

Incorporating Sliding Mode and Fuzzy Controller with Bounded Torques for Set-Point Tracking of Robot Manipulators

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Introduction

A robot manipulator is a nonlinear system including high coupling between its dynamics and friction phenomena at its joints [1]. One constraint on controlling of robots is saturation nonlinearities of actuators which less considered in control systems for robot manipulators.

A well improved control method is sliding mode control which is capable in the control of nonlinear systems, MIMO systems and even discrete-time systems [2, 3]. A good survey in this realm is provided by [4]. Prominent trait of SMC is its robustness against parameter uncertainties and disturbances. In spite of these merits, SMC suffers from chattering problem which can excite unmodeled dynamics and harm the control system. To avoid this problem, a traditional approach is the use of boundary layer around sliding surface, but this causes steady state tracking error. Additionally, performance of sliding mode control of robot manipulator is declined by applying limitation on control input magnitude. On the other hand, in absence of parameter uncertainties and external disturbance, fuzzy control is appropriate for many control designs [5-8]. At a good study which has been accomplished in [9], a fuzzy controller with bounded torques has been designed for set-point tracking of robot manipulator in where the friction phenomenon has been considered; Also stability of control system was proved by Lyapunov method and Lasalle's theorem. Moreover, because of being similarities between fuzzy logic and SMC [8, 10], fuzzy logic is widely used to enhance SMC performance such as chattering elimination that generally has been called "fuzzy sliding mode control" [11-13].

In this note, a combined controller includes SMC term and fuzzy term is proposed for set-point tracking of robot manipulators. Some practical issues, such as existence of joint frictions, restriction on input torque magnitude due to saturation of actuators, and modeling uncertainties have been considered here. Design procedure

contains two steps. First, SMC design is accomplished and system stability in this case is provided by Lyapunov direct method. When the tracking error would be less than predefined value then a sectorial fuzzy controller (SFC), [14], is responsible for control action. Designing of this kind of fuzzy controller is exactly same as in which has performed in [9]. This proposed controller has following advantages. 1) There are less tracking error versus traditional SMC in condition that the control input is limited, 2) the chattering is avoided, 3) convergence of tracking error is more rapid than fuzzy controller designed in [9] and modeling uncertainty is considered here.

Mathematical Model for robot manipulator and problem formulation

The dynamical equation of an n-link robot manipulator in the standard form is as follows [1]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}, \tau) = \tau, \quad (1)$$

where $M(q) \in R^{n \times n}$ is a symmetry and bounded positive definite matrix which is called inertial matrix. Moreover, $q, \dot{q}, \ddot{q} \in R^n$ are the position, velocity, and angular acceleration of robot joints, respectively. The matrix $C(q, \dot{q}) \in R^{n \times n}$ is the matrix of Coriolis and centrifugal forces such that the matrix $\dot{M}(q) - 2C(q, \dot{q})$ is asymmetry, i.e., for a nonzero $n \times 1$ vector x we will have: $x^T [\dot{M}(q) - 2C(q, \dot{q})]x = 0$. Also, $G(q) \in R^n$ is the gravity vector and $F(\dot{q}, \tau) \in R^n$ stands for friction vector which is as follows [15]

$$f_i(\dot{q}, \tau_i) = b_i \dot{q}_i + f_{ci} \operatorname{sgn}(\dot{q}_i) + [1 - |\operatorname{sgn}(\dot{q}_i)|] \operatorname{sat}(\tau_i; f_{si}), \quad (2)$$

where $f_i(\dot{q}, \tau_i)$, $i = 1, 2, \dots, n$, denotes the i-th element of

$F(\dot{q}, \tau)$ vector. b_i , f_{ci} and f_{si} are the viscous, Coulomb and static friction, respectively.

In the following, $M(q)$, $C(q, \dot{q})$ and $G(q)$ might be shown by M , C , and G , respectively in where it would be requisite.

