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An Efficiency Optimization Model for 802.11 Wireless Communication Channels

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The basic model

The communication channel efficiency is mainly reflected by the link utilization factor. This parameter is defined as the ratio between the utilization time for sending the payload and the total time while the channel is busy. In terms of transferred bits, a more appropriate parameter is the channel efficiency, defined as the ratio between the number of useful data bits (payload) and the total number of bits which are sent. If no errors occur, the efficiency is

$$Ef = \frac{L}{L+H}. \quad (1)$$

Knowing the channel data rate *R*, it is possible to calculate the channel throughput multiplying channel efficiency with the transmission rate R as in equation (2):

$$Th = R \cdot \frac{L}{L+H}. \quad (2)$$

The basic model is considering a communication channel with no FEC (Frame Error Check) function. Some packets are not error free and they are considered lost packets. Starting from here, simple communication models can be developed [7]. From this point of view, we can calculate the efficiency, Ef_0, if we do know the error probability for a given channel. Depending on the modulation technique and the channel performance, BER - Bit Error Rate, *p* in notations below, is usually considered [4, 5].

At the PHY level a data packet contains the payload (user data, *L* bytes length) and different overheads: high level protocol overheads, security overheads, PCLP overheads. All of them are considered to have a total length of *H* bytes.

If *p* is the bit error probability, then we can calculate the bit successful transmission probability as $s = 1 - p$. For a data packet with a length of $L+H$ bytes, we have a packet success probability *S* as in equation (3).

$$S = (1-p)^{8(L+H)}. \quad (3)$$

Now PER - Packet Error Rate (notation *P*) can be calculated:

$$P = 1-(1-p)^{8(L+H)}. \quad (4)$$

In Fig. 1 is shown the data packages stream, each packet having a payload and overheads. The shadowed packets have erroneous bits and they became useless, being considered lost packets. No recovering technique is considered at this stage.

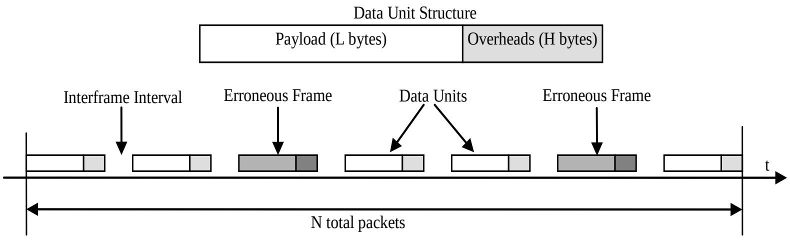

Fig. 1. Data packets flow with packet loss and no retransmissions

Knowing that each packet is carrying *8L* useful bits, we can easy determine the total number of the payload successfully sent bits, *S x N x 8L*, and the total number of the transmitted bits *N x 8(L+H)*. Supposing that *N* packets are passing through the channel in a unit of time and if part of them are successfully received, by dividing the amount of successfully received bits to the total number of the transmitted bits included into N packets, the channel efficiency, with no retransmissions, Ef_0 , is (5):

$$Ef_0 = \frac{N \cdot 8L \cdot (1-p)^{8(L+H)}}{N \cdot 8(L+H)} = \frac{L}{L+H} \cdot (1-p)^{8(L+H)} . \quad (5)$$

For ideal channel with no errors, $p = 0$, and the efficiency formula became as in equation (1).

For this simple model the maximum efficiency can be classically calculated as extreme point of the function which occurs when the derived is zero:

$$\frac{dEf}{dL} = 0 . \quad (6)$$

The optimum payload length L_{opt} is therefore:

$$L_{opt} = \frac{-2H\log(1-p) + \sqrt{2}\sqrt{-H\log(1-p) + 2H^2\log(1-p)^2}}{4\log(1-p)} . \quad (7)$$

Mathematically, there is also a secondary solution of equation (6), but with negative values, which can not be considered as a payload field length.

