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Fifth Order Butterworth Low Pass Square-Root Domain Filter Design

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Introduction

Interest in translinear circuits has rapidly increased recently[1, 2].Translinear circuits have low supply voltage, low power consumption, high dynamic range and low noise performance [2, 3].

Square-root domain (SRD) circuits are subclass of translinear circuits and externally linear internally nonlinear (ELIN) circuits. ELIN circuits are suitable for low voltage applications and low power dissipation [3, 4]. SRD circuits use the MOSFET devices in the strong inversion region of operation, and thus are based on the quadratic relationship between gate-to-source voltage and drain current [5, 6]. SRD circuits use compounding in signal processing. Therefore, SRD circuits have large dynamic ranges at low supply voltages [1, 3]. In the last decades, SRD filtering has become very popular and many studies have been presented in this issue [7–10].

In this study, a fifth order low pass Butterworth filter is designed with two different approaches having same transfer function by using state space synthesis method. First of all, fifth order state space system equations obtained from the transfer function are considered. Secondly, system is considered as compose of two second order and one first order transfer functions. Therefore, three set of system equations are cascaded to obtain a fifth order filter. Both filters are designed by using state space synthesis method and designed circuits are analyzed with PSpice. Obtained frequency, THD and noise analysis results are presented.

Fifth order SRD Butterworth filter state space synthesis

For SRD circuits, state space synthesis method is based on a nonlinear mapping on input and state variables of state space description of a particular transfer function. Input and state variables are equal to simple functions of node voltages. In design procedure, firstly, an appropriate state space description is considered for the filter; next a mapping function is applied to the input and state variables. Mapping function is a kind of square-root function similar to a MOSFET's current-voltage relation. After circuit equations are obtained, the circuit is designed using square-root blocks, current mirrors, grounded capacitors, and current sources.

Let us consider a fifth order Butterworth low pass filter transfer function as given in (1). In this equation ω_0 is the pole frequency of the filter. According to state space synthesis method [11, 12] initially a state space description of this transfer function is obtained as shown in (2).

In (2), u is input, y is output, x_1, x_2, x_3, x_4 and x_5 are the state variables and u_2 is DC dummy input. Note that u_2 is added to the system equations in order to synthesize the filter properly in SRD circuits.

$$H(s)=\frac{Y(s)}{U(s)}=\frac{{\omega_0}^5}{s^5+3.24\omega_0 s^4+5.24{\omega_0}^2 s^3+5.24{\omega_0}^3 s^2+3.24{\omega_0}^4 s+{\omega_0}^5}, \tag{1}$$

$$\begin{cases} \dot{x_1}=\omega_0 x_2-\omega_0 u_2, \\ \dot{x_2}=\omega_0 x_3-\omega_0 u_2, \\ \dot{x_3}=\omega_0 x_4-\omega_0 u_2, \\ \dot{x_4}=\omega_0 x_5-\omega_0 u_2, \\ \dot{x_5}=-3.24\omega_0 x_5-5.24\omega_0 x_4-5.24\omega_0 x_3-3.24\omega_0 x_2-\omega_0 x_1+\omega_0 u+16.96\omega_0 u_2. \\ y=x_1. \end{cases} \tag{2}$$

Nonlinear square-root transformation is shown in (3):

$$\begin{cases} x_n = v_n = \sqrt{\dfrac{I_n}{\beta}} + V_{th}, \\ u = \sqrt{\dfrac{I_u}{\beta}} + V_{th}, \end{cases} \tag{3}$$

where $\beta = \frac{\mu_0 C_{ox} W}{2L}$.

Nonlinear mapping shown in (3) is applied to the state space equations (2). After some manipulation nodal equations are obtained as shown in (4).

Fifth order SRD Butterworth filter circuit can be realized using square-root blocks, current mirrors capacitors and current sources. Square-root blocks which are used in proposed fifth order circuits, were proposed in [9].

