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# Estimation of Winding Factors of Two-layer Three-phase Fractional Windings

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### Introduction

Two-layer former three-phase windings are most widely used in the alternating-current electrical machines in the present time. These windings have some certain advantages compare to three-phase windings of other types. Their winding span $y$ may be shortened by required amount. Two-layer former three-phase windings can be also wound under integer number or some certain defined fractional number of poles and slots $q$. These windings are manufactured as distributed type ($q$ > 1), furthermore their span is shortened ( $y$ < $\tau$); here $y$ – winding span, $\tau$ – pole pitch. Therefore factors of these windings $k_{w\nu}$ are equal to the product of winding distribution and pitch factors ($k_{w\nu} = k_{p\nu}\, k_{y\nu}$); here $k_{p\nu}$ – winding distribution factor, $k_{y\nu}$ – pitch factor, $\nu$ – number of harmonics. Factors of two-layer former windings are typically calculated using analytical expressions [1–3]. There are some reservations concerning exact estimation of winding factors of two-layer three-phase fractional windings by using these equations. It is stated in [5] that „the analytical expression can not be used for calculation of winding factors of two-layer three-phase fractional windings, since section numbers forming section groups of these windings are not equal to the number of pole and phase slots $q$". But this proposition is unproved and it will be accomplished in this paper.

### Object of research

In this work two-layer three-phase fractional winding is investigated. The parameters of it are as follows: number of phases $m$ = 3; number of slots $Z$ = 9; number of poles $2p$ = 2; number of pole and phase slots $q$ = 3/2; pole pitch $\tau$ = 4,5; winding span $y$ = 4; slot span expressed in electrical degrees $\alpha = 40^{\circ}$. Any two adjacent coil sections of this winding consist of three coils, thus the number of coils in adjacent sections is 2 and 1, or 1 and 2.

Distribution of elements of this winding is presented in Table 1.

**Table 1.** Distribution of elements of two-pole two-layer three-phase winding with $q$ = 3/2

| Phase alteration | | U1 | W2 | V1 | U2 | W1 | V2 |
|---|---|---|---|---|---|---|---|
| Number of coils in a section | | 1 | 2 | 1 | 2 | 1 | 2 |
| Slot No. | $Z$ | 1 | 2; 3 | 4 | 5; 6 | 7 | 8; 9 |
| | $Z'$ | 5 | 6; 7 | 8 | 9; 1 | 2 | 3; 4 |

Electrical circuit layout for the considered two-layer three-phase fractional winding is charted according to the data from Table 1 (Fig. 1, a).

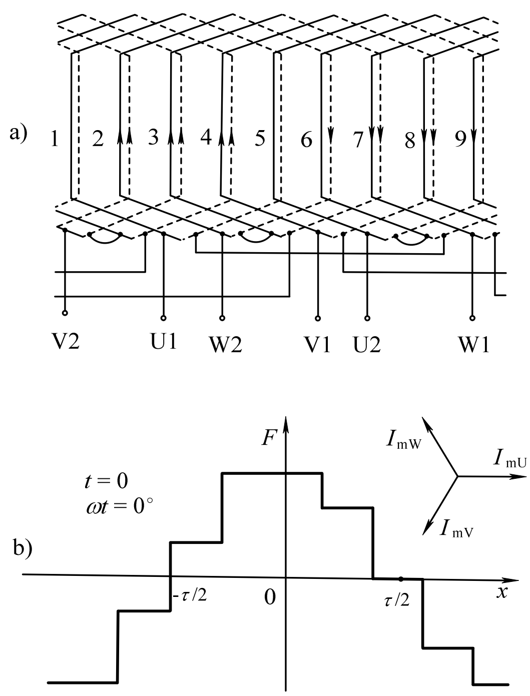


**Fig. 1.** Electrical circuit layout of the two-pole two-layer three-phase winding with $q$ = 3/2 (a) and distribution of rotating magnetomotive force in time moment $t$ = 0 (b)

Relative magnitude of effective conductors available at the slots of analysed two-layer three-phase fractional winding is related to the single-layer concentrated three-phase winding by $N^*_{2_g} = N^*_1 / q = 1/3/2 = 0{,}667$. Relative magnitude of the number of single coil turns $N^*_2 = N^*_{2_g} / 2 = 0{,}667/2 = 0{,}333$. Relative magnitudes of instantaneous current values of this winding in time moment $t$ = 0: $i^*_{\mathrm{mU}} = 0$; $i^*_{\mathrm{mV}} = -0{,}866$; $i^*_{\mathrm{mW}} = 0{,}866$. Conditional changes of magnetomotive force $\Delta F$ in slots in time moment $t$ = 0 are calculated by applying the electrical circuit layout of the considered winding (Fig. 1, a) and using estimated number of coil turns and relative current magnitudes (Table 2).

