Properties of Helical Structures Containing Periodical Inhomogeneities

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Introduction

Helical structures are applied for retardation of electromagnetic waves in traveling-wave tubes, traveling-wave cathode-ray tubes, delay lines and other electronic devices [1–4]. Models of helical systems are proposed and their properties are described in [5–12] and other monographs and papers. According to [11], the pass-band of a helical system containing periodical inhomogeneities is limited because the stop-band appears when the phase angle between voltages or currents on neighbor helix turns approaches to π with increase of frequency.

The nature of periodical inhomogeneities can be different. They can appear as a result of variation of dimensions (width) of helical conductor, variation of distance between the helical conductor and shields of the helix, periodical variation of characteristic admittance due to dielectric holders, and other reasons.

Sometimes it is impossible to avoid inhomogeneities in microwave devices. For this reason it is important to reveal the influence of various inhomogeneities onto frequency properties of periodical helical structures.

The model of the helical structure containing periodical inhomogeneities

Let us consider a system, consisting of a helix with rectangular cross-section, shielded by internal and external shields. Let us assume that there are four homogeneous sections along a turn of the helix. The dimensions of the helical conductor and the gaps between the helix and their
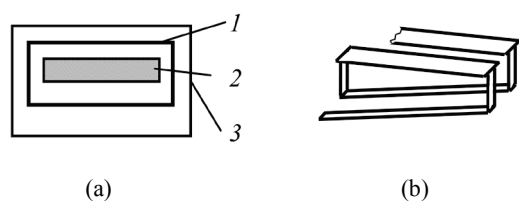
shields can be different in the sections.

The model of the system is presented in Fig. 2. The segments of multiconductor lines are used for modeling, like in [8, 11, 12]. Because a turn of the helix contains four homogeneous parts, four segments of multiconductor lines are used in the model.

Using the quasi-TEM approximation and taking into account normal modes, we have the following expressions [5–8] for voltages and currents of the conductors in the multiconductor line:

$$\underline{U}_{in}(x) = \left(\underline{A}_{i1} \sin kx + \underline{A}_{i2} \cos kx \right) e^{-jn\theta}, \qquad (1)$$

$$\underline{I}_{in}(x) = jY_i(\theta) \left(\underline{A}_{i1} \cos kx - \underline{A}_{i2} \sin kx \right) e^{-jn\theta}, \quad (2)$$

where $\underline{A}_{in}$ are coefficients, i is the number of the segment of the multiconductor line ($i = 1$ at $0 < x < x_1$; $i = 2$ at $x_1 < x < x_2$; $i = 3$ at $x_2 < x < x_3$; $i = 4$ at $x_3 < x < x_4$), n is the number of the conductor in the row, k is the wave number, θ is the phase angle between the voltages on the adjacent conductors of the multiconductor line, $Y_i(\theta)$ is the characteristic admittance of the segment of the multiconductor line.

The structure shown in Fig. 2 models the helical system if these boundary conditions are satisfied:
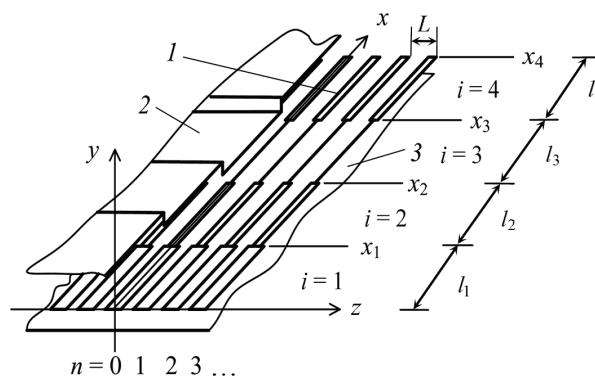

Fig. 1. (a) The cross-section of a helical system, and (b) the view of the helix: 1 – helix; 2, 3 – shields