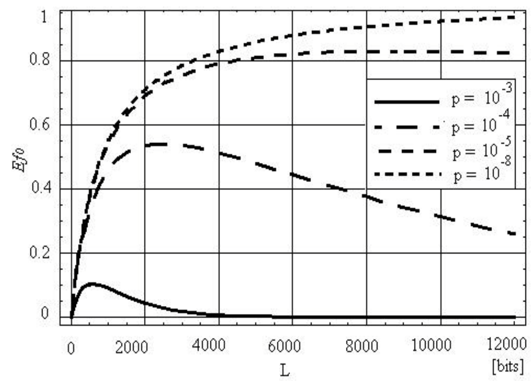

Fig. 2. Efficiency versus payload for different BER values and one retransmission

Based on obtained result, in Fig. 2 is shown the channel efficiency versus the payload length for different bit errors probabilities (BER). The considered values are from 0 to 1500 bytes for *L*, while 1500 bytes is the maximum length for Ethernet originated frames. *H* is 102 bytes in length for all protocol overheads and no security but 14 bytes for ACK back confirmation can also be added without major modification into the function shape and final result. ACK message is associated with every transmitted frame, is not directly carrying payload and is occupying the same transmission medium. From this point of view, ACK frames could be considered acting as the overheads in efficiency evaluation. As we can see in Fig. 2, the maximum point of the efficiency is migrating to the right (increase the payload) when the error probability decrease. Obviously, fewer errors allow longer frames and more errors needs shorter frames in order to capture less errors.

Starting from the formula for Ef_0, we can note that, even when no recovery retransmissions occur, the efficiency have a maximum for a certain length of the transmitted frame. Starting from equation (7), some optimal payload lengths for different bit error probabilities were computed and they are presented in Table 1.

Table 1. Optimal Payload Length

Bit Error Probability	Optimal Payload Length, L [bits]
10^{-2}	90
10^{-3}	583
10^{-4}	2477
10^{-5}	8634
10^{-8}	285249

Assuming that *H* can not be changed being imposed by the communication standard, we can work around the payload length *L*. A larger payload *L* means that a bigger number of bits belonging to the frame are changed (erroneous bits) and the probability of having a wrong received frame is increase accordingly. The efficiency goes to zero for large payloads, more quickly as the bit error probabilities are bigger. A second effect is expected when retransmissions are involved, pursuant to the fact that a longer frame means more retransmitted bits and needs longer retransmission time. This effect should induce a stronger dependency with the payload length in retransmission based communication model as we will see further. The optimization can also be done by forcing the use of compressed overheads as stated in IEEE 802.11n standard [8] or modifying the interframe intervals or the contention window CW [1], [3]. In terms of consumed time, these intervals have the same contribution to the transmission time budget as the overheads and, to some extent, and they can be treated on the same principle when time analysis (link utilization factor) is evaluated.

Retransmission based model

Frame error check procedure will generate a negative ACK confirmation to the transmitter when an erroneous packet arrives at the destination. Accordingly, the transmitter will retransmit that frame. Since part of the frames are recovered due to these retransmissions, we can evaluate now the channel efficiency considering that all *N* frames arrive correctly at the destination, but part of them are transmitted twice: a frame with errors and a successfully retransmitted frame. The new total number of the transmitted frames is bigger then the successfully only transmitted frames *N* with an amount equal with the number of wrong frames, *N x P*.

In Fig. 3 is suggested the transmission process with retransmitted frames due to negative ACK confirmations. The shadowed frames are travelling as they are (erroneous) and therefore two times retransmitted.

The channel efficiency for one retransmission, Ef_1 is:

$$Ef_1 = \frac{N \cdot 8L}{8 \cdot (L+H) \cdot \{N + N \cdot [1-(1-p)^{8(L+H)}]\}} =$$
$$= \frac{L}{L+H} \cdot \frac{1}{1+[1-(1-p)^{8(L+H)}]} = \frac{L}{L+H} \cdot \frac{1}{1+P} . \quad (8)$$

In Fig. 4 the variation of the channel efficiency versus the payload length *L* is presented, for different bit errors probabilities and one retransmission. As we can see, for a given BER value, the efficiency finally goes to a constant non-zero amount, when the payload L is increased over a certain limit. Obviously, a longer frame means more errors and more retransmissions, so less efficiency. The efficiency will never decrease to zero since the model is considering that all frames are finally arriving to the destination.

