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Simulation Evaluation of BER Characteristics for M-PSK and M-QAM Modulations used in the Reverse Channel of Cable TV Nets

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Introduction

The frequency range 0 ÷ 44 MHz in CATV networks is used for making interactive systems for signal transmission and is referred to as reverse or return channel. Frequency separation of the direct and reverse channel is effected through the inclusion of frequency diplex filters at the input and output of the cable amplifiers [3,7].

It is a common practice to denote digital broadcast of TV programs in cable distribution networks by DVB-C. The following conditions are set in order to enable this transmission:

- available cable networks to be usable without changing their structure, their equipment and frequency separation;
- possibility to transmit both digital and analogue signals along the network without causing mutual disturbances;
- selected method of modulation is to allow programs' transmission from a certain satellite (or studio) in the cable network without complicated conversions of digital traffic;
- methods used for processing and encoding of signal should be as close as possible to the methods used with DVB-S and DVB-T;
- selected methods for encoding and modulation should allow for transmission in the network of digital sequence with error density of 10^{-11}

In the case of DVB-C quadratic amplitude modulation is used which requires high noise to signal ratios, but allows the transmission of larger number of bits per symbol i.e the coefficient of the frequency band is higher. As a rule 64-QAM is used (since the spectrum relationship is three times more effective), however the standard allows the use of 16-QAM, 32-QAM … M-QAM [1, 2, 3].

Analytical determination of BER for M-PSK и M-QAM modulations

It is known that the disposition of the vector tip for signals I and Q at the output of the modulator for 4-QAM (also called QPSK) and 64-QAM coincide [2, 4, 5]. Apart from that, the theoretically determined BER – Bit Error Rate at ideal conditions for 4-QAM (QPSK) to 64-QAM as a function of the signal to noise ratio (SNR – Signal to Noise Ratio), QPSK is with higher priority of 12dB[2] than 64-QAM .

Calculation of BER for M-PSK and M-QAM is done either by estimation of symbol error probability or by evaluation of upper and lower boundaries of noise level. Accurate analysis of system's BER often appears to be too complex and, usually, there should be further processing of results. However, the use of boundaries cannot always guarantee sufficient accuracy as long as the transformation of SER (Symbol Error Rate) into BER is generally not quite true. Thus the mathematical model can reach maximum agreement with the properties of the physical object under investigation [5,6].

Let us consider the simplest possible case of 4-PSK (QPSK). Fig.1 shows the "signal-site" diagram for 4-PSK where axes "a" and "b" stand for the basic co-ordinates. The structure using Gray's code a_0a_1 is also shown as vector rendition. It is clear that for the upper semi-circle or the one above axis "a", a_0=0 whereas for the lower semi-circle i.e below axis "a", a_0=1.Similarly, a_1 can be obtained through a solution toward axis "b" (a_1=0, on the right and a_1=1 on the left) [4].

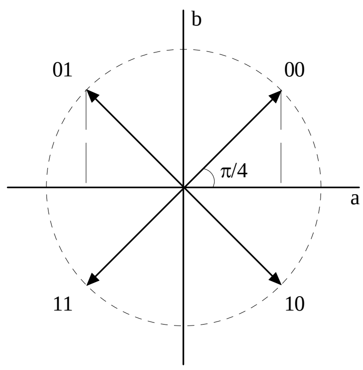


Fig. 1. "Star" chart for QPSK with signal transmission by Gray's code

$P_e(a_i)$ is defined:

- P_r (Error | a_i which is defined); P_X – error probability related to the solution and based on axis "x".

Due to symmetry: $P_a=P_b$. Therefore, equally probable symbols:

$$P_{e,4-PSK} = \frac{1}{2}\left[P_e(a_0) + P_e(a_1)\right] = \frac{1}{2}(1+1)P_a . \qquad (1)$$

By way of analogy we can get the diagram (chart) for 8, 16 … M-PSK. For example, for 16-PSK, the diagram is shown on Fig. 2. Here there are 4 bits ($a_0a_1a_2a_3$), by means of which a symbol (character) is represented.