Proposed fifth order low pass filter, which is a block diagram of (4), is shown in Fig. 1. In this circuit current steering and current mirrors blocks are designed in usual ways. Numbers above current mirror blocks in Fig. 1 imply to the (W/L) ratios of transistors.

$$\begin{cases} C_1\dot{v}_1 = \sqrt{I_f I_2} - \sqrt{I_f I_{u2}}, \\ C_2\dot{v}_2 = \sqrt{I_f I_3} - \sqrt{I_f I_{u2}}\,, \\ C_3\dot{v}_3 = \sqrt{I_f I_4} - \sqrt{I_f I_{u2}}, \\ C_4\dot{v}_4 = \sqrt{I_f I_5} - \sqrt{I_f I_{u2}}\,, \\ C_5\dot{v}_5 = -3.24\sqrt{I_f I_5} - 5.24\sqrt{I_f I_4} - 5.24\sqrt{I_f I_3} - 3.24\sqrt{I_f I_2} - \sqrt{I_f I_1} - \sqrt{I_f I_u} + 8.48\sqrt{I_{f2} I_{u2}}, \\ V_{out} = v_1, \end{cases} \tag{4}$$

where

$$I_f = \frac{{\omega_0}^2 C^2}{\beta}, \quad I_{f2} = \frac{4{\omega_0}^2 C^2}{\beta}. \tag{5}$$

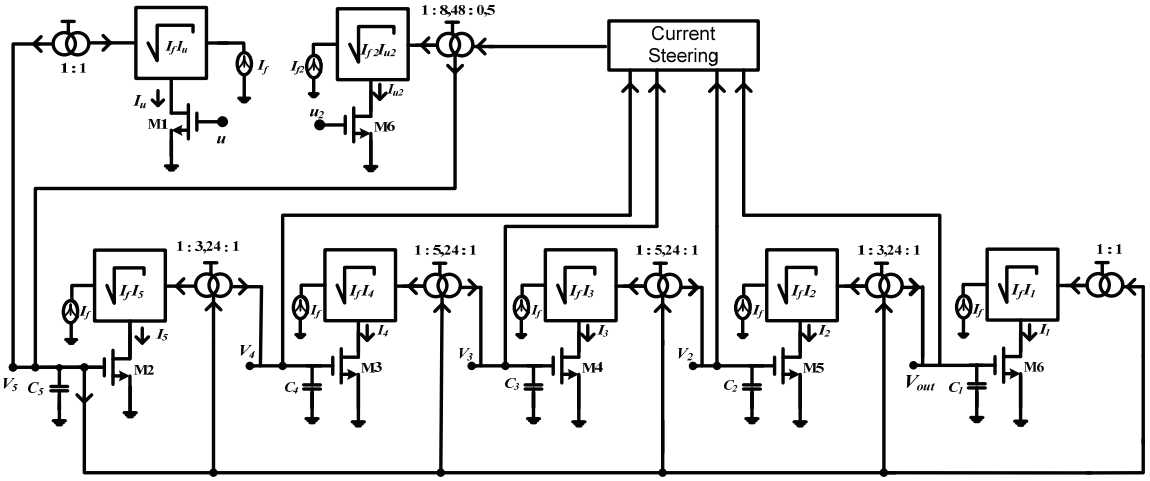

Fig. 1. Fifth order SRD Butterworth filter circuit

Fifth order SRD Butterworth filter cascade design

In this section, a fifth order SRD filter is obtained by cascade connection of one first order and two second order filters rather than direct implementation as previous section. This concept is formulized in (6)

$$H(s) = H_1(s)H_{21}(s)H_{22}(s). \tag{6}$$

First order SRD filter transfer function and nodal equations are given in (7)–(9):

$$H_1(s) = \frac{\omega_0}{s+\omega_0}, \tag{7}$$

$$C_1\dot{v}_1 = -\sqrt{I_f I_1} + \sqrt{I_f I_u}\,, \tag{8}$$

$$V_{out1} = v_1. \tag{9}$$

This circuit equation is realized by using square-root blocks, current mirrors, current sources and capacitors as shown in Fig. 2.