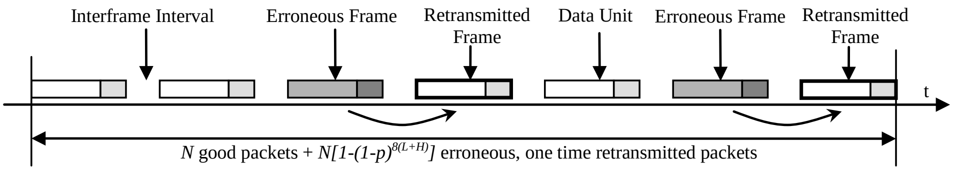

Fig. 3. Data packets flow with one retransmission for erroneous packets

Multiple retransmissions communication

When more then one retransmission is possible, we can extend the procedure described above. If N frames are finally correctly transmitted, then part of them arrives to be correctly delivered after one retransmission and other part of them are correctly received after two retransmissions. The frames which are suffering two retransmissions are part from the frames already retransmitted one time, specifically that part which was affected by errors during first retransmission. Considering that the transmission conditions are unchanged, it is possible to calculate the total number of transmitted frames, including those transmitted two times and three times and based on that the channel efficiency for 2 retransmissions can be calculated as follows:

$$Ef_2 = \frac{N \cdot 8L}{8 \cdot (L+H) \cdot [N + N \cdot P + N \cdot P \cdot P]} = \frac{L}{L+H} \cdot \frac{1}{1 + P + P^2}, \quad (9)$$
$$if \quad P = 1 - (1-p)^{8(L+H)}.$$

The equation is the ratio between the total number of payload bits included into N frames and the total number of travelling bits, including the retransmitted ones.

We can observe know the equation expanding rule for k retransmissions:

$$Ef_k = \frac{L}{L+H} \cdot \frac{1}{\sum_{i=0}^{k} [1-(1-p)^{8(L+H)}]^i}, \quad (10)$$
$$if \quad k \neq 0, e \neq 0.$$

We can recognize as denominator for the second ratio a geometric series:

$$\sum_{i=0}^{k} P^i = \begin{cases} \dfrac{1-P^{k+1}}{1-P}, & P \neq 1, \\ (k+1) \cdot P, & P = 1, \end{cases} \quad (11)$$

where

$$P = 1 - (1-p)^{8(L+H)}. \quad (12)$$

The variable P is the already used packet error rate and has the dimension of a probability, $0 \le P \le 1$. $P = 1$ is equivalent with no errors, $p = 0$.

For an infinite number of retransmissions, the series converge:

$$\lim_{k \to \infty} \sum_{i=0}^{k} P^i = \frac{1}{1-P} = \frac{1}{(1-p)^{8(L+H)}}. \quad (13)$$

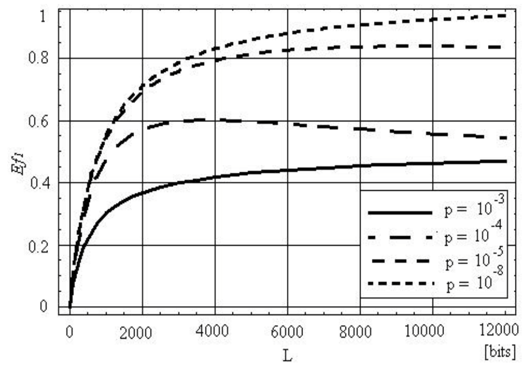

Fig. 4. Efficiency versus payload for different BER values and one retransmission

An infinite number of retransmissions is not a realistic situation to be considered for practical approaches.

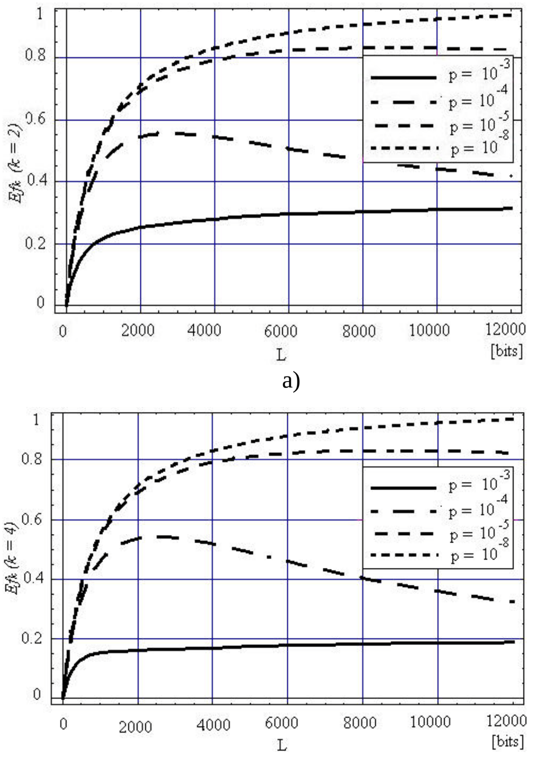

Fig. 5. Efficiency versus payload for different BER values, for channels with 2 (a) and 4 (b) retransmissions