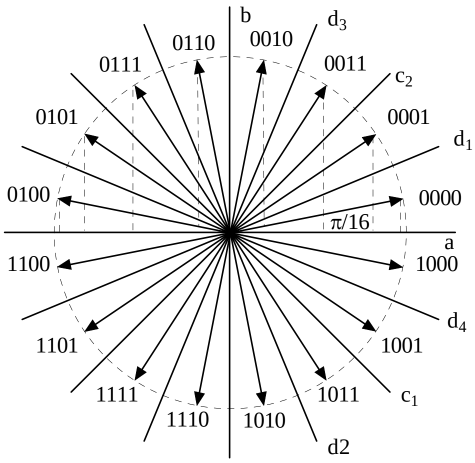


Fig. 2. "Star" diagram for 16-PSK

The same method of defining is applied for $\hat{a}_0$, $\hat{a}_1$ и $\hat{a}_2$ as is the case with 4 and 8-PSK. Determining of $\hat{a}_3$, demands the application of another two coordinate systems rotated in relation to their basic axes "a" and "b" – at an angle $\frac{2\pi}{16}$ so that new axes "d_1"and "d_2" be obtained; and at an angle $\frac{6\pi}{16}$, so that the other axes "d_3" and "d_4" be obtained.

Therefore for equally probable symbols we get:

$$P_{e,16-PSK} = \frac{1}{4}\left[P_e(a_0) + P_e(a_1) + P_e(a_2) + P_e(a_3)\right] \cong$$

$$\cong \frac{1}{4}(1+1+2+4)P_a . \qquad (2)$$

Summarizing for M-PSK signal it can be written for the general case:

$$P_{e,M-PSK} =$$

$$= \frac{1}{\log_2 M}\left(1+1+2+4+...+2^{\log_2 M-2}\right)P_a =$$

$$= \frac{M}{2\log_2 M} P_a . \qquad (3)$$

By using (3), BER for M-PSK can be calculated after P_a.is determined. This represents the error probability in performing a solution which is based on a certain coordinate axis (for example, axis "a"). Vertical distance is counted from every point of the diagram to the axis "a" and then is individually related to P_a.

Then P_a can be defined in the following way:

$$P_a = \frac{4}{M} Q\left[\sqrt{\frac{2E_{TP}}{N_0}} \sin\frac{(2i-1)\pi}{M}\right], \; i = 1,2,..., M/4, \qquad (4)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{t^2}{2}} dt$ – Gaussian (normal) probability distribution (Rayleigh distribution), and N_0 is single aspect power of noise spectrum density. The use of classic normal distribution is suitable for fluctuation noises of top line type.

Having in mind the respective diagram of the corresponding manipulation for $E_{тр}=E_b\log_2M$, it follows that:

$$P_a = \frac{4}{M}\sum_{i=1}^{M/4} Q\left[\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\frac{(2i-1)\pi}{M}\right]. \qquad (5)$$

By substituting (5) in (3) we get unified approximation for BER of coherent M-PSK modulation for random values of M which is represented as follows:

$$P_{e,M-PSK} \approx \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max\left(\frac{M}{4},1\right)} Q\left[\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\frac{(2i-1)\pi}{M}\right] \qquad (6)$$

This formula could be regarded as approximation for M-PSK signals. For the particular case when *M>4*, *E_b/N_0>>1*, the first member (i=1) will dominate in BER. Then:

$$P_{e,M-PSK} \cong \frac{2}{\log_2 M} Q\left[\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\frac{\pi}{M}\right], \quad for\ M \geq 4 \qquad (7)$$

Block Schematic for computer simulation

The above theoretical models have been examined by means of real time simulation. The complexity of the problem and the application of mathematical statistics implies an extensive use of numeric methods for computer analysis. Due to the great variety of types of manipulations and the identical common structure of digital signals transmission systems, it is deemed expedient to develop a whole program application for real time behavior simulation of these systems. [3].

To facilitate work with random complexity structure systems a program language has been used instead of system and circuit level simulators (such as Electronics Workbench or Simulink of MATLAB). In the latter case the block schematics could find some applicability for the simplest types of digital manipulations, low accuracy, poor legibility of results and most of all absolutely low flexibility (no practical possibility for a change of the type of manipulation).

The following generalized block schematic is proposed which served as a base for the development of the computer simulation system; block designations in Fig. 3 are as follows: FSA –frequency standard assembly; RNG – random number generator; DM – digital manipulator; CDD – coherent digital detector; T – tester; NLA – noise level amplifier; CU –control unit; NG –noise generator.