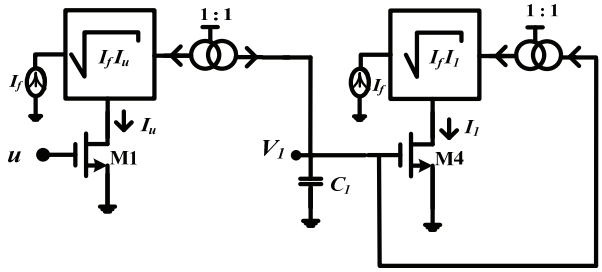

Fig. 2. First order SRD Butterworth filter

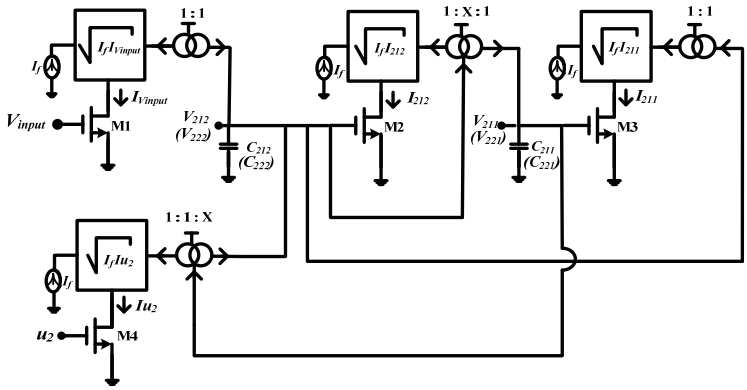

Fig. 3. Second order SRD Butterworth filter

Two second order filter transfer functions and circuit equations are:

$$H_{21}(s) = \frac{{\omega_0}^2}{s^2+0.618\omega_0 s+{\omega_0}^2}, \quad (10)$$

$$C_{211}\dot{v_{211}} = \sqrt{I_f I_{212}} - \sqrt{I_f I_{u2}}, \quad (11)$$

$$C_{212}\dot{v_{212}} = -\sqrt{I_f I_{211}} - 0.618\sqrt{I_f I_{212}} + \\ +\sqrt{I_f I_1} + 0.618\sqrt{I_f I_{u2}}, \quad (12)$$

$$V_{out2} = v_{211}, \quad (13)$$

$$H_{22}(s) = \frac{{\omega_0}^2}{s^2 + 1.618\omega_0 s + {\omega_0}^2}, \quad (14)$$

$$C_{221}\dot{v_{221}} = \sqrt{I_f I_{222}} - \sqrt{I_f I_{u2}}, \quad (15)$$

$$C_{222}\dot{v_{222}} = -\sqrt{I_f I_{221}} - 1.618\sqrt{I_f I_{222}} + \\ +\sqrt{I_f I_{211}} + 1.618\sqrt{I_f I_{u2}}, \quad (16)$$

$$V_{out} = v_{221}. \quad (17)$$

These circuit equations are obtained by using square-root blocks, current mirrors, current sources and capacitors as shown in Fig. 3.

Two second order filter circuits have been same topology given in Fig. 3. In this circuit, X coefficient is equal to 0.618 and 1.618 respectively for (12) and (16).

Simulation results

Proposed both direct and cascade fifth order SRD Butterworth filters are simulated in PSpice. Both circuits use supply voltage of 3 V. These filters are set to 517 kHz pole frequency. The simulations are performed using TSMC 0.35 µm CMOS model parameters in PSpice. The values of capacitances of the circuit are selected to be 50 pF. The bias currents of these circuits, I_f and I_{f2}, are set to be 30 µA and 120 µA respectively.

Proposed both fifth order low pass filters gain and phase responses are given in Fig. 4, Fig. 5 respectively.

Then, the output signal's THD are measured for input voltage amplitude values from 25 mV to 145 mV for both designed filters. Obtained THD results for both circuit topologies are given in Fig. 6.

