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Performance Analysis of μ-law Companding for Laplacian Source with Transmission over Rayleigh Fading Channel

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Introduction

Recently, there has been considerable interest in source transmission over wireless networks. There has also been some theoretical interest in evaluating source fidelity over a multi-hop channel, and in comparing source and channel diversity for various channel conditions. Compression and transmission of data signal have made the stringent demand on the quality of the reconstructed signal. Multipath fading can seriously degrade quantization system performances of wireless transmission. Therefore, upgrading transmission reliability and increasing channel capacity withhout icreasing transmission power and bandwith are the main problems. Optimizaton of quantization, coding and transmission techniquesis are the basic mission of modern digital signal processing in wireless communications[1, 2].

In a point-to-point link, when a discrete source is transmitted through a discrete channel, the optimal tradeoff between (channel input) cost and (source reconstruction) distortion can be achieved by separate source and channel coding. Despite its conceptual beauty, in practice, to approach the optimal pair of cost and distortion, the separate source and channel coding leads to high complexity and long delay when block length increases. Although joint source-channel coding does not have the separation property, it sometimes can lead to simple but optimal coding strategy. A well-known example is when a memoryless Gaussian source transmitted through an *AWGN* channel, an amplify-and-forward transmission strategy achieves the optimal power-distortion tradeoff. The perfect match between the source and channel leads to a very simple but optimal coding strategy which is both theoretically and practically appealing. Unfortunately, when source and channel do not come up with such a natural match, the simple but optimal coding is not easy to find. Let us analyze performances of of Laplacian source transmission over wirelles Rayleigh fading channel, for the purpuse of finding optimal coding technique. We will take into account the effect of a Rayleigh fading channel into nonuniform scalar quantization system based on the μ-law companding over Laplacian source. We shall assume how random errors are introduced in the bits that convey information about quantizer output level. System performance analysis is based on the statistical approach, where capitalizing on derived closed-form expressions for receiving signal-to-noise ratio (*SNR*), average bit error probabilty (*ABEP*) is efficiently evaluated for several modulation transmission techniques. Numerical results for this performance criterion are presented and discussed in the function of various system parameters.

This paper is organized as follows: A general analysis of non-uniform scalar quantization and μ-law companding of Laplacian source is given in Section 2. In Section 3, Rayleigh fading cannel is presented. Section 4 analised influence of Rayleigh fading cannel on the nonuniform scalar quantization system through quality of transmission. Section 5 concludes the paper by summarizing the key features of the transmission error effect design and its applications.

Scalar quantizer and μ-law companding

The L-point scalar quantizer $Q^{(L)}$ is characterized by the set of real numbers $x_1^{(L)}$, $x_2^{(L)}$, ..., $x_L^{(L)}$ called decision thresholds, which satisfy $-\infty = x_1^{(L)} < x_2^{(L)} < ... < x_L^{(L)} < x_{L+1}^{(L)} = +\infty$. Also quantizer has set $y_1^{(L)}$, $y_2^{(L)}$,..., $y_L^{(L)}$, called representation levels satisfying $y_k^{(L)}\ \varepsilon\ \alpha_k^{(L)} =(x_k^{(L)}, t_{k+1}^{(L)}]$, where $\alpha_k^{(L)}$, k=1, 2,..., L, denote quantization cells. The quantizer is defined as many-to-one mapping Q: $R \rightarrow R$, $Q^{(L)}(x)=y_k$ if $x\ \varepsilon\ \alpha_k^{(L)}$. The companding technique, recommended by ITU-T G.711 standard, is used for construction of nearly optimal quantizers for large number of quantization levels. The compression and expansion characteristics are piecewise linear approximation to μ-law (μ=255), with 8 bits/sample are adopted, leading to a bit rate of 64 kbps at 8 kHz of sampling frequency. Companding procedure consists of : 1) compressing the input signal x using nonuniform compressor characteristic $c(x)$; 2) quantizing the compressed signal $c(x)$ employing a uniform quantizer Q_u in the interval [-1,1]; 3) expanding the quantized version of the compressed signal using a inverse nonuniform transfer characteristic $c^{-1}(x)$. The companding quantizer can be represented as $Q(x)=c^{-1}(Q_u(c(x)))$, where $Q_u(x)$ denote uniform quantizer with decision tresholds $c(x_k)$ and representation levels $c(y_k)$. Coresponding values for the companding quantizer $Q(x)$ could be determined as the solutions of the following eqations

$$c(x_k) = -1 + \frac{2(k-1)}{L}, \quad c(y_k) = -1 + \frac{2(k-\frac{1}{2})}{L}. \tag{1}$$

