$ of the saccade. Probability distribution along target direction $l$ is shifted to the initial eye position by mean $\mu_l$ and has standard deviation $\sigma_l$. Probability distribution around target direction $\varphi$ has mean $\mu_\varphi$ and standard deviation $\sigma_\varphi$. Parameters of the distributions (example for subject RZ eliciting 10 degree saccades shown in the figure 5) empirically defined by the equations:

$$\begin{cases}\mu_l = 0.95A - a(A_x + 1.5A_y), & \mu_\varphi = \varphi,\\ \sigma_l = 0.1 + b(A_x + 1.5A_y), & \sigma_\varphi = 0.1 + cA.\end{cases} \quad (4)$$

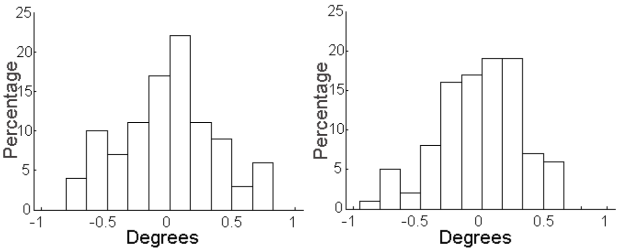

**Fig. 5.** Distribution of the scatter of the landing places of the 10 degree primary saccades: along target direction (on the left) and around target direction (on the right)

In this equations $A_x$ and $A_y$ are components of the amplitude of the saccade $A$ in the horizontal and vertical direction respectively. Obtained results of the parameters a, b and c for all five subjects are placed in the Table 1.

**Table 1.** Experimentally defined values of a, b and c parameters of the equations 4

| Subj. | RZ | GD | NR | BD | VL | Average |
|---|---|---|---|---|---|---|
| a | 0.021 | 0.024 | 0.021 | 0.022 | 0.026 | 0.023 |
| b | 0.012 | 0.015 | 0.014 | 0.015 | 0.017 | 0.015 |
| c | 0.013 | 0.014 | 0.015 | 0.016 | 0.017 | 0.015 |

## Conclusions

1. Bayesian decision theory was applied to the human saccadic eye movements control system, which elicit them in the double-step mode.
2. Influence of the noisy visual estimation of the target position by the visual field receptors of the eye and eye globe muscles errors made bringing eye to the new target position were evaluated.
3. Analytical model of the two-dimensional probability distribution of the landing positions of the primary saccades was proposed.
4. Means and standard deviations of the two-dimensional scatter of the landing places of the primary saccades were defined experimentally.
5. Obtained results could be used developing algorithms of the targeted movements of the robots.

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**V. Laurutis, R. Zemblys. Bayesian Decision Theory Application for Double-step Saccades // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 4(92). – P. 99–102.**

Bayesian decision theory applied to the human saccadic eye movements elicited in double-step mode: primary and corrective saccade. Primary saccade performs large amplitude but not precise eye jump towards new target position and small amplitude corrective saccade brings line of sight precisely on the target. This behavior mach two-step Bayesian decision making process: first with large uncertainty and second, after getting additional information, – more precise. It explains how saccadic eye movements control system performs precise eye jump in the conditions of the noisy visual estimation of the new target position and eye jump errors made by eye globe muscles. Scatter of the landing places of the primary saccades was investigated theoretically using Bayesian statistics approach and experimentally. Ill. 5, bibl. 6 (in English; summaries in English, Russian and Lithuanian).

**В. Лаурутис, Р. Земблис. Применение теории Байеса принятия решений при формировании двухступенчатых саккад // Электроника и электротехника. – Каунас: Технология, 2009. – № 4(92). – С. 99–102.**

Теория Байеса принятия решений была применена к двухступенчатым саккадам, формируемым двумя этапами: первичной и корректирующей саккадами. Первичная саккада поворачивает глаз к цели с большой амплитудой, но не точно, а небольшой амплитуды корректирующая саккада переводит взгляд точно на цель. Это соответствует принятию решений по теории Байеса: первое решение с большой неопределенностью и второе, после получения дополнительной информации, более точное. Так в системе контроля саккадических движений глаз выполняет точный скачок при наличии неточного визуального определения положения новой цели и ошибке глазных мыщц. Разбросы амплитуд первичных саккад исследованы теоретически и экспериментально. Ил. 5, библ. 6 (на английском языке; рефераты на английском, русском и литовском яз.).

**V. Laurutis, R. Zemblys. Bajeso sprendimų teorijos taikymas dviem etapais formuojamoms sakadoms // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 4(92). – P. 99–102.**

Bajeso sprendimų teorija buvo pritaikyta žmogaus sakadiniams akių judesiams, formuojamiems dviem etapais – pirmine ir korekcine sakadomis. Pirminė sakada, suformuojanti akies posūkį link taikinio, yra didelės amplitudės, bet netiksli, o mažos amplitudės korekcinė sakada perkelia žiūros liniją tiksliai į taikinį. Tai atitinka dviejų pakopų Bajeso sprendimų priėmimą: pirmas yra labai neapibrėžtas, o antras, gavus papildomos informacijos, – tikslesnis. Tai paaiškina, kaip sakadinių akių judesių kontrolės sistema suformuoja tikslų akies šuolį, esant netiksliam regos sistemos naujo taikinio padėties įvertinimui ir akies raumenų paklaidoms. Pirminių sakadų paklaidų sklaida buvo ištirta teoriškai, naudojant Bajeso modelį, ir eksperimentiškai. Il. 5, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).