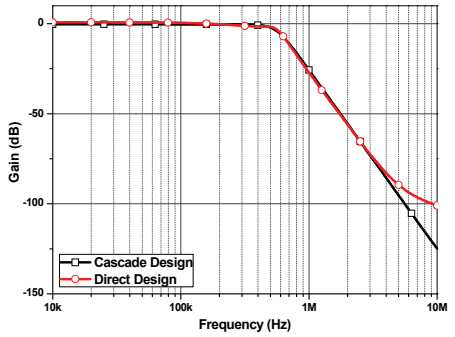

Fig. 4. Gain responses of fifth order filter for both cases

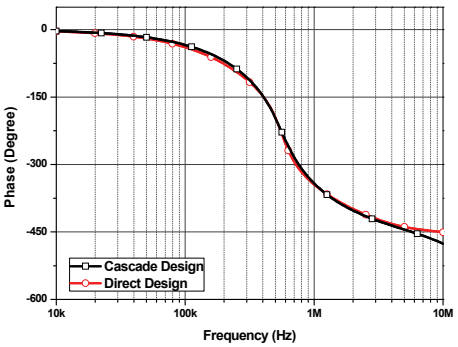

Fig. 5. Phase responses of fifth order filter for both cases

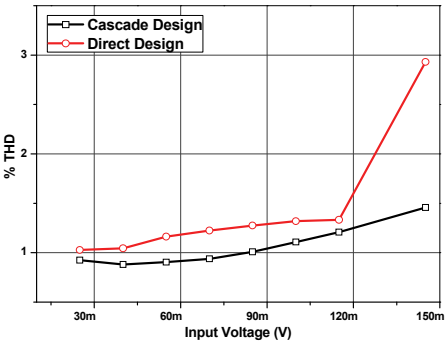

Fig. 6. THD results of the proposed both filters

Noise analysis is also performed. For both cases noise response shows a low pass filter characteristics. For direct design's noise is 207.016 $nV/\sqrt{Hz}$ and for cascade connection's noise is 76.240 $nV/\sqrt{Hz}$ for inband frequencies.

Conclusions

In this work, starting from one transfer function, two different fifth order low pass Butterworth filters are designed in SRD by using state space synthesis method. Proposed filters use square-root blocks, current mirrors, current sources and capacitances. First one use a fifth order system equations whereas second one use cascade connected one first order and two second order system equations. Proposed filters are simulated by PSpice using 3V supply voltage, 0.35μm CMOS technology parameters. Current sources and capacitances values are selected to 30μA and 50pF respectively. The pole frequency is equal to 517KHz. PSpice simulations are confirmed that both filters are working as expecting. Both filters frequency, THD and noise analysis are performed and presented. According to analysis results, both filters work similarly in terms of frequency responses. However, cascade design has better THD and noise performances due to using less valued current sources.

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In this study, two different fifth order square-root domain low pass Butterworth filters are designed. First filter is designed directly from fifth order transfer function. Second filter is obtained by cascading one first order and two second order filters. Both filters are designed by using state space synthesis method and constituted by current mirrors, square-root circuits, current sources and capacitors. Proposed filters are simulated by PSpice using 3V supply voltage and 0.35μm CMOS technology parameters. Both filters simulation results are compared. . Ill. 6, bibl. 12 (in English; abstracts in English and Lithuanian).

S. Surav Yilmaz, A. T. Tola. Siaurajuosčio šakninio penktos eilės Batervorto filtro projektavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 9(115). – P. 55–58.

Suprojektuoti du siaurajuosčiai šakniniai penktos eilės Batervorto filtrai. Pirmasis filtras suprojektuotas naudojantis penktos eilės perdavimo funkcija, antrasis – kaskadiškai jungiant vieną pirmos ir du antros eilės filtrus. Abu filtrai suprojektuoti taikant erdvės būsenos sintezės metodus. Atliktas pasiūlytų filtrų modeliavimas taikant programų paketą „Pspice“ (maitinimo įtampa – 3V, KMOP užtūros plotis – 0,35 mm). Palyginti abiejų filtrų modeliavimo rezultatai. Il. 6, bibl. 12 (anglų kalba; santraukos anglų ir lietuvių k.).