Now, the boundedness properties are defined as below

$$\sup_{q \in R^n} \{g_i(q)\} \leq \bar{g}_i, \quad i=1, \dots, n, \quad (3)$$

where g_i stands for the i -th element of $G(q)$ and \bar{g}_i is finite nonnegative constant. Assume, the maximum torque that joint actuator can supply is τ^{\max} . Therefore

$$|\tau_i| \leq \tau_i^{\max}, \quad i=1, \dots, n \quad (4)$$

and each actuator satisfies the following condition

$$\tau_i^{\max} > \bar{g}_i + f_{si}. \quad (5)$$

In robot modeling, one can well determine the terms $M(q)$ and $G(q)$ but it is difficult in most cases obtaining the parameters of $C(q, \dot{q})$ and $F(\dot{q}, \tau)$ exactly. So, in present paper, the matrix C is considered as follows

$$C = \hat{C} + \Delta C, \quad (6)$$

where \hat{C} denotes estimation of C and ΔC is bounded estimation error which has following relation

$$|\Delta C_{i,j}| \leq 0.1 |C_{i,j}|, \quad (7)$$

where $C_{i,j}$ stands for elements of the matrix C . Also the vector F is supposed as an external disturbance with following unknown upper bound

$$\|F\| \leq F_{up}, \quad (8)$$

where the operator $\|\cdot\|$ denotes Euclidean norm.

If one considers the desired point which joint position must be held on it as q_d , then the position error could be defined as

$$\tilde{q} = q_d - q. \quad (9)$$

The set-point tracking problem refers to define the control law such that error e would be driven toward the inside of an arbitrary small region around zero with maintaining the torques within the constraints (4). In succeeding sections this aim will be attained.

Sliding mode control

In order to design SMC controller, the following sliding surface is considered

$$s = \dot{e} + \lambda e, \quad (10)$$

where $e = -\tilde{q} = q - q_d$ is error vector and λ is supposed symmetric positive definite matrix such that $s=0$ would

become a stable surface. The reference velocity vector " \dot{q}_r " is defined as in [3]

$$\dot{q}_r = \dot{q}_d - \lambda e. \quad (11)$$

Thus, one can interpret sliding surface as

$$s = \dot{q} - \dot{q}_r. \quad (12)$$

In order to reach the system states (e, \dot{e}) to the sliding surface $s=0$ in a limited time and remain there, the control law should be designed such that the following sliding condition is satisfied

$$\frac{1}{2} \frac{d}{dt} [s^T M s] < -\eta (s^T s)^{1/2}, \quad s \neq 0, \quad (13)$$

where η is positive definite. Here, the SMC controller design is expressed by following lemma.

Lemma 1. Consider the system with dynamic equation (1) and sliding surface and reference velocity defined by (10) and (11), respectively. If one chooses the control law below

$$\tau = \hat{\tau} - K \operatorname{sgn}(s), \quad (14)$$

such that

$$\hat{\tau} = M \ddot{q}_r + \hat{C} \dot{q}_r + G \quad (15)$$

and

$$K_i \geq \|\Delta C \dot{q}_r\| + \Gamma_i, \quad (16)$$

then the sliding condition (13) is satisfied. In the last inequality, K_i denotes the element of sliding gain vector K and Γ is design parameter vector which must be selected such that $\Gamma_i \geq F_{up} + \eta_i$.

Proof. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2} s^T M s. \quad (17)$$

Since M is positive definite, for $s \neq 0$ we have $V > 0$ and by taking time derivative of the relation (17) and regarding the symmetric property of M , it can be written

$$\dot{V} = s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s. \quad (18)$$

From (12), gives

$$\dot{V} = s^T (M \ddot{q} - M \ddot{q}_r) + \frac{1}{2} s^T \dot{M} s. \quad (19)$$

By substituting (1) in (19) and considering asymmetry property $s^T (\dot{M} - 2C) s = 0$, we have

$$\dot{V} = s^T (\tau - C \dot{q}_r - G - F - M \ddot{q}_r). \quad (20)$$

Now, applying (14) and (15) yields

$$\dot{V} = s^T (\Delta C \dot{q}_r + F) - \sum_{i=1}^n K_i |s_i|. \quad (21)$$

Finally, from relation (16) it can be concluded that

$$\dot{V} \leq -\sum_{i=1}^n \eta_i |s_i|. \quad (22)$$

This indicates that V is a Lyapunov function and the sliding condition (13) has been satisfied.