Fig. 2. The model of the helical line: 1 – conductor of the multiconductor line; 2, 3 – shields

$$\underline{U}_{1n}(x_1) = \underline{U}_{2n}(x_1), \tag{3}$$

$$\underline{I}_{1n}(x_1) = \underline{I}_{2n}(x_1), \tag{4}$$

$$\underline{U}_{2n}(x_2) = \underline{U}_{3n}(x_2), \tag{5}$$

$$\underline{I}_{2n}(x_2) = \underline{I}_{3n}(x_2), \tag{6}$$

$$\underline{U}_{3n}(x_3) = \underline{U}_{4n}(x_3), \tag{7}$$

$$\underline{I}_{3n}(x_3) = \underline{I}_{4n}(x_3), \tag{8}$$

$$\underline{U}_{4n}(x_4) = \underline{U}_{1(n+1)}(0), \tag{9}$$

$$\underline{I}_{4n}(x_4) = \underline{I}_{1(n+1)}(0). \tag{10}$$

Substituting (1) and (2) into (3)–(10), we arrive at a set of algebraic equations. Using the principles of the matrix algebra and iterations we can find the value of the wave number, corresponding to the given phase angle θ and zero determinant of the set of equations.

After that we can find values of retardation factor K_r and frequency f:

$$K_r = c_0 / v_{ph} = \theta / kL, \tag{11}$$

$$f = kc_0 / 2\pi, \tag{12}$$

where v_{ph} is the phase velocity of the traveling-wave, c_0 is the light velocity and L is the step of the conductors of the helix and the multiconductor lines in the model of the system.

During calculations, values of characteristic admittances $Y_i(\theta)$ and characteristic impedances $Z_i(\theta) = 1/Y_i(\theta)$ can be found using formula [5, 8, 10]:

$$Y_i(\theta) = Y_i(0) + [Y_i(\pi) - Y_i(0)] \sin^2 (\theta / 2). \tag{13}$$

Values of characteristic admittances $Y_i(0)$, $Y_i(\pi)$ and corresponding characteristic impedances $Z_i(0) = 1/Y_i(0)$, $Z_i(\pi) = 1/Y_i(\pi)$ can be determined using numerical (finite difference or finite element) methods [13, 14].

The input impedance $\underline{Z}_{IN}$ of the system is dependent on the coordinate x. At $x = 0$, according to (1) and (2)

$$\underline{Z}_{IN}(0) = \frac{\underline{U}_{10}(0)}{\underline{I}_{10}(0)}. \tag{14}$$

Usually characteristics of helical systems (retardation factor and input impedance versus frequency) are considered in the frequency range corresponding to the phase angle θ changing from 0 to $\pi/2$. For this reason, there is not enough information about stop-bands and rejection properties of inhomogeneous helical structures.

According to [11] rejection properties depend on ratio of characteristic admittances or impedances of homogeneous sections. Unfortunately, variation of these characteristic parameters with frequency are not taken into account in [11]. Besides that it is important that variation of characteristic admittances with frequency and their ratio

at $\theta = 0$ and $\theta = \pi$ depend on different factors. For instance, characteristic impedances at low frequency (small phase angle θ) depend on the cross-section dimensions of the multi-conductor line. If frequency increases electromagnetic field obtains surface character, and the characteristic impedance becomes less dependent on the shields of the system. At the same time, it strongly depends on the step of the helix and gaps between neighbor helix turns.

Taking this into account, let us examine how frequency properties of inhomogeneous helical systems depend on variation of characteristic impedances along the turn of the helix and with frequency.

Results of simulation

Results of simulation of inhomogeneous helical system (retardation factor and input impedance versus frequency) are presented in Fig. 3–6.

