

# Monte Carlo Based Wireless Node Localization

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**Abstract**—In this paper we propose the WNMCL algorithm that allows iteratively locate unknown wireless nodes. The algorithm requires only one mobile node, which collects RSS signal from other nodes with unknown position and approximates their distance by path-loss model. These data are used for the probability occurrence correction of the nodes in the space modelled by Monte Carlo. The WNMCL algorithm was validated using simulations and the results are presented in this paper.

**Index Terms**—Localization, mobile nodes, Monte Carlo methods, wireless sensor networks.

## I. INTRODUCTION

Currently, the wireless node localization plays an essential role in many engineering areas and methods, which are developed for localization purposes. Monte Carlo localization (MCL) algorithm is widely used nowadays.

If the location is not known a priori, a robot solves the problem of position tracking (continuous localization). With the chosen representation of the environment model, MCL is able to represent the position with multimodal probability distributions and work with multiple hypotheses about the final position. With the robot's movement, position is refined and eventually only one hypothesis about the position outweighs. At this point, the robot is located absolutely.

Absolute position obtained posteriori can be a serious problem in applications where we need to know the absolute position sooner or very soon after the robot starts moving. In such cases it is possible to use the location based on wireless networks. Like GPS, this localization is able to provide absolute position directly assuming knowledge of the position of its nodes. In unknown environments it is a useful to implement method that is able to locate these nodes.

Many existing algorithms are limited to GPS, requires some beacon nodes or hand-placing method merely. Other methods have some limited usage in short range networks like indoor Wi-Fi.

Paper [1] proposed Offline LEGMM and Online LEGMM methods, which can approximate position of unknown wireless networks nodes, assuming that the position of RSS-

collector is known. The methods are based on the calculation of the node's distance to the RSS-collector and path that RSS-collector travels. These methods work above the grid that manages the probability of occurrence of the located nodes.

This paper proposes the Monte Carlo based algorithm for location of wireless network nodes and it indirectly benefits from MCL approach. Such benefit is e.g. concentration of computing power to areas in which the high probability of occurrence of the searched nodes is. The proposed method like LEGMM manages with the minimal knowledge of the searched nodes and it is reduced only to compute with the received signal intensity, without node's explicit resolution (e.g. using MAC addresses).

The ability to obtain the position of nodes online opens up new possibilities, where the mobile system can during its movement use nodes, which have been already localized, for backward localization and due to this fact the robustness of the existing localization system can be increased.

## II. SIGNAL MODELLING

The relationship between the RSS and the distance between transmitter and receiver is defined by the path-loss model. Signal propagation is affected by losses and needs to be filtered. Filtration is carried out by Gaussian filter and it calculates the actual position with multilateration.

### A. Modelling Radio Connection Power Balance

The radio signal is being attenuated by its propagation and is affected by a few physical mechanisms. This power balance is demonstrated in (Fig. 1).

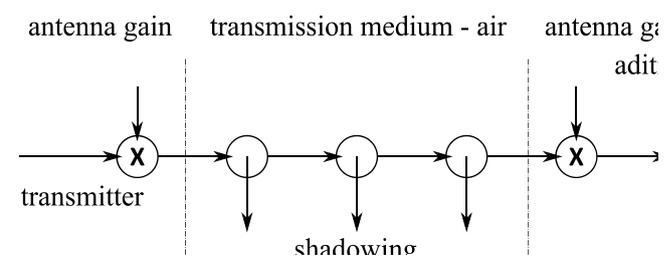


Fig. 1. Model of signal propagation.

All the noise caused by interference and the own noise of the receiver and transmitter are considered as an additive noise of the receiver. The own attenuation of the signal consists of three components [2]:

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– *Shadowing*: attenuation caused by obstacles. In terms of wavelength, it is relatively slow fluctuations and decline may be very large.

– *Path loss*: attenuation depends on the distance and the type of communication environment. It is time-invariant and from the localization point of view, it is the component, which bears the information.

– *Fast fading*: causes very rapid and deep fluctuations of the signal power. It occurs mainly due to multipath signal propagation and Doppler shift that arises from the movement of mobile antennas and surrounding objects.