Note that, in general, the sign function is replaced by saturation function as $\text{sat}(s/\varphi)$, where φ denotes boundary layer thickness.

Fuzzy controller design

In this section, the SFC class of fuzzy controller studied in [9] is considered which has two-input one-output rules used in the formulation of the knowledge base. These IF-THEN rules have following form

$$\text{If } x_1 \text{ is } A_1^{l_1} \text{ and } x_2 \text{ is } A_2^{l_2} \text{ then } y \text{ is } B^{l_1 l_2}, \quad (23)$$

where $x = [x_1 \ x_2]^T \in U = U_1 \times U_2 \subset \mathfrak{R}^2$ and $y \in V \subset \mathfrak{R}$. For each input fuzzy set $A_j^{l_j}$ in $x_j \in U_j$ and output fuzzy set $B^{l_1 l_2}$ in $y \in V$ exist an input membership function $\mu_{A_j^{l_j}}(x_j)$ and output membership function $\mu_{B^{l_1 l_2}}(y)$ shown in Fig. 1 and Fig. 2, respectively.

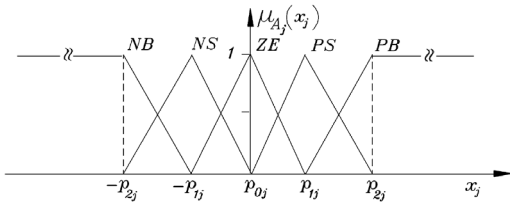


Fig. 1. Input membership functions

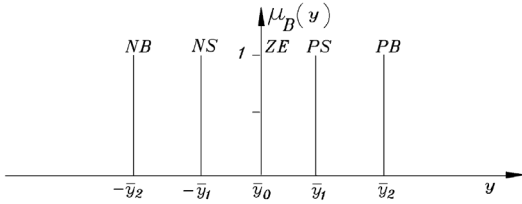


Fig. 2. Output membership functions

The fuzzy system considered here has following specifications: Singleton fuzzifier, triangular membership functions for each inputs, singleton membership functions for the output, rule base defined by (23), (see Table. 1), product inference and center average defuzzifier. Thus, one can compute the output y in terms of inputs as follows [5]

$$y(x) = \phi(x_1, x_2) = \frac{\sum_{l_1} \sum_{l_2} \bar{y}^{l_1 l_2} \left(\prod_{j=1}^2 \mu_{A_j^{l_j}}(x_j) \right)}{\sum_{l_1} \sum_{l_2} \left(\prod_{j=1}^2 \mu_{A_j^{l_j}}(x_j) \right)}. \quad (24)$$

Special properties of this input-output mapping $y(x)$ for x_1, x_2 are given in [9].

Table 1. The fuzzy rule base for obtaining output y

$x_2 \backslash x_1$	NB	NS	ZE	PS	PB
NB	NB	NB	NS	ZE	ZE
NS	NB	NB	NS	ZE	ZE
ZE	NS	NS	ZE	PS	PS
PS	ZE	ZE	PS	PB	PB
PB	ZE	ZE	PS	PB	PB

Lemma 2. For the system with dynamic equation (1), if one chooses the following control law

$$\tau = \phi(\tilde{q}, \dot{\tilde{q}}) + G(q), \quad (25)$$

where \tilde{q} is defined as (9) and $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$ is velocity error vector, then the closed-loop system shown in Fig. 3 becomes stable.

Proof. The stability analysis is based on the study performed in [14] and is fully discussed in [9], so it is omitted here. Note that for constant set-point $\dot{q}_d = 0$ hence $\dot{\tilde{q}} = -\dot{q}$.