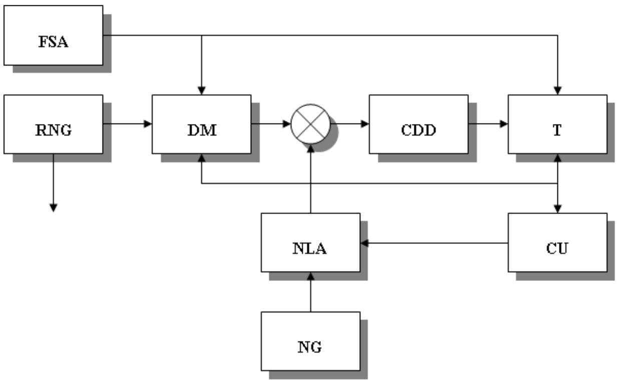


Fig. 3. Generalized block schematic of proposed system

Apart from the type of manipulation, the elements common for the structure are the signal source (data) RNG, frequency standard assembly FSA – stable frequency generator which ensures the reception of signals at the modulator output DM and synchronization of the coherent detector CDD. Tester T compares signals transmitted by RNG with those which have been detected by CDD. Error probability data is recorded in the control unit CU. The latter not only reads the result from tester's comparison but also controls the signal amplitudes (in M) and the noise level by means of NLA. In this way various signal to noise ratios are assigned.The noise added to the output signal is generated by means of NG. The blocks designated by an arrow allow the possibility for tuning, that is, a change of the type of manipulation and the type of noise (a change in the utilized subroutine – function in the computer simulation application).

Because of symmetry considerations the computer investigation is done for just one signal: the first quadrant from the diagram in Fig.1, namely for 4-PSK. For the purpose of theoretical determination of BER, formula 6 for M=4 is to be used. Members in the row which are of higher order are neglected. They have considerably less impact especially in counting the system's At the start an initialization is performed in the program; the kind of modulation is assigned as well as the type of the system employed. Following that we assign the values of the specific parameters of the investigated problem. At the same time the simulation of the real physical system is run through its computer model thereby calculating the theoretical expectation for system's BER.

During simulation the Monte Carlo method is applied. It concerns the generation of sequences of random numbers with definite probable distribution and consequent statistic processing of the result. This method is very powerful and finds a wide range of scientific and engineering applications. Owing to its effectiveness it is possible to obtain high accuracy results even for very complex systems. The basic requirements to the simulation medium are the necessity of using standard generators of pseudo-random numbers with assignment according to the law of probable distribution on one hand and the availability of procedures for processing of large data arrays on the other. Results are presented both numerically and graphically. After the comparison of both experimental and theoretical results it is possible to get the estimation of the deviation.

Program medium for determining error probability in using digital modulations–BER Calculator

The program used for computer simulation has been developed on Delphi which is similar to MATLAB; however, it has no strictly defined scientific orientation. On the other hand this language is compiled completely which is why performing a large number of calculations (needed in achieving the assigned accuracy in the statistical analysis) does not pose a problem concerning the time necessary for calculations. The program uses individual user's interface (Fig. 4), however, for the purpose of compatibility with MATLAB numeric results are written in a data file (BER.dat).

Fig.4 presents the main window of a BER Calculator – module of Delphi. This program is implemented with the purpose of demonstrating the principle of building up a larger program application which allows complete computer analysis of the information system in use and uses a large data base of structures (designs) and parameters. Nevertheless, the presented program allows for drawing up of analysis of systems' performances based on M-PSK and M-QAM manipulations.