There are several ways to chose the compressor function for compresion law. The μ-law companding is used for *PCM* systems in the North America, with the standard value of μ = 255, and μ-law compression characteristic is given:

$$c(x) = \begin{cases} \frac{x_{max}}{\ln(1+\mu)} ln\left(1 + \mu\frac{x}{x_{max}}\right), & 0 \le x \le x_{max}, \\ -\frac{x_{max}}{\ln(1+\mu)} ln\left(1 - \mu\frac{x}{x_{max}}\right), & -x_{max} \le x \le 0. \end{cases} \tag{2}$$

Output signal from simplest kind of information source, memoryless and identically distributed, is characterized by continous random variable X, with probability density function (*PDF*) $p(x)$. One of the approximations to the long-time-averaged *PDF* of amplitudes is provided by Laplacian model. Laplacian source can be used for modeling of the speech signal and difference signal for an image waveform [1]. In the rest of paper, we assume that information source is Laplacian source with memoryless property and zero mean value. The *PDF* of such source, where x denotes zero-mean statistically independent Laplacian random variable of variance σ^2, is given by

$$p(x) = \frac{1}{\sqrt{2\sigma^2}} exp\left(-\frac{\sqrt{2}|x|}{\sigma}\right). \tag{3}$$

The quality of the scalar quantizer is measured by distortion of resulting reproduction in comparision to the original one. The total distortion $D(Q)$ is defined as

$$D(Q) = E\big(X - Q(X)\big)^2 = \sum_{k=1}^{L} \int_{x_k}^{x_{k+1}} (x - y_k)^2 p(x) dx \tag{4}$$

and it consists of two components, the granular $D_g(Q)$ and the overload $D_{ol}(Q)$ distortion:

$$D_{\mathrm{g}}(Q) = \sum_{k=2}^{L-1} \int_{x_k}^{x_{k+1}} (x - y_k)^2 p(x) dx, \tag{5}$$

$$D_{ol}(Q) = \int_{-\infty}^{x_2} (x - y_1)^2 p(x) dx + \int_{x_L}^{\infty} (x - y_L)^2 p(x) dx. \tag{6}$$

Substituting (3) into (5) and (6) we can obtain expression:

$$\begin{aligned} D_{\mathrm{g}}(Q) = \sum_{k=2}^{L/2} \tfrac{1}{2}\Big[-\Big(x_k{}^2 + y_k{}^2 + \sqrt{2}\sigma y_k + \sigma^2 - \\ -\sqrt{2}x_k\big(\sqrt{2}y_k + \sigma\big)\Big) exp\left(\tfrac{\sqrt{2}x_k}{\sigma}\right) + \Big(x_{k+1}{}^2 + y_k{}^2 + \\ +\sqrt{2}\sigma y_k + \sigma^2 - \sqrt{2}x_{k+1}\big(\sqrt{2}y_k + \sigma\big)\Big) exp\left(\tfrac{\sqrt{2}x_{k+1}}{\sigma}\right)\Big] + \\ + \sum_{k=\frac{L}{2}+1}^{L-1} \tfrac{1}{2}\Big[\Big(x_k{}^2 + y_k{}^2 - \sqrt{2}\sigma y_k + \sigma^2 - \\ -\sqrt{2}x_k\big(\sqrt{2}y_k - \sigma\big)\Big) exp\left(-\tfrac{\sqrt{2}x_k}{\sigma}\right) - \Big(x_{k+1}{}^2 + y_k{}^2 - \\ -\sqrt{2}\sigma y_k + \sigma^2 - \\ -\sqrt{2}x_{k+1}\big(\sqrt{2}y_k - \sigma\big)\Big) exp\left(-\tfrac{\sqrt{2}x_{k+1}}{\sigma}\right)\Big], \end{aligned} \tag{7}$$