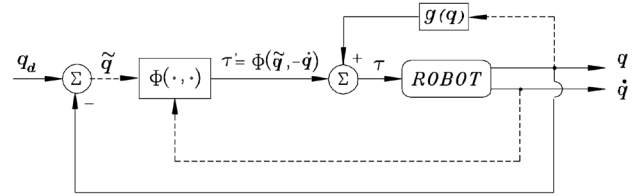


Fig. 3. Closed-loop system in the case of fuzzy control [9]

Incorporating SMC and SFC

Each of the two controllers explained in last two sections drives the robot joint angles to desired set-point in finite time and according to the Lemma 1 and 2 the closed-loop system is stable in both cases. In this paper, for obtaining advantages of both sliding mode and sectorial fuzzy controllers and also minimizing the drawbacks of the both of them, the following control law is proposed:

$$\tau = \begin{cases} \hat{\tau} - K \text{sgn}(s), & \text{when } |q_e| \geq \alpha, \\ y(q_e, \dot{q}_e) + G(q), & \text{when } |q_e| < \alpha, \end{cases} \quad (26)$$

where α is strictly positive small parameter which can be determined adaptively or set to a constant value. So, while the magnitude of error is greater than or equal to α , SMC drives the system states, errors in our case, toward sliding surface and as soon as the magnitude of error becomes less than α , then the SFC which is designed independent of initial conditions, controls the system. Since the SMC has faster transient response, the response of the system controlled by (26) is faster than the case of SFC. Additionally, in spite of the torque boundedness, since the SFC controls the system in the steady state, the proposed controller (26) has less set-point tracking error. Also, since near the sliding surface the proposed controller switch from SMC to SFC, therefore the chattering is avoided here.

The case study and simulation results

In order to show the effectiveness of the proposed control law, it is applied to a two-link direct drive robot arm with the following parameters [9]:

$$\begin{cases} M(q) = \begin{bmatrix} 2.351 + 0.168 \cos(q_2) & 0.102 + 0.084 \cos(q_2) \\ 0.102 + 0.084 \cos(q_2) & 0.102 \end{bmatrix}, \\ \hat{C}(q, \dot{q}) = \begin{bmatrix} -0.084 \sin(q_2) \dot{q}_2 & -0.084 \sin(q_2) (\dot{q}_1 + \dot{q}_2) \\ 0.084 \sin(q_2) \dot{q}_1 & 0 \end{bmatrix}, \\ G(q) = 9.81 \begin{bmatrix} 3.921 \sin(q_1) + 0.186 \sin(q_1 + q_2) \\ 0.186 \sin(q_1 + q_2) \end{bmatrix}, \\ F(\dot{q}) = \begin{bmatrix} 2.288 \dot{q}_1 + 8.049 \operatorname{sgn}(\dot{q}_1) + [1 - |\operatorname{sgn}(\dot{q}_1)|] \operatorname{sat}(\tau_1; 9.7) \\ 0.186 \dot{q}_2 + 1.734 \operatorname{sgn}(\dot{q}_2) + [1 - |\operatorname{sgn}(\dot{q}_2)|] \operatorname{sat}(\tau_2; 1.87) \end{bmatrix}, \\ C = \hat{C} + \Delta C. \end{cases} \quad (27)$$

According to the actuators manufacturer, the direct drive motors are able to supply torques within the following bounds:

$$\begin{cases} |\tau_1| \leq \tau_1^{\max} = 150 [\text{Nm}], \\ |\tau_2| \leq \tau_2^{\max} = 15 [\text{Nm}]. \end{cases} \quad (28)$$

The desired set-point is

$$q_d = [\pi \quad -\pi]^T, \quad (29)$$

which is applied as a step function at time zero. The SMC design parameters are as below

$$\lambda = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 140 \\ 8 \end{bmatrix} \quad \text{and} \quad \varphi = 5. \quad (30)$$