Fig. 3 illustrates how retardation factor and input impedance of the helical system containing two homogeneous segments depend on the ratio of characteristic impedances $Z_i(0)$ and lengths of homogeneous segments in the period at $Z_i(\pi) = $ const. In this instance (at $Z_i(\pi) = $ const), variation of retardation
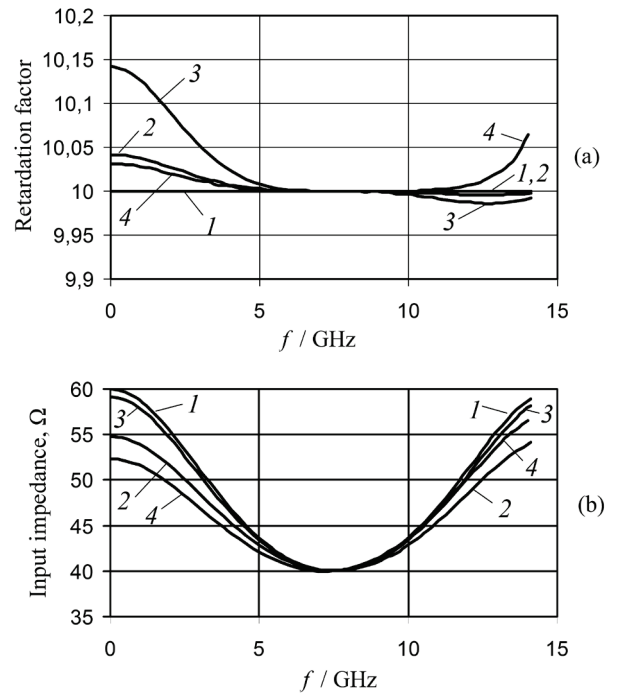

Fig. 3. (a) Retardation factor and (b) input impedance versus frequency at $x_4 = l_1 + l_2 + l_3 + l_4 = 20$ mm, $L = 2$ mm and other data given in the table

Table 1. Simulation related data for Fig. 3

Cur-ve	$l_1 + l_2 + l_3 + l_4$ mm	$Z_1(0)/Z_1(\pi)$	$Z_2(0)/Z_2(\pi)$	$Z_3(0)/Z_3(\pi)$	$Z_4(0)/Z_4(\pi)$
1	10+0+10+0	60/40	60/40	60/40	60/40
2	10+0+10+0	60/40	60/40	50/40	50/40
3	10+0+10+0	70/40	70/40	50/40	50/40
4	5+0+15+0	60/40	60/40	50/40	50/40

50

factor is small in the wide frequency range. Periodical variation of characteristic impedance $Z_i(\theta)$ along the helix turn causes increase of retardation factor. If $Z_i(\pi) = \text{const}$, we can reduce dispersion of retardation factor decreasing variation of characteristic impedance $Z_i(0)$ and shortening the section with less length (dispersion is maximal at $l_3 / l_1 = 1$ when $l_2 = l_4 = 0$). At constant characteristic impedance along the helix turn, retardation factor is constant. These results coincide with [8].

The input impedance of the system depends on phase angle θ. It is minimal at $\theta = \pi$ because capacitances between neighbor turns of the helix are maximal at this phase angle.

Variation of characteristic impedance along the helix turn at values θ close to π cause radical changes of frequency characteristics of helical systems (Fig. 4). Retardation factor and input impedance rapidly change when θ approaches to π (when the half of the wavelength in the system approaches the length of the helix turn), and the stop-band appears. The central frequency of the stop-band depends on the delay time t_d in the period of the system and is given by
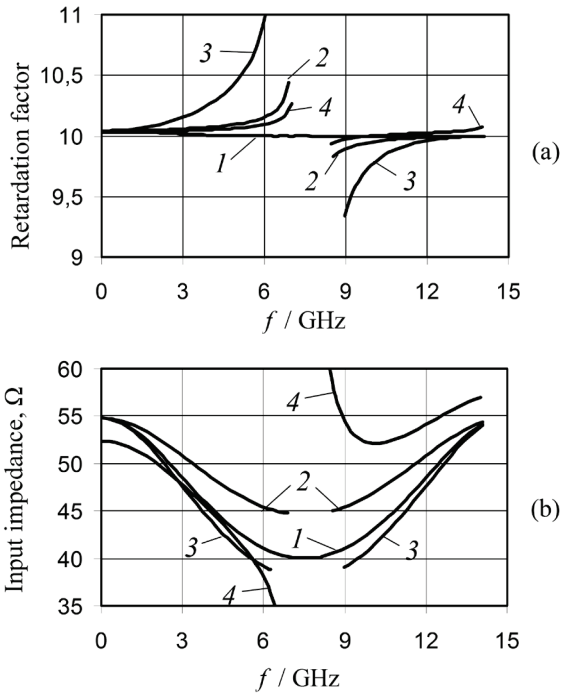