$$\begin{aligned} D_{ol}(Q) = \tfrac{1}{2}\big[(x_2 - y_1)^2 + \sqrt{2}\sigma(-x_2 + y_1) + \\ +\sigma^2\big] exp\left(\tfrac{\sqrt{2}x_2}{\sigma}\right) + \tfrac{1}{2}\big[(x_L - y_L)^2 + \sqrt{2}\sigma(x_L - y_L) + \\ +\sigma^2\big] exp\left(-\tfrac{\sqrt{2}x_L}{\sigma}\right), \end{aligned} \tag{8}$$

with decision tresholds x_k and representation levels y_k, obtained from exspression (1) and (2),and presented by:

$$x_k = \begin{cases} \frac{x_{max}}{\mu}\left[(1+\mu)^{\frac{2(k-1)-L}{L}} - 1\right], & 0 \le x \le x_{max}, \\ \frac{x_{max}}{\mu}\left[1 - (1+\mu)^{\frac{L-2(k-1)}{L}}\right], & -x_{max} \le x \le 0, \end{cases} \tag{9}$$

$$y_k = \begin{cases} \frac{x_{max}}{\mu}\left[(1+\mu)^{\frac{2(k-1/2)-L}{L}} - 1\right], & 0 \le y \le x_{max}, \\ \frac{x_{max}}{\mu}\left[1 - (1+\mu)^{\frac{L-2(k-1/2)}{L}}\right], & -x_{max} \le y \le 0, \end{cases} \tag{10}$$

which are nessesery equation for further transmission error effect analyse.

Rayleigh fading cannel

Radio-wave propagation over wireless channels is impaired by a number of effects, including the multipath fading effect. It originates due to the constructive and destructive combination of randomly delayed, reflected, scattered, and diffracted signal components. There are different models describing the statistical behavior of the multipath fading envelope depending on the nature of the radio propagation environment. Rayleigh distribution is frequently used to model multipath fading where no direct line-of-sight (*LOS*) path exists. The instantaneous *SNR* per symbol of the channel, $\gamma = \alpha^2 Es/N_0$, in the case of a Rayleigh channel becomes

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \gamma \ge 0, \tag{11}$$

where the average *SNR* per symbol is denoted by $\bar{\gamma} = \Omega Es/N_0$, Es represents the energy per symbol, channel fading amplitude α is a random variable with mean-square value $\Omega = \overline{\alpha^2}$ and N_0 (W/Hz) is one-sided power spectral density of noise. The generic form of the expression for the error probability, when characterizing the performance of coherent digital communications, involves form of Gaussian Q-function

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2sin^2\theta}\right) d\theta. \tag{12}$$

To compute the average error probability one must evaluate an integral whose integrand consists of the product of the above-mentioned Gaussian Q-function and fading *PDF*, that is

$$I = \int_0^\infty Q(a\sqrt{\gamma}) p_\gamma(\gamma)\, d\gamma, \tag{13}$$

where a is a constant that depends on the specific modulation/detection combination. Substituting (11) and (12) into (13), and using the Laplace transform of the Rayleigh PDF (moment-generating function) which can be expressed in closed form with the result $M_\gamma(-s) = \int_0^\infty p_\gamma(\gamma)\exp(-s\gamma)\,d\gamma = (1+s\overline{\gamma})^{-1}$ gives

$$I = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{a^2\overline{\gamma}}{2sin^2\theta}\right)^{-1} d\theta = \frac{1}{2}\left(1-\sqrt{\frac{a^2\overline{\gamma}/2}{1+a^2\overline{\gamma}/2}}\right). \quad (14)$$

Denoting conditional bit error probability (*BEP*) by $P_b(E|\gamma)$, the average BEP in the presence of fading is obtained from

$$P_b(E) = \int_0^\infty P_b(E|\gamma)p_\gamma(\gamma)\,d\gamma. \quad (15)$$

Combining (11)-(15), we obtain average *BEP* for three well known specific modulation/detection combination:

1) average BEP of 2-AM over a Rayleigh fading channel for the binary case

$$P_b(E) = \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right), \quad (16)$$

2) average BEP of orthogonal BFSK over a Rayleigh fading channel

$$P_b(E) = \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}/2}{1+\overline{\gamma}/2}}\right), \quad (17)$$

3) average BEP of binary DPSK over a Rayleigh fading channel

$$P_b(E) = \frac{1}{2(1+\overline{\gamma})}. \quad (18)$$

These tree caracteristics are represented on Fig. 1.