For SFC case, according to Fig. 1 and Fig. 2, $p_{x_j} = \{-p_{2j}, -p_{1j}, p_{0j}, p_{1j}, p_{2j}\}$ is fuzzy partition of the input universe of discourse and $p_{y_j} = \{-\bar{y}_2, -\bar{y}_1, \bar{y}_0, \bar{y}_1, \bar{y}_2\}$ is for output universe of discourse. Now, SFC design parameters are given by following equations [9]:

$$\begin{aligned} p_{\dot{q}_1} &= \{-180, -4, 0, 4, 180\}, \\ p_{\dot{q}_2} &= \{-180, -2, 0, 2, 180\}. \end{aligned} \quad (31)$$

$$\begin{aligned} p_{\ddot{q}_1} &= \{-360, -270, 0, 270, 360\}, \\ p_{\ddot{q}_2} &= \{-360, -270, 0, 270, 360\}. \end{aligned} \quad (32)$$

$$\begin{aligned} p_{y_1} &= \{-109, -90, 0, 90, 109\}, \\ p_{y_2} &= \{-13, -9, 0, 9, 13\}. \end{aligned} \quad (33)$$

For our proposed controller (26), the constant $\alpha = 0.3$ is supposed. Additionally, to show the improvement achieved from applying the proposed method of this paper (incorporating SMC and SFC), the simulation results of applying this method are compared with the related results of the SMC case and SFC case, separately. The error vector and control law in the case of conventional SMC have been shown in Fig. 4 and Fig. 5, respectively.

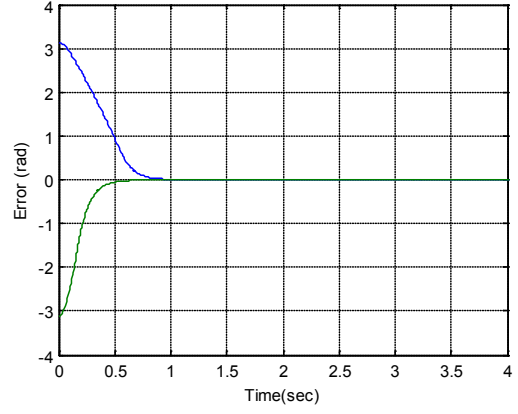


Fig. 4. Error vector in the case of SMC

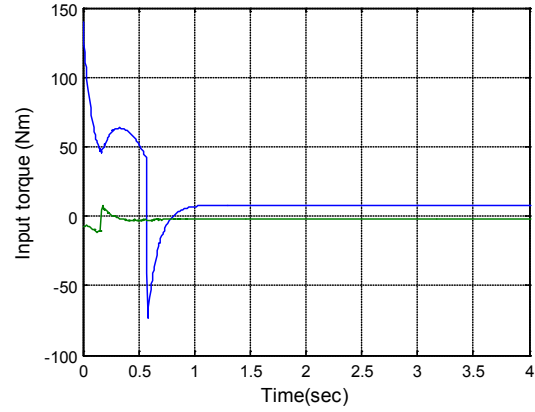


Fig. 5. The control torques in the case of SMC

The tracking error in this case is about 0.1(rad) and when one choose the thinner boundary layer to decrease this error, chattering will be occurred. The corresponding graphs for the case of applying SFC are also provided in Fig. 6, and Fig. 7.

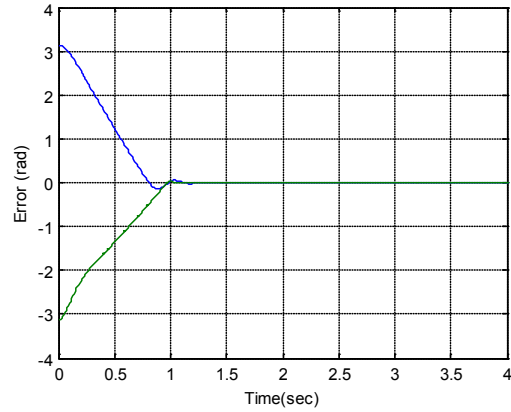


Fig. 6. Error vector in the case of SFC

In the case of control law proposed in the present paper, Fig. 8 and Fig. 9 illustrate the error vector and control law, respectively. The tracking error is about 0.002 in this state of affairs.

As it can be seen from these results, the proposed incorporating SMC and SFC controller has faster response and less tracking error in comparison with SMC and also the error vector converge toward zero faster than SFC.