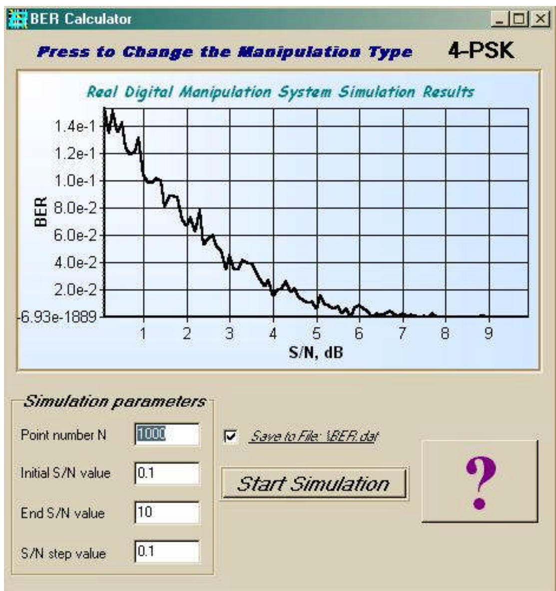


Fig. 4. User's graphic interface of BER Calculator

By default 4-PSK is set in the module (currently selected type of manipulation is written in the upper right hand corner of the window). Both type and multiplicity of the modulation are assigned in a separate window after pressing button "Press to Change the Manipulation Type":

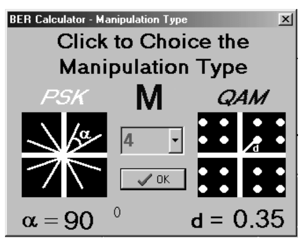


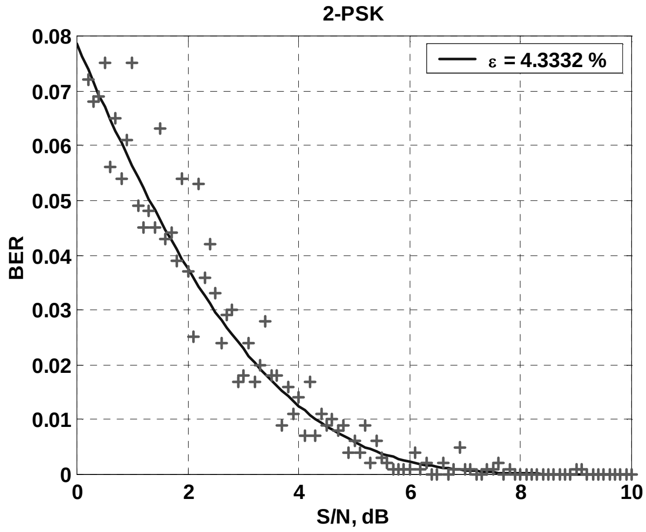
Fig. 5. Window of BER Calculator for selection of type and multiplicity of manipulation

Selected manipulation is displayed by way of illumination. The figure shows the general appearance of the signal vector space and the characteristic parameter of the selected manipulation is written. Multiplicity M is assigned through shortlisting (selection from a list of numbers – power rates of 2) as parameters α and d. Are automatically re-calculated. The rest of the data is assigned on the main window at the time of initialization; count of random numbers employed in the Monte Carlo method (in this case N=1000), initial/end speed and a step of change of the signal to noise ratio (by default S/N = 0.1 : 0.1 : 10). BER=BER(S/N), graphic dependency is drawn by the program and the numeric outcome is written in a table in BER.dat file (this function can be overruled with the note *Save to file: \BER.dat*). The latter is used by the second program module. In the core of the MATLAB module are the subroutines for numeric integration of elliptical integrals from the analytical formulae – gaus (), kramp () etc. [7]. As a result there is a graphic representation of theoretically estimated BER (thick line) and the one obtained during the computer experiment (points are designated).If necessary. If needed, there could made an additional processing of random complexity plus an analysis of the results.

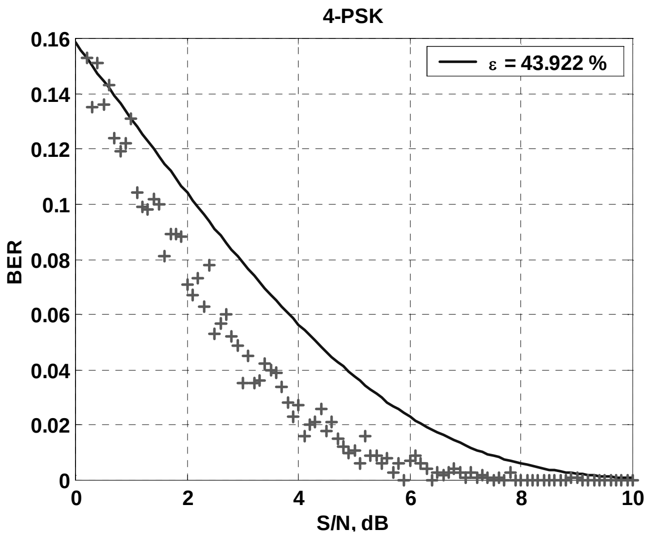
Results from the computer experiment and comparative analysis of outcomes for BER in using PSK