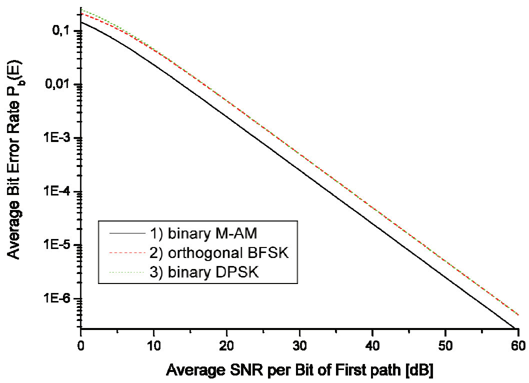

Fig. 1. Average BEP performance for suboptimum reception of 2-AM, orthogonal BFSK, and binary DPSK over Rayleigh fading

Transmission error effect

We examine in more detail the effect of a Rayleigh fading channel on the nonuniform scalar quantization system. A block fading channel is considered that remains constant over a block length and changes along different block lengths based on a Rayleigh distribution. We shall assume that random errors are introduced in the bits that convey information about quantizer output level. We assume a binary symmetric channel, which implies that information about quantizer output level will be transmitted as a sequence of R=log$_2L$, and we denote the average bit error probability, or bit error rate, by P_b. The above channel is assumed to be stationary memoryless channel, which implies that it´s properties do not change with time, and mapping between input and output at any given time is independent of previous outcomes.

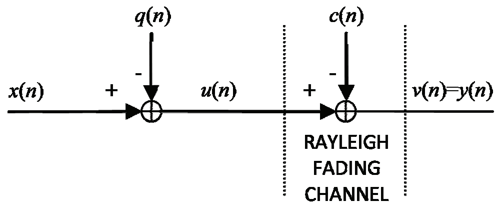

Fig. 2. Transmission of quantized amplitudes over Rayleigh fading channel

Referring to Fig. 2, we find that in the presence of transmission errors, reconstructed output signal $y(n)$ will include the effect of both quantization error $q(n)$ and channel error $c(n)$, resulting in a total reconstruction error $r(n)$ that is the sum of $q(n)=x(n)-u(n)$ and $c(n)=u(n)-v(n)$

$$r(n) = x(n) - y(n) = q(n) + c(n). \quad (19)$$

For exemple, the midrise quantizer maps each input sample $x(n)$ into one of a set of L rational numbres $u(n)\varepsilon\{y_k\}$; k=1,2,...,L. The representation level y_k is chosen if $x_{k+1}\geq x(n) > x_k$. The index k of input simbol y_k of the transmission system is transmitted to the receiver in a binary format, as the channel codeword. The received channel codeword is interpreted as one of the L output symbols $v(n)$)$\varepsilon\{z_k\}$; k=1,2,...,L, with $z_k = x_k$. A change in amplitude $\Delta_{k,j}=|y_k - z_j|$ results if the transmitted quantizer index k is changed to j because of channel errors. If this happens for input sample n, the corresponding channel error is $c(n)=\Delta_{k,j}$. The codewords in natural binary code (*NBC*) are merely the so-called binary representations of decimal numbers 0 to 2^R-1

$$codeword[b_1 \ldots b_R] decimal\ equvalent \sum_{r=1}^{R} 2^{R-r}\, b_r. \quad (20)$$