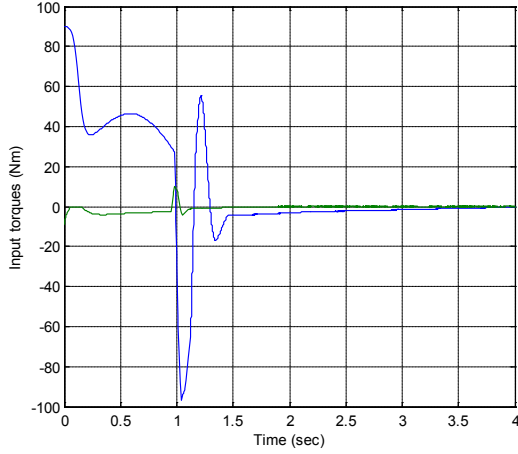


Fig. 7. The control torques in the case of SFC

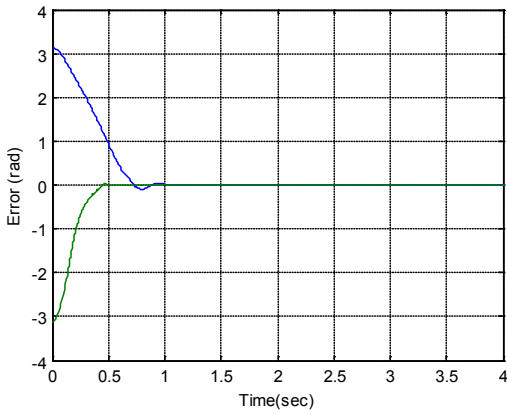


Fig. 8. Error vector in the case of incorporating SMC and SFC

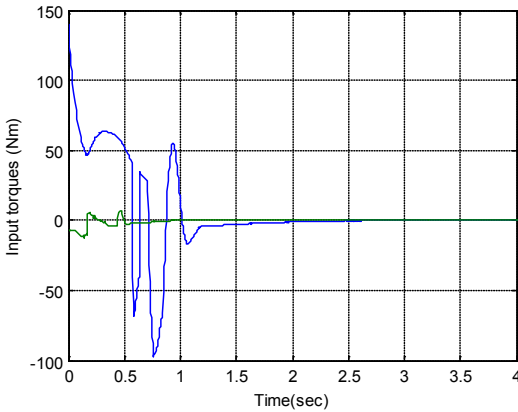


Fig. 9. The control torques in the case of incorporating SMC and SFC

In order to show the robustness of the proposed method, the inertia and torque perturbations are considered as following. The elements of inertia matrix are supposed to increase fifty percent after 2 sec. It can be a weight that added to the mass of 2nd link. Also, disturbance torque is considered with the following equation.

$$\tau_d = [3 \sin 2\pi t \quad 3 \sin 2\pi]^T. \quad (34)$$

In this case, the vector of joint errors is shown in Fig. 10. The errors are as good as previous case. Fig. 11

illustrates the control torques which are not change significantly, and because of existing perturbations, they alter trivially after 2 sec. these two recent results verify the robustness of the presented approach.