Fig. 4. (a) Retardation factor and (b) input impedance versus frequency at $x_4 = l_1 + l_2 + l_3 + l_4 = 20$ mm, $L = 2$ mm and other data given in the table

Table 2. Simulation related data for Fig. 4

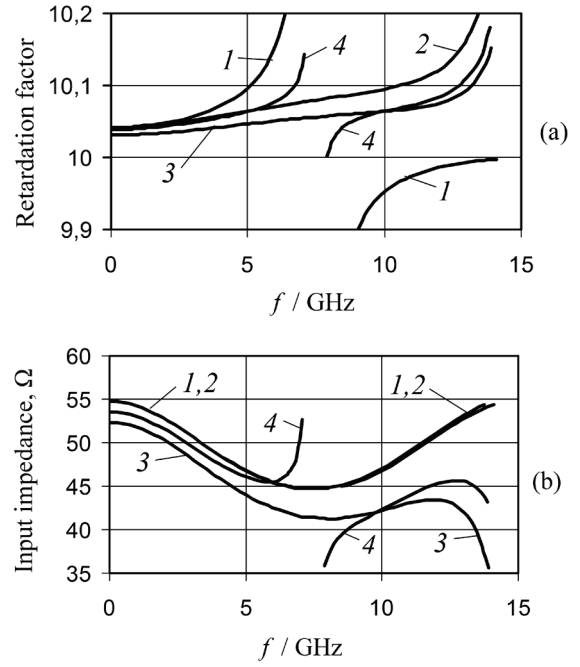| Cur-ve	$l_1 + l_2 + l_3 + l_4$ mm	$Z_1(0)/$ $Z_1(\pi)$	$Z_2(0)/$ $Z_2(\pi)$	$Z_3(0)/$ $Z_3(\pi)$	$Z_4(0)/$ $Z_4(\pi)$
1	10+0+10+0	60/40	60/40	50/40	50/40
2	10+0+10+0	60/50	60/50	50/40	50/40
3	10+0+10+0	60/50	60/50	50/30	50/30
4	5+0+15+0	60/50	60/50	50/40	50/40

Fig. 5. (a) Retardation factor and (b) input impedance versus frequency at $x_4 = l_1 + l_2 + l_3 + l_4 = 20$ mm, $L = 2$ mm and other data given in the table

Table 3. Simulation related data for Fig. 5

Cur-ve	$l_1 + l_2 + l_3 + l_4$ mm	$Z_1(0)/$ $Z_1(\pi)$	$Z_2(0)/$ $Z_2(\pi)$	$Z_3(0)/$ $Z_3(\pi)$	$Z_4(0)/$ $Z_4(\pi)$
1	10+0+10+0	60/50	60/50	50/40	50/40
2	5+5+5+5	60/50	50/40	60/50	50/40
3	2.5+7.5+2.5+7.5	60/50	50/40	60/50	50/40
4	2.5+7.5+5+5	60/50	60/50	50/40	50/40

$$f_c = \frac{1}{2t_d}. \qquad (15)$$

The width of the stop-band depends on ratio of characteristic impedances $Z_i(\pi)$ of homogeneous sections and ratio of their lengths. In order to reduce dispersion of retardation and width of the stop-band, we must reduce the ratio of characteristic impedances of homogeneous sections at $\theta = \pi$ and reduce the length of the section with less length.

The input impedance of the helical system containing periodical inhomogeneities strongly depends on coordinate x and ratio of lengths of homogeneous sections (Fig. 4 (b), curve 4).

Frequency characteristics of helical systems containing two and four homogeneous sections along the helix turn are presented in Fig. 5. Comparing curves 1 and 2 in Fig. 5 (a, b) we see that reducing the length of the period of inhomogeneities we can avoid the stop-band at $\theta = \pi$, improve dispersion properties of helical systems and reduce the change of input impedance. If the period of inhomogeneities is two times less, the stop-band appears at twice higher frequency (at $\theta \cong 2\pi$).

Curves 2 and 3 (Fig. 6 (a)) illustrate that dispersion in the system containing two periods along the helix turn decreases if the length of the section with the less length

decreases.