The bit with the highest weighting is b_1, the most significant bit (*MSB*). The least significant bit (*LSB*) is b_R. A usuful tool in channel error analyses is the Hamming distance D_{kj}, the number of codeword letters that are different in the representation of intended output level y_k and actual output level y_j. It can be seen that with a binary symmetric channel, and an R-bit code, the conditional probability that y_j will be received when y_k was sent, is given by

$$P_{kj} = P\left[V(n) = y_j | U(n) = y_k\right] = P_b^{D_{kj}}(1-P_b)^{R-D_{kj}}. \quad (21)$$

The total reconstruction error variance, in this case, is:

$$D(Q) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-v)^2 p_{xv}(x,v)\,dxdv. \quad (22)$$

Since $p_{xv}(x,v)=p(v|x)p_x(x)$ and $p(v|x) = \sum_{k=1}^{L} P[V = y_k|x]\,\delta(v-y_k)$, then

$$D(Q) = \sum_{j=1}^{L}\sum_{k=1}^{L} P_{kj}\int_{x_k}^{x_{k+1}}\left(x-y_j\right)^2 p_x(x)dx. \quad (23)$$

In this equation we should keep in mind know that when j,k ε [1,L/2], decision tresholds and representation levels x_k, y_j ε [-x$_{max}$,0] and it will be replace by appropriate expressions obtained in (9) and (10). The same stands for the case when j,k ε $[1+L/2,L]$, thus x_k, y_j ε $[0, x_{max}]$. Substituting (3) into (23), we can obtain closed form expression for total distortion

$$\begin{aligned} D(Q) = {} & \sum_{j=1}^{L}\Big\{\sum_{k=2}^{L/2}\frac{P_{kj}}{2}\Big[-\Big(x_k^2+y_j^2+\sqrt{2}\sigma y_j+\sigma^2 - \\ & -\sqrt{2}x_k(\sqrt{2}y_j+\sigma)\Big)exp\left(\frac{\sqrt{2}x_k}{\sigma}\right)+\Big(x_{k+1}^2+y_j^2+ \\ & +\sqrt{2}\sigma y_j+\sigma^2-\sqrt{2}x_{k+1}(\sqrt{2}y_j+\sigma)\Big)exp\left(\frac{\sqrt{2}x_{k+1}}{\sigma}\right)\Big]+ \\ & +\sum_{k=\frac{L}{2}+1}^{L-1}\frac{P_{kj}}{2}\Big[\Big(x_k^2+y_j^2-\sqrt{2}\sigma y_j+\sigma^2- \\ & -\sqrt{2}x_k(\sqrt{2}y_j-\sigma)\Big)exp\left(-\frac{\sqrt{2}x_k}{\sigma}\right)-\Big(x_{k+1}^2+y_j^2\mp \\ & +\sqrt{2}\sigma y_j+\sigma^2-\sqrt{2}x_{k+1}(\sqrt{2}y_j-\sigma)\Big)exp\left(-\frac{\sqrt{2}x_{k+1}}{\sigma}\right)\Big]+ \\ & \frac{P_{1j}}{2}\left[(x_2--y_j)^2+\sqrt{2}\sigma(-x_2+y_j)+\sigma^2\right]exp\left(\frac{\sqrt{2}x_2}{\sigma}\right)+ \\ & +\frac{P_{Lj}}{2}\Big[(x_L-y_j)^2+\sqrt{2}\sigma(x_L-y_j)+ \\ & +\sigma^2\Big]exp\left(-\frac{\sqrt{2}x_L}{\sigma}\right)\Big\}. \end{aligned} \tag{24}$$

By using well known relationship between signal power and total distortion, $SQNR$ can be calculated from

$$SQNR[dB] = 10log\frac{\sigma^2}{D(Q)}. \tag{25}$$

Let us compare $SQNR$ values obtained by using different value of R and P_b of Rayleigh fading channel. Fig. 3 sugest that at high bit rates, the quantizer system is more sensitive to significant multipath fading effect. It can be seen from Table 1 and Fig. 4 that the lower input powers generally require higher average SNR per simbol to achive nessesery $SQNR$ values.