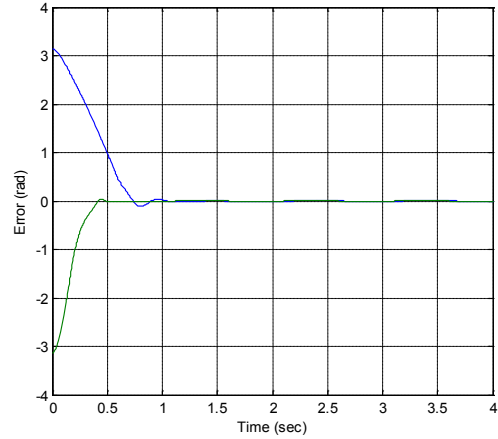


Fig. 10. Error vector in the case of torque and inertia perturbations

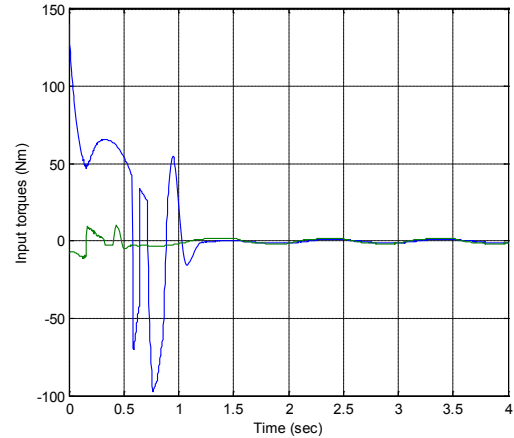


Fig. 11. The control torques in the case of torque and inertia perturbations

Conclusions

In this note, a new combination of sliding mode control and fuzzy control is proposed which is called incorporating sliding mode and fuzzy controller. Three practical aspects of robot manipulator control are considered here, such as restriction on input torque magnitude due to saturation of actuators, friction and modeling uncertainty. In spite of these features, the designed controller can improve the sliding mode and fuzzy controller performance in the tracking error and faster transient points of view, respectively. Finally, the simulation results of applying the proposed methodology and other two cases to a two-link direct drive robot arm were provided. Comparing these results demonstrate the success of the proposed method.

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Received 2010 01 27

S. E. Shafiei, S. Sepasi. Incorporating Sliding Mode and Fuzzy Controller with Bounded Torques for Set-Point Tracking of Robot Manipulators // *Electronics and Electrical Engineering*. – Kaunas: Technologija, 2010. – No. 8(104). – P. 3–8.

In this paper, an incorporating sliding mode and fuzzy controller is designed for set-point tracking of robot manipulators with friction and restriction on input torque magnitude due to saturation of actuators. Sliding mode control (SMC) has fast response and is robust to uncertainties and disturbances, but it suffers from chattering problem, in addition, its performance is decreased by prescribing limits on control input. On the other hand, the fuzzy control which is used here has very small tracking error but its response is slower than SMC case. Therefore, in order to profit from both merit of these controllers, at first, the sliding mode controller constraints the states to reach sliding surface. When the tracking error would be less than predefined value, then the fuzzy controller is responsible for control action. Comparative simulation results demonstrate the efficiency of this combination. Ill. 11, bibl. 15, tabl. 1 (in English; abstracts in English, Russian and Lithuanian).

С. Э. Шафей, С. Сепаси. Исследование состояния объединенного движения и торможения датчиков контроля в роботах и манипуляторах // *Электроника и электротехника*. – Каунас: Технология, 2010. – № 8(104). – С. 3–8.

Дан анализ движения и торможения датчиков контроля, применяемых для сложения точек движения в роботах. Указано, что предлагаемые датчики отличаются малой величиной шумов и отказов в работе, но чувствительны к вибрациям. Приводятся результаты моделирования. Ил. 11, библи. 15, табл. 1 (на английском языке; рефераты на английском, русском и литовском яз.).

S. E. Shafiei, S. Sepasi. Jungtinės judėjimo ir stabdymo būsenos ir neraiškiosios kontrolės valdiklių taikymo robotuose manipulatoriuose su trinties ir sukimo momento apribojimais tyrimas // *Elektronika ir elektrotechnika*. – Kaunas: Technologija, 2010. – Nr. 8(104). – P. 3–8.

Apžvelgta roboto manipulatoriaus su trinties ir sukimo momento apribojimais jungtinė judėjimo ir stabdymo būsenos ir neraiškiosios kontrolės valdikliai, suprojektuoti roboto judėjimo taškams sekti. Jungtinės judėjimo ir stabdymo būsenos valdikliams būdinga greita reakcija ir atsparumas trikdžiams ir sutrikimams, tačiau jie kenčia nuo vibracijų. Jungtinės judėjimo ir stabdymo būsenos roboto manipulatoriaus našumas mažėja didėjant įeinančių valdymo signalų skaičiui. Kita vertus, šiuo metu plačiai paplitusi neraiškioji logika pasižymi labai mažu sekimo klaidų skaičiumi, bet atsakymai gaunami lėčiau nei esant jungtinei judėjimo ir stabdymo būsenai. Pateikti modeliavimo rezultatai. Il. 11, bibl. 15, lent. 1 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).