**Table 2.** Conditional changes of magnetomotive force in slots of analyzed three-phase winding in time moment $t = 0$

| Slot No. | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\Delta F$ | 0 | 0,577 | 0,577 | 0,577 |

**Table 2 (continued)**

| Slot No. | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| $\Delta F$ | 0 | –0,289 | –0,577 | –0,577 | –0,289 |

Spatial distribution of rotating magnetomotive force in the defined moment of time was formed according to results presented in Table 2 (Fig. 1, b).

## Research method

Winding factors of two-layer former three-phase winding are calculated using analytical expression of winding factors given in [1–3] and which evaluates winding distribution and shortening of its span:

$$k_{\mathrm{w}_2 \nu'} = k_{\mathrm{p}\nu'}\, k_{\mathrm{y}\nu'} = \\ = \sin\left(\nu' \frac{\pi}{6}\right) \sin\left(\nu' \frac{\pi\, y}{6q}\right) / \left[ q \sin\left(\nu' \frac{\pi}{6q}\right)\right]. \qquad (1)$$

This expression will be used to determine the factors of the analyzed fractional winding also. Since winding factors estimate not only the reduction of the rotating magnetomotive forces and harmonics of internal voltages induced by them, but also reflect the relative magnitudes of amplitude values of harmonics of these variable quantities in respect of amplitude values of the same corresponding harmonics of the concentrated three-phase winding, for this reason winding factors of the odd harmonics of the analyzed winding calculated according to expression (1) will be verified using formula (2):

$$k_{\mathrm{w}_2 \nu'} = F_{2\nu'} / F_{1\nu'}; \qquad (2)$$

here $F_{2\nu'}$ – conditional amplitude value of the rotating magnetomotive force of the $\nu'$-th harmonics of the two-layer tree-phase fractional winding with $q$ = 3/2; $F_{1\nu'}$ – conditional amplitude value of the rotating magnetomotive force of the $\nu'$-th harmonics of the concentrated three-phase winding.

On the basis of Fig. 1, b, the amplitude value of the rotating magnetomotive force of the $\nu$-th harmonics $F_{2\nu}$ is calculated according to these analytical expressions [4]:

$$F_{\nu'} = \frac{4}{\pi\, \nu'} \sum_{i=1}^{k} F_{\mathrm{m}i} \sin\left(\nu' \beta_i / 2\right) \cos\left(\nu' \gamma_i\right); \qquad (3)$$

$$F_{\nu''} = \frac{4}{\pi\, \nu''} \sum_{i=1}^{k} F_{\mathrm{m}i} \sin\left(\nu'' \beta_i / 2\right) \sin\left(\nu''' \gamma_i\right); \qquad (4)$$

here $k$ – number of rectangles forming the half-period of the rotating magnetomotive force; $F_{\mathrm{m}\,i}$ – height of the $i$-th rectangle of the half-period of the stair-shaped magnetomotive force; $\beta_i$ – width of the $i$-th rectangle of the stair-shaped magnetomotive force, expressed in electrical degrees of the fundamental space harmonic; $\gamma_i$ – asymmetry of the $i$-th rectangle of the half-period of the stair-shaped magnetomotive force in respect of the reference axis expressed in electrical degrees of the fundamental space harmonics; $\nu'$ – sequence number of the odd harmonics; $\nu''$ – sequence number of the even harmonics.

Results of the harmonic analysis of the instantaneous periodic function of the rotating magnetomotive force of the concentrated three-phase winding will be taken from [4].

If winding factors of the odd harmonics calculated by expressions (1) and (2) do not match it will be possible to assume that expression (1) can not be used to determine the winding factors of the two-layer three-phase fractional windings. Then a question would arise how to calculate winding factors of even harmonics of the particular analyzed windings, since only winding factors of odd harmonics can be determined using expression (2)? It is offered to solve this problem in the following way. It is obtained from the results of harmonic analysis of the periodic function of instantaneous rotating magnetomotive force of the two-layer fractional three-phase winding with $q$ =1/2, that the relative magnitudes $f_{3\nu}$ of the rotating magnetomotive force of odd harmonics of this winding are equal to the relative magnitudes $f_{1\nu}$ of the respective odd harmonics of the rotating magnetomotive force of the concentrated three-phase winding $f_{1\nu}$ [4]:

$$f_{1\nu'} = F_{1\nu'} / F_{11} = F_{3\nu'} / F_{31} = f_{3\nu'}; \qquad (5)$$

here $F_{1\nu'}$ and $F_{11}$ – conditional amplitude values of $\nu'$-th and fundamental harmonics of rotating magnetomotive force of the concentrated three-phase winding; $F_{3\nu'}$ and $F_{31}$ – conditional amplitude values of $\nu'$-th and fundamental harmonics of rotating magnetomotive force of the two-layer three-phase fractional winding with $q$ = 1/2.