Some digital systems for signal transmission, which have been observed in the previous sections, are investigated here. Apart from the check of the theory results, which is done through critical comparative analysis, an investigation of the behavior of some types of digital systems for signal transmission is also carried out by means of mathematical models. Maximum relative error between theory and computer results (ε) is used as evaluation criterion for comparative analysis.

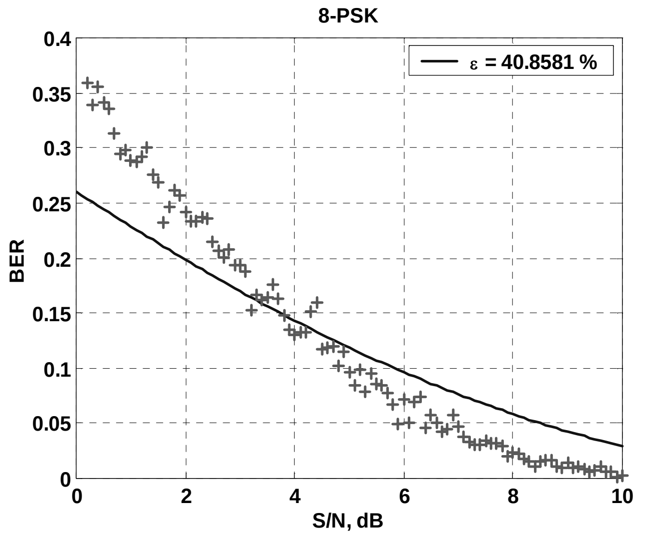
This is the simplest kind of digital manipulation. For the purpose of symmetry the computer investigation is done for one signal that is the first quadrant of the "star diagram". For theoretical determination of BER formula (6) is used. The members from the row which are of higher order are usually neglected. They have considerably lesser impact especially in reading the system's symmetry. Experiments show that practically acceptable approximation is the use of (7) or the calculation of the row up to the second member. Fig.6 shows the results of computer simulation for PSK with multiplicity 2÷16. Evidently the error probability intensifies quickly along with the increment of the multiplicity of manipulation M. The underlying reason is the shortened distance between signals - vectors of various words (i.e decrease of angle α). The graphics in Fig. 6,a and Fig. 6,b present the dependence of BER on signal to noise ratio for PSK with multiplicity M=2 and M=4 match those for QAM with multiplicity M=2 and M=4. This peculiarity is clarified in the previous sections of this work. For multiplicity of manipulation 2 and 4 PSK matches QAM.



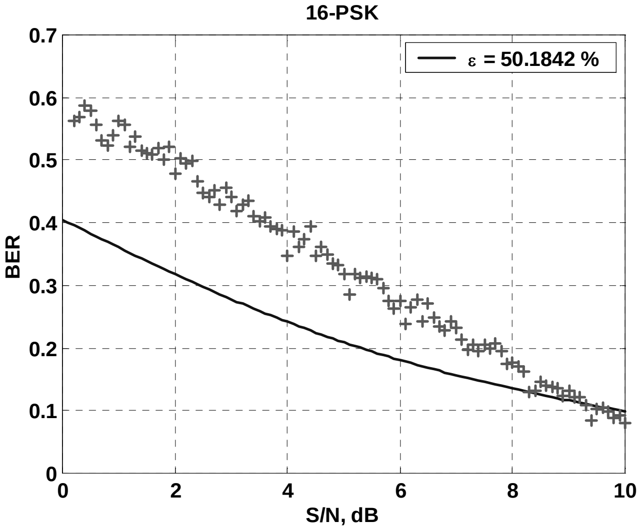
a)



b)



c)



d)

Fig. 6. Comparative analysis of theoretically and experimentally determined result for BER in case of using PSK

It is seen from the graphics that the dependency of BER on the signal to noise ratio in practice completely matches the one determined by (6). The slight offset along the vertical axis is due to the approximations used in obtaining analytical expression (6), and for M>8 as well as to the neglect of the last few members in the row (6).