It is also important that in the helical system containing four sections along the helix turn the stop-band exists at $\theta = \pi$ if $Z_1(\theta) \neq Z_3(\theta)$, $Z_2(\theta) \neq Z_4(\theta)$, or $l_1 \neq l_3$ and $l_2 \neq l_4$ (curve *4* in Fig. 4 (a, b)). In this instance, the period of inhomogeneities becomes the same as the length of the helix turn, and the central frequency of the stop-band can be found using equation (15).

Conclusion

Due to periodical inhomogeneities and multiple reflections from them helical systems obtain properties of the stop-band filters. In the system containing two homogeneous sections along the helix turn, the stop-band appears when the phase angle between the voltages or currents on the neighbor conductors of the helix approach π.

Dispersion properties of the helical system containing periodical inhomogeneities can be improved reducing variation of characteristic impedance along the helix turn at $\theta = \pi$ and reducing the length of the inhomogeneous section (section with less length) of the helix turn.

Besides that there is the other possibility to improve the dispersion properties. We can increase the width of the pass-band of the helical system containing periodical inhomogeneities decreasing the period of inhomogeneities (increasing the number of the periods of inhomogeneities along the helix turn).

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Rejecting properties of inhomogeneous helical systems are considered. The model of the systems based on the multi-conductor line method is used. Examples of calculated characteristics are presented, and properties of the non-homogeneous helical systems are revealed. In the system containing two homogeneous sections along the helix turn, the stop-band appears when the phase angle between the voltages on the neighbor conductors of the helix approach π. Dispersion properties of the helical system containing periodical inhomogeneities can be improved reducing variation of characteristic impedance along the helix turn at $\theta = \pi$ and reducing the length of the inhomogeneous section of the helix turn. Besides that we can increase the width of the pass-band of the helical system containing periodical inhomogeneities decreasing the period of inhomogeneities (increasing the number of the periods of inhomogeneities along the helix turn). Ill. 5, bibl. 14 (in English, summaries in English, Russian and Lithuanian).

С. Штарас. А. Каткевичюс. Свойства неоднородных спиральных систем //Электроника и электротехника. – Каунас: Технология, 2010. – № 3(99). – С. 49–52.
Рассматриваются свойства спиральных систем, содержащих периодические неоднородности. Для анализа систем выбран метод многопроводных линий. Приведены и обсуждаются расчетные зависимости коэффициента замедления и входного сопротивления от частоты. Показано, что периодические неоднородности вдоль витка могут привести к появлению заграждающей полосы. В общем случае центральная частота этой полосы соответствует фазовому углу θ между напряжениями соседних витков, равному π. Для улучшения дисперсионных свойств и расширения полосы пропускания следует уменьшать изменение волнового сопротивления вдоль витка при $\theta = \pi$, уменьшать длину неоднородного участка витка и уменьшать длину периода неоднородного участка витка (увеличивать число периодов неоднородностей вдоль витка). Ил. 5, библ. 14 (на английском языке, рефераты на английском, русском и литовском яз.).

S. Štaras, A. Katkevičius. Spiralinių sistemų, turinčių periodinių netolygumų, savybės // Elektronika ir elektrotechnika – Kaunas: Technologija, 2010. – Nr. 3(99). – P. 49–52.
Nagrinėjamos nevienalytės spiralinės lėtinimo sistemos užtvarinės savybės. Sistema modeliuojama vieneilės vienpakopės nevienalytės daugialaidės linijos atkarpa. Pateikiamos ir nagrinėjamos apskaičiuotos lėtinimo koeficiento ir įėjimo varžos dažninės charakteristikos. Parodyta, kad dėl nevienalytiškumo spiralinės sistemos įgyja užtvarinių filtrų savybių, kai, didėjant dažniui, fazės kampas tarp gretimų spiralės vijų įtampų ar srovių artėja prie π. Parodyta, kad norint pagerinti spiralinių sistemų dispersines savybes reikia mažinti banginės varžos kitimą išilgai spiralės vijos, kai fazės kampas tarp gretimų vijų įtampų arba srovių artimas π, ir trumpinti nevienalytę vijos atkarpą. Be to sistemos praleidžiamų dažnių juostą galima išplėsti trumpinant netolygumų periodą (didinant periodų skaičių išilgai spiralės vijos). Il. 5, bibl. 14 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).