From equation (5) we have:

$$F_{1\nu'} = F_{11}\, F_{3\nu'} / F_{31} = F_{11}\, f_{3\nu'}. \qquad (6)$$

Expression (6) is substituted into (2) and the obtained equation is multiplied and divided by the conditional amplitude value of the fundamental harmonic of the rotating magnetomotive force $F_{21}$ of the two-layer three-phase fractional winding with $q$ = 3/2:

$$k_{w_2\nu} = \frac{F_{21}}{F_{11}} \frac{F_{2\nu}}{F_{21}} \frac{1}{f_{3\nu}} = k_{w_2 1} \frac{f_{2\nu}}{f_{3\nu}}; \qquad (7)$$

here $f_{2\nu}$ – relative magnitudes of the odd and even harmonics of rotating magnetomotive force of the two-layer three-phase fractional winding with $q = 3/2$ .

Winding factors of not only odd but also even harmonics will be calculated according to the obtained expression (7).

**Research results**

Winding factors of the analyzed two-layer three-phase fractional winding calculated according to analytical expression (1) of winding factors are presented in Table 3.

**Table 3.** Winding factors of odd harmonics of the two-layer three-phase fractional winding with $q = 3/2$ and $y = 4$, calculated according to expression (1)

| $\nu$ | 1 | 5 | 7 | 11 | 13 | 17 | 19 |
|---|---|---|---|---|---|---|---|
| $k_{w_2\nu}$ | 0,960 | 0,218 | 0,1774 | 0,1774 | 0,218 | 0,960 | 0,960 |

Harmonic analysis of instantaneous rotating magnetomotive force function (Fig. 1, b) of the two-layer three-phase fractional winding (Fig. 1, a) was accomplished using expressions (3) and (4) and by applying the estimated parameters of half-period of the rotating magnetomotive force ($k$ = 3; $F_{1s} = 0{,}289$; $F_{2s} = 0{,}289$; $F_{3s} = 0{,}289$; $\beta_1 = 160^\circ$; $\beta_2 = 120^\circ$; $\beta_3 = 80^\circ$; $\gamma_1 = 10^\circ$; $\gamma_2 = -10^\circ$; $\gamma_3 = 10^\circ$) (Table 4).

**Table 4.** The results of harmonic analysis of instantaneous rotating magnetomotive force function of the two-layer three-phase fractional winding with $q = 3/2$ and $y = 4$

| $\nu$ | 1 | 2 | 4 | 5 | 7 | 8 |
|---|---|---|---|---|---|---|
| $F_{2\nu}$ | 0,9036 | 0,0290 | 0,0334 | -0,0267 | -0.0083 | -0,1130 |
| $f_{2\nu}$ | 1 | 0,0321 | 0,0370 | 0,0296 | 0,0092 | 0,1250 |

**Table 4 (continued)**

| 10 | 11 | 13 | 14 | 16 | 17 | 19 |
|---|---|---|---|---|---|---|
| 0,0904 | -0,0053 | -0,0103 | -0,0095 | -0,0036 | 0,0532 | -0,0476 |
| 0,10 | 0,0058 | 0,0114 | 0,0106 | 0,0040 | 0,0588 | 0,0526 |

In this table $\nu$ – number of the harmonic; $F_{2\nu}$ – conditional amplitude value of the $\nu$-th harmonic of the rotating magnetomotive force; $f_{2\nu}$ – relative magnitude of the $\nu$-th harmonic of the rotating magnetomotive force $\left(f_{2\nu} = F_{2\nu} / F_{21}\right)$.

Results of harmonic analysis of instantaneous periodic rotating magnetomotive force function of the concentrated three-phase winding are given in Table 5 [4].