The reasons for the comparatively large difference between theoretical and experimental results are:

- Allowances have been made while analytical expressions have been obtained;

- The reason for the kind of the observed result is in the statistical character of the employed numeric method for analysis which is distinguished by heightened accuracy when the number of members within the sample is larger. In this particular case a small quantity of random numbers is used in the Monte Carlo method (N=1000) which extremely insufficient for this particular method. Monte Carlo accuracy is good for very large values of N (in theory $\varepsilon \rightarrow 0$ with $N \rightarrow \infty$) [6];

Conclusions

Deviation between theoretically determined value of BER and that of the real simulation is evidently within the permissible limits for engineering calculations. This allows to use (7) in the methodology of designing systems which are based on that particular type of digital manipulation and ensures high reliability in signal transmission.

It is evident from the graph that the dependency of BER on the signal to noise ratio matches in practice the one determined by (7). The slight offset along the vertical axis is due to the neglect of the rest of the members from the row in the “exact formula” (6).

Investigations show that it is expedient to apply formula (6) up to the second member in the row.

Similar results are obtained for PSK, QAM and hierarchical QAM with random M as the program takes into account the respective boundaries of the change in the de-phasing angle (variable Fi in the program), and for QAM and the amplitude of the received signal (vectors norm s). In the various studied cases the error in comparing theoretical and experimental results is $\varepsilon \cong 10\%$, and it goes down with the increase of the quantity of the pseudo-random numbers used in the Monte Carlo method. The reason for this type of observed result is the statistical character of the employed numerical method for analysis which is distinguished by heightened accuracy as the number of members within the sample increases.

Acknowledgement

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A profile is drawn for the various types of square modulations, which are used in transmitting digital signals in the reverse channel of cable TV networks (CATV). The latter are attacked by disturbances, which deteriorate the quality of transmitted signals and cause errors in the digital flow. 32, 64, 128-QAM are also analyzed. Signal to noise ratio is calculated as a function of error probability. Based on the obtained results, it is possible to establish the dependence of the cable length sector on the transmission speed featured by different types of cables provided we have an allowable error coefficient. Simulation and experimental results are compared. Ill. 6, bibl. 12 (in English; summaries in English, Russian and Lithuanian).

С. Садинов, К. Койчев, П. Пенчев, К. Ангелов. Моделирование оценки BER характеристик M-PSK и M-QAM модуляций, используемых в обратном канале сети кабельного телевидения // Электроника и электротехника. – Каунас: Технология, 2009. – № 7(95). – С. 71–76.

Описываются профили различных типов квадратных модуляций, которые используются при передаче цифрового сигнала в обратном канале сети кабельного телевидения (CATV). Такие модуляции ухудшают качество передаваемых сигналов и являются причинами ошибок в цифровом потоке. Отношение сигнал/шум определяется как функция вероятности ошибок. Основываясь на полученных результатах, установлена зависимость скорости передачи от длины сектора кабеля. Представлены результаты экспериментов. Ил. 6, библ. 12 (на английском языке; рефераты на английском, русском и литовском яз.).

S. Sadinov, K. Koichev, P. Penchev, K. Angelov. Kabelinės televizijos tinkluose naudojamų M-PSK ir M-QAM moduliacijų BER charakteristikų imitavimo įvertinimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 7(95). – P. 71–76.

Pateikti įvairių tipų kvadratūrinių moduliacijų, naudojamų skaitmeniniams signalams perduoti atgaliniuose kabelinės televizijos tinkluose (CATV), profiliai. Įvairūs trukdžiai blogina kanalais perduodamų signalų kokybę, todėl skaitmeniniame sraute atsiranda klaidų. Analizuotos 32, 64 ir 128 QAM tipų moduliacijos. Signalo ir triukšmo santykis skaičiuotas pagal klaidos tikimybės funkciją. Remiantis gautais rezultatais galima nustatyti kabelio segmento ilgio ir perdavimo spartos priklausomybę skirtingų tipų kabeliams, kai žinomas leistinas klaidų koeficientas. Palyginami imitavimo ir eksperimentiniai rezultatai. Il. 6, bibl. 12 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).