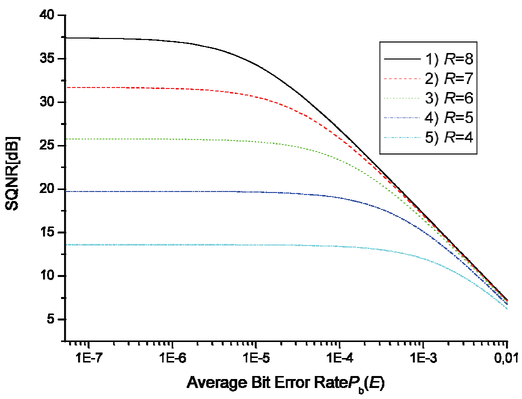

Fig. 3. Transmission quality ($SQNR$) of μ-law companding model versus $ABEP$ of 2-AM for Rayleigh fading

Table 1. Comparration of input power range, $SQNR$, and average SNR per symbol of 2-AM for Rayleigh fading channel

20log(σ/σ0)	$SQNR$[dB]($\bar{\gamma}$=49.8dB)	$SQNR$[dB]($\bar{\gamma}$=79.8dB)
-22	1.52	26.27
-16	7,53	30.92
0	26.87	36.82
16	36.38	37.43
22	23.69	23.71

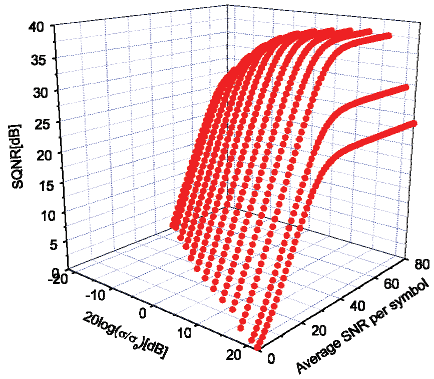

Fig. 4. Comparison of input power range, quality of transmission ($SQNR$), and average SNR per symbol of 2-AM for Rayleigh fading channel

Conclusions

An approach to the performance analysis of Laplacian source transmission over wirelles Rayleigh fading channel, for the purpose of finding optimal coding technique is presented.. Standard performance criterioun measure, ABEP is efficiently evaluated for several modulation transmission techniques. Numerical results for ABEP and output SNR and are presented and discussed in the function of various system parameters.

References

1. **Jayant N. S., Noll P.** Digital Communications over fading channels. – Prentice–Hall, New Jersey, Pb., 1984.
2. **Simon M. K., Alouini M. S.,** Digital Coding of Waveforms. – John Wiley & Sons, Inc, New York, Pb., 2000.
3. **Perić Z., Mosić A., Panić S.** Robust and switched nonuniform scalar quantization of Gaussian source in a wide dynamic range of power // AVT Journal, 2008. – No. 6. – P. 74–84.
4. **Perić Z.** Quantization Optimizations of Speech Signal in Wide Volume Range // Electronics and Electrical Engineering. – Kaunas: Technologija, 2003. – No. 3(45). – P. 41–48.
5. **Paulikas Š., Karpavičius R.** Application of Linear Prediction Coefficients Interpolation in Speech Signal Coding // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 8(80). – P. 39–42.
6. **Anskaitis A., Kajackas A.** The Tool for Quality Estimation of Short Voice Segments // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 8(104). – P. 103–108.

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Performance analysis of the Laplacian source wireless transmission over Rayleigh fading channels is presented. System performance analysis is based on the statistical approach, where capitalizing on derived closed-form expressions for receiving *SNR*, average bit error probabilty (*ABEP*) is efficiently evaluated for several modulation transmission techniques. Numerical results for this performance criterion are presented and discussed as the function of various system parameters. Ill. 4, bibl. 6, tabl. 1 (in English; abstracts in English and Lithuanian).

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Pateikta Laplaso šaltinio bevielio perdavimo Reilėjaus slopinimo kanalais našumo analizė, pagrįsta statistiniu požiūriu. Apskaičiuota kelių moduliacijos perdavimo būdų vidutinė bito klaidos tikimybė. Skaitiniai našumo kriterijaus rezultatai pateikiami ir nagrinėjami kaip įvairių sistemos parametrų funkcija. Il. 4, bibl. 6, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).