**Table 5.** The results of harmonic analysis of instantaneous rotating magnetomotive force function of the concentrated three-phase winding

| $\nu$ | 1 | 5 | 7 | 11 | 13 | 17 | 19 |
|---|---|---|---|---|---|---|---|
| $F_{1\nu}$ | 0,955 | – 0,1910 | 0,1364 | – 0,0868 | 0,0735 | – 0,0562 | 0,0503 |
| $f_{1\nu}$ | 1 | 0,20 | 0,1429 | 0,0909 | 0,0769 | 0,0588 | 0,0526 |

Winding factors of odd harmonics given in Table 6 are calculated using results from Tables 4 and 5 and by applying equation (2).

**Table 6.** Winding factors of odd harmonics of the two-layer three-phase fractional winding with $q = 3/2$ and $y = 4$ calculated according to expression (2)

| $\nu$ | 1 | 5 | 7 | 11 | 13 | 17 | 19 |
|---|---|---|---|---|---|---|---|
| $k_{w_2\nu}'$ | 0,946 | 0,1398 | 0,0607 | 0,0607 | 0,1398 | 0,946 | 0,946 |

It is obvious that winding factors of odd harmonics given in Table 6 do not coincide with these factors given in Table 3. This means that the assumption that the analytical expression (1) can not be used to estimate winding factors of the two-layer three-phase fractional windings.

In order to determine winding factors of the considered three-phase winding not only for odd harmonics but also for even harmonics, results of the harmonic analysis of instantaneous rotating magnetomotive force function of the two-layer three-phase fractional winding with $q = 1/2$ [4, 5] are used (Table 7).

**Table 7.** The results of harmonic analysis of instantaneous rotating magnetomotive force function of the two-layer three-phase fractional winding with $q = 1/2$

| $\nu$ | 1 | 2 | 4 | 5 | 7 | 8 |
|---|---|---|---|---|---|---|
| $F_{3\nu}$ | 0,827 | -0,4135 | 0,2067 | 0,1654 | -0,1181 | 0,1034 |
| $f_{3\nu}$ | 1 | 0,50 | 0,250 | 0,20 | 0,1429 | 0,125 |

**Table 7 (continued)**

| 10 | 11 | 13 | 14 | 16 | 17 | 19 |
|---|---|---|---|---|---|---|
| -0,0827 | -0,0752 | 0,0636 | -0,0591 | 0,0517 | 0,0486 | -0,0436 |
| 0,10 | 0,0909 | 0,0769 | 0,0714 | 0,0625 | 0,0588 | 0,0526 |

Then winding factors of all harmonics are calculated from the expression (7) using results presented in Tables 4 and 7. These factors are shown in Table 8.

**Table 8.** Winding factors of the two-layer three-phase fractional winding with $q = 3/2$; $y = 4$, calculated according to expression (7)

| $\nu$ | 1 | 2 | 4 | 5 | 7 | 8 |
|---|---|---|---|---|---|---|
| $k_{2\nu}$ | 0,946 | 0,0607 | 0,140 | 0,140 | 0,0606 | 0,946 |

**Table 8 (continued)**

| 10 | 11 | 13 | 14 | 16 | 17 | 19 |
|---|---|---|---|---|---|---|
| 0,946 | 0,0606 | 0,140 | 0,140 | 0,0602 | 0,943 | 0,940 |

Winding factors of the odd harmonics (Table 8) calculated by expression (7) (Table 8) are basically equal to the respective winding factors of the odd harmonics obtained from the expression (2) (Table 6).

Electromagnetic efficiency factors $k_{ef}$ of the three-phase windings were calculated using the results of harmonic analysis from the Tables 4, 5 and 7 [4]. For the analyzed two-layer former three-phase fractional winding $k_{ef_2} = 0{,}7958$; for the concentrated three-phase winding $k_{ef_1} = 0{,}6987$. For the two-layer former three-phase fractional winding with $q = 1/2$, $k_{ef_3} = 0{,}3284$.

## Conclusions

1. Two layer three-phase fractional windings induce rotating magnetic fields of odd and even harmonics except for multiples of three due to the asymmetry of half-periods of the periodic instantaneous rotating magnetomotive force functions in respect of their symmetry axes.

2. The analytical expression of winding factors of the two-layer former three-phase windings can not be applied for the two-layer three-phase fractional windings.

3. Winding factors of odd and even harmonics of the two-layer three-phase fractional windings can be estimated using the results of harmonic analysis of rotating magnetomotive force functions of these windings and of the winding with $q = 1/2$.

4. Winding factors of the magnetic circuit teeth harmonics ($\nu$ = 8; 10; 17; 19; ...) of the analyzed two-layer three-phase fractional winding are of the same magnitude (0,946) as the winding factor of the fundamental harmonic. Winding factors of other higher odd and even harmonics are considerably less (0.0607 and 0.140) than the winding factor of the fundamental harmonic.