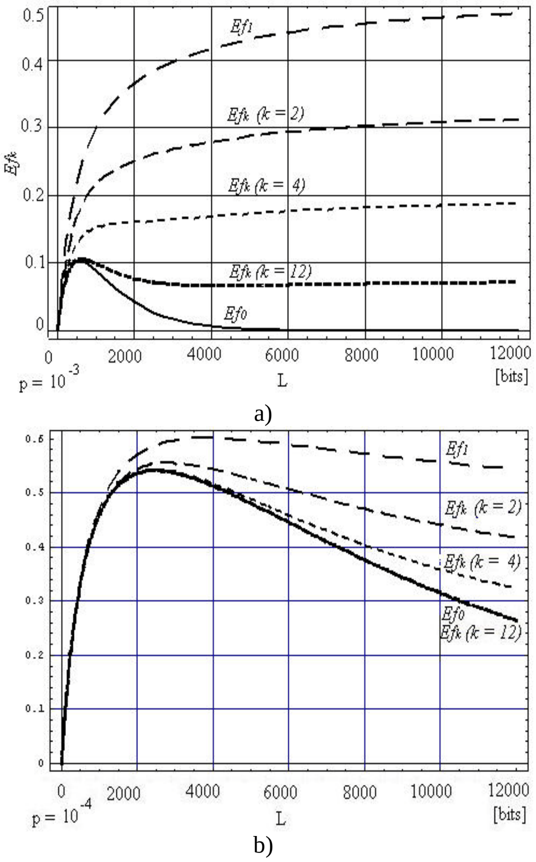

Fig. 6. Efficiency versus payload for different retransmissions number with constant BER (10^{-4})

The channel efficiency for k retransmissions is therefore:

$$Ef_k = \frac{L}{L+H}\cdot\frac{1-P}{1-P^{k+1}} = \frac{L}{L+H}\cdot\frac{1-[1-(1-p)^{8(L+H)}]}{1-[1-(1-p)^{8(L+H)}]^{k+1}} = \frac{L}{L+H}\cdot\frac{(1-p)^{8(L+H)}}{1-[1-(1-p)^{8(L+H)}]^{k+1}}, \quad (14)$$

$$if\ \ k \neq 0,\ p \neq 0.$$

The equation above (14) is describing how the efficiency evolve versus the payload L, the overheads H and a finite number of retransmissions, k.

In Fig. 5 we can see the maximum efficiency point migration for different errors rate, versus the payload.

When the number of retransmissions is $k = 1$, the general equation above becomes

$$Ef_{k=1} = \frac{L}{L+H}\cdot\frac{(1-p)^{8(L+H)}}{1-[1-(1-p)^{8(L+H)}]^2} = \frac{L}{L+H}\cdot\frac{1-[(1-p)^{8(L+H)}]}{[1-(1-p)^{8(L+H)}]\cdot[1+(1-p)^{8(L+H)}]} = \frac{L}{L+H}\cdot\frac{1}{2-(1-p)^{8(L+H)}} = \frac{L}{L+H}\cdot\frac{1}{1+P}; \quad (15)$$

which is exactly the form (8).

For $i = 0$, we have the case with no retransmissions and no loss, as in equation (1).

Based on the obtained equation, adaptive channel algorithms can be implemented. Depending on the bit error probability, the channel efficiency has a maximum value for a certain payload length, L_{opt}. Adaptive communications based on SNR can be developed considering predefined transmissions strategies [2], [6]. Bit error probability is usually indirectly evaluated starting from SNR [4], [6].

Fig. 6 shows how the efficiency of the transmission channel modify versus the payload amount L, for different number of retransmissions, k, at a two constant error probability rates, $p = 10^{-4}$ and $p = 10^{-5}$. The maximum of the efficiency is obtained for larger payloads only when the retransmissions number decrease because retransmitting long frames overloads the transmission. Smaller error probability rate means better efficiency, whatever the retransmissions number is.