5. Electromagnetic efficiency factor (0.7958) of the analyzed two-layer three-phase fractional winding is considerably greater than of the concentrated three-phase winding (0.6987) and exceeds the electromagnetic efficiency factor of the two-layer former three-phase fractional winding with $q = 1/2$ (0.3284) more than twice.

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**J. Bukšnaitis. Estimation of Winding Factors of Two-layer Three-phase Fractional Windings // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 8(96). – P. 23–26.**

Estimation method of winding factors of the two-layer three-phase fractional windings is analyzed. It was proved by theoretical calculations that the analytical expression dedicated for the calculation of winding factors of the two-layer former three-phase windings can not be applied to calculate these factors of the odd and even harmonics of the two-layer former three-phase fractional windings. It can be explained by the fact that the numbers of the three-phase fractional winding coils in the sections are not equal and they do not match the number of the pole and phase slots. Since the single-layer concentrated and the two-layer former fractional ($q$ = 1/2) three-phase windings are similar because the relative magnitudes of the respective $\nu$-th odd harmonics of their rotating magnetomotive forces are equal, thus the winding factors for all harmonics of two-layer three-phase fractional windings can be estimated only on the basis of the results of harmonic analysis of instantaneous rotating magnetomotive force function of the considered windings and two-layer former fractional ($q$ = 1/2) three-phase winding. Ill. 1, bibl. 5, tabl. 8 (in English; abstracts in English, Russian and Lithuanian).

**Ю. Букшнайтис. Определение обмоточных коэффициентов двухслойных трёхфазных обмоток с дробным числом пазов на полюс и фазу // Электроника и электротехника. – Каунас: Технология, 2009. – № 8(96). – С. 23–26.**

Рассматривается метод определения обмоточных коэффициентов двухслойных трёхфазных обмоток с дробным числом пазов на полюс и фазу. Теоретическими расчётами доказано, что обмоточные коэффициенты нечётных и чётных гармоник двухслойных трёхфазных обмоток с дробным числом пазов на полюс и фазу при использовании аналитических выражений, назначенных для определения этих коэффициентов двухслойных трёхфазных обмоток, определить невозможно. Это можно объяснить тем, что числа секций в их группах являются неодинаковыми. Кроме того, они несоответствуют числам пазов на полюс и фазу. Так как однослойная сосредоточенная и двухслойная шаблонная с дробным числом пазов на полюс и фазу ($q$ = 1/2) трёхфазные обмотки, ввиду того что их относительные величины соответствующих нечётных гармоник вращающихся магнитодвижущих сил являются равными, поэтому обмоточные коэффициенты нечётных и чётных гармоник рассматриваемых трёхфазных обмоток возможно определить на основе резулътатов гармонического анализа мгновенных функций вращающейся магнитодвижущей силы этих обмоток и обмотки с дробным числом пазов на полюс и фазу ($q$ = 1/2). Ил. 1, библ. 5, табл. 8 (на английском языке; рефераты на английском, русском и литовском яз.).

**J. Bukšnaitis. Dvisluoksnių trifazių trupmeninių apvijų koeficientų nustatymas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 8(96). – P. 23–26.**

Nagrinėjamas dvisluoksnių trifazių trupmeninių apvijų apvijos koeficientų nustatymo metodas. Teoriniais skaičiavimais įrodyta, kad nelyginių ir lyginių harmonikų dvisluoksnių forminių trifazių trupmeninių apvijų koeficientams apskaičiuoti negalima taikyti analizinės išraiškos, skirtos dvisluoksnių forminių trifazių apvijų šiems koeficientams nustatyti. Tai galima paaiškinti tuo, kad trifazių trupmeninių apvijų ričių skaičiai sekcijose yra nevienodi, taip pat jie neatitinka poliaus ir fazės griovelių skaičiaus. Kadangi viensluoksnė sutelktoji ir dvisluoksnė forminė trupmeninė ($q$ = 1/2) trifazės apvijos yra artimos, nes jų sukamųjų magnetovarų atitinkamų $\nu$-ųjų nelyginių harmonikų santykiniai dydžiai yra vienodi, todėl dvisluoksnių trifazių trupmeninių apvijų koeficientus visoms harmonikoms galima nustatyti tik remiantis nagrinėjamų apvijų ir dvisluoksnės forminės trupmeninės ($q$ = 1/2) trifazės apvijos sukamosios magnetovaros akimirkinių funkcijų harmoninės analizės rezultatais. Il. 1, bibl. 5, lent. 8 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).