For k=4 retransmissions, a mathematical evaluation of the optimum packet length, based on equation (14), is presented in the table 2, for different usual bit error rates.

Table 2. Optimal Payload Length for 4 retransmissions

Bit Error Probability	Optimal Payload Length, L [bits]
10^{-4}	2511
10^{-5}	8636
10^{-8}	285249

For a limited retransmissions number it is possible that some packets can not be recuperated after consuming all the permitted retransmissions and there still are loss packets after a number of retransmissions. Eliminating the frames which are still not recovered after the first k retransmissions, $N \times P^{k+1}$ (P is the packet error rate) by subtracting this amount from the total number of sent frames N and dividing to the total number of transmitted frames, including k retransmissions, we have

$$Ef_{k,loss} = \frac{L}{L+H}\cdot\frac{1-P^{k+1}}{\frac{1-P^{k+1}}{1-P}} = \frac{L}{L+H}\cdot(1-P) = \frac{L}{L+H}\cdot(1-p)^{8(L+H)}, \quad (16)$$

$$if \quad P = [1-(1-p)^{8(L+H)}], k \neq 0,\ p \neq 0.$$

Since P is the packet error rate, $1-P$ has the significance of packet success rate. As we can infer from the beginning, for this case the result is not any more related with the number of retransmissions. A number of data packets are always lost whatever the retransmission number is. The correct transmitted frames are only related with the packets success rate P. Any supplementary retransmission is diminishing the efficiency adding more transmitted bits and, at the same time, is improving the efficiency by recuperating erroneous frames using the retransmissions, but the two effects compensate each other.

For packet switched networks the throughput can be further calculated as $Th = R \times Ef_k$. Obviously, in the equation obtained for the efficiency, if $p = 0$, we arrive at the ideal channel ratio, $L/(L+H)$.

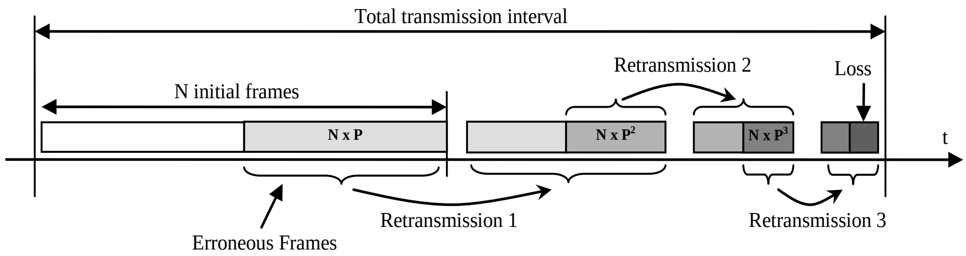

Fig. 7. Data packets flow with three retransmission and loss packets

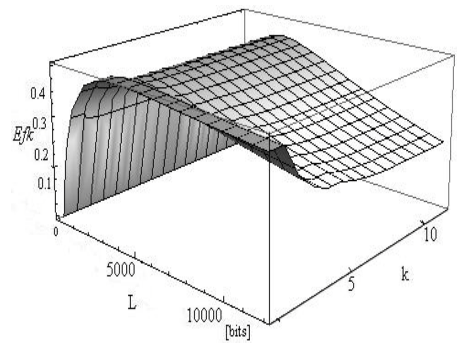

Fig. 8 Efficiency versus payload and retransmissions number for constant BER (10^{-4})

Some limitations of the presented model are due to the considered simplified communication principle. Not all the data packets are traveling at the same rate, so, trying to include all overheads in a unique processing is not trivial. The overheads concept can be however extended to include, out of the usual protocol overheads, the ACK back confirmation messages. Since some overheads (PLCP preamble and header, for example) are transmitted at different rate (the basic rate), the efficiency evaluation based on the transmitted number of bits could be less suggestive then using the link utilization factor. The link utilization factor is based on the necessary transmission time for each sequence. The interframe intervals (DIFS, SIFS intervals) and the variable transmission contention windows (CW, considered as an average constant value) are factors which act on the transmission time budget and they are imposed by the communication standard [2, 3]. This initial proposed model is supposing a point-to-point communication, ad-hoc like. When a larger number of stations is involved, the efficiency needs to be corrected with a factor which has to take into consideration the transmission probability for each client of the WLAN infrastructure [2].

Conclusions

Having a mathematical model for 802.11 communications, it allows finding and developing an efficient adaptive optimization technique. In Fig. 8 a 3D representation of the efficiency versus payload length L and retransmissions number k is shown.

The results presented in this paper can directly be used for improving the efficiency of a communication channel by using the optimum value for the payload length. Further work could also consider the use of reduced or compressed overheads [8] or optimizing the interframe interval or the contention window CW, [1]. Also, multiple stations can be considered or RTS/CTS effect as well [2]. As direct implementation, knowing the mechanism through which the payload length is acting on the efficiency for a given BER amount, L value (frame payload length) could be optimized in order to have the maximum efficiency for a limited and defined number of retransmissions.

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In 802.11 data communications is often necessary to develop communication models in order to calculate the quality parameters and to develop adaptive communication techniques. The particularities of the radio communications make quite difficult to state a complete model due to different factors which act on the 802.11 throughput: radio interferences, channel overlap, variable SNR value, CSMA/CA medium access control techniques or legacy support. The main information reflecting the quality of a transmission channel is related with bit error rate, BER. The Bit Error Rate is depending on signal to noise ratio and is related to the employed modulation technique. For this reason it is useful to have a communication model starting from a given channel bit error rate, able to allow the calculation of channel efficiency and throughput. However, the most important advantage of this type of model is related with the possibility of defining adaptive algorithms for dynamic maximization of the QoS parameters, the throughput in particular. The paper is proposing an analytical model for AWGN communication channels efficiency optimization. The proposed model allows calculating the data packet length in order to obtain the best efficiency on a communication channel in terms of useful transferred data bits or throughput. The principle is useful in implementing adaptive communications. Ill. 8, bibl. 8, tabl. 2 (in English; abstracts in English, Russian and Lithuanian).

А. Д. Поторак, А. Онофрей, А. Балан. Исследование эффективности оптимизированной модели канала связи типа 802.11 // Электроника и электротехника. – Каунас: Технология, 2010. – № 1(97). – С. 67–72.

Приведена технология адаптивной связи при использовании протокола 802ю11 передачи данных. Основным параметром выбрано качество работы системы, связанное со степенью точности двоичного кода. На основе теоретических исследований установлены зависимости QoS параметров от максимального значения пропускной способности. Оптимизация параметров канала связи осуществлена применением AWGN система. Доказано, что предлагаемая модель канала связи обеспечивает наилучшую эффективность и пропускную способность. Ил. 8, библ. 8, табл. 2 (на английском языке; рефераты на английском, русском и литовском яз.).

A. D. Potorac, A. Onofrei, A. Balan. 802.11 protokolo bevielio ryšio kanalų optimizavimo modelio efektyvumo tyrimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 1(97). – P. 67–72.

Svarbu sukurti ryšio modelius, kurie padėtų skaičiuoti kokybės parametrus ir parinkti adaptyvią ryšio užmezgimo technologiją naudojant 802.11 duomenų perdavimo protokolą. 802.11 duomenų perdavimo protokolo pralaidumą veikiantys veiksniai (radijo trikdžiai, kanalų sanklota, SNR kitimas, CSMA/CA vidutinės prieigos kontrolė) apsunkina modelio kūrimą. Pagrindinė informacija, atspindinti ryšio kanalo kokybę, yra susijusi su dvejetainės klaidos laipsniu. Dvejetainės klaidos laipsnis susijęs su taikomos moduliacijos technologija ir priklauso nuo signalo ir triukšmo santykio. Dėl šios priežasties naudinga turėti tokį ryšio modelį, kuris padėtų skaičiuoti kanalo efektyvumą ir pralaidumą, dvejetainį klaidos laipsnį. Pats didžiausias šio modelio privalumas yra susijęs su galimybe nustatyti ir parinkti adaptyvius algoritmus, siekiant užtikrinti maksimalų QoS parametrų dinamiškumą ir atitinkamą pralaidumą. Čia išanalizuotas AWGN ryšio kanalo efektyvumo optimizavimas analitiniame modelyje. Siūlomas modelis ryšio kanale leis apskaičiuoti duomenų paketo ilgį, kad būtų galima nustatyti didžiausią efektyvumą ar pralaidumą. Šį principą naudinga taikyti adaptyviosiose ryšio technologijose. Il. 8, bibl. 8, lent. 2 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).