### B. Path-Loss Model

The path-loss model can be used to predict distance to transmitter, in environments without obstacles, where we expect a smooth sloping curve. The path-loss model can be derived from the basic empirical model (see Fig. 2), which can be expressed according to (1) [3]

$$r = t - l_0 - 10 \cdot \alpha \cdot \log \frac{d}{d_0} - S, \quad (1)$$

where  $d \geq d_0$ ,  $r$  is the power at the receiver side, including the antenna gain of the receiver (dBm);  $t$  is the effective radiated power, antenna gain, including transmitter (dBm);  $l_0$  refers to the path-loss at  $d_0$  (dBm);  $\alpha$  is the path-loss exponent, which depends on the characteristics of the transmission channel;  $d$  is the distance between the transmitter and receiver (m)  $d_1$  is the reference distance (m) and  $S$  stands for slow leaks, log-normal shadow fading (dB).

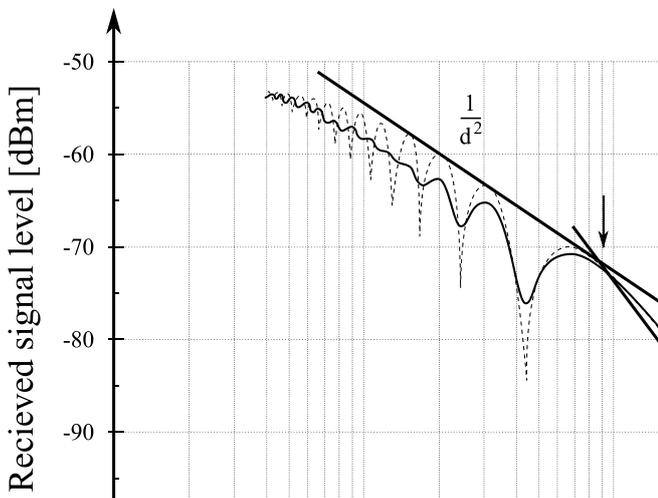


Fig. 2. Path-loss model.

### C. Gaussian Filter

The measured value of RSS is unstable and time-variant. The error caused by *fast fading* can be very high. In order to evaluate the RSS, it is necessary to filter those fading. The Gaussian filter is suitable for the filtration of those signals, and can be described as discrete convolution with one-dimensional sampled Gaussian kernel [4]

$$L(k, t) = \sum_{n=0}^k \frac{1}{\sqrt{2\pi}t} \exp\left(-\frac{n^2}{2t}\right) \times r(k-n), \quad (2)$$

where  $k \in \mathbb{Z}$ ,  $t > 0$ .

### D. Multilateration

The multilateration is a mathematical apparatus based on the distance from known reference nodes to the position of unknown node (Fig. 3). Although it is possible to approximate the position based on the distance from the wireless network nodes, e.g. the usage of these data in the MCL correction phase, multilateration can be particularly preferred for its speed.

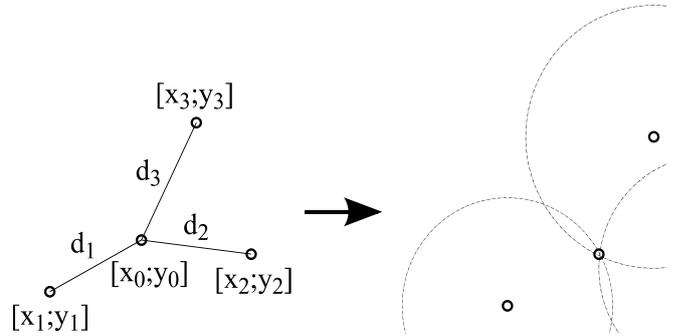


Fig. 3. Special case of multilateration with 3 reference nodes [3].

The mathematical description of the multilateration is an analytical system of equations, describing circles around the reference nodes. The formula for  $n$ -nodes, which allows calculating the coordinates of unknown node directly, has the form [3], [5]:

$$\mathbf{H} \cdot \bar{\mathbf{x}} = \bar{\mathbf{b}}, \quad (3)$$

$$\text{where } \bar{\mathbf{x}} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}.$$

$$\mathbf{H} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \\ \vdots & \vdots \\ x_n - x_1 & y_n - y_1 \end{bmatrix}, \quad (4)$$

$$\bar{\mathbf{b}} = \frac{1}{2} \begin{bmatrix} (x_2^2 + y_2^2 - d_2^2) \cdot (x_1^2 + y_1^2 - d_1^2) \\ (x_3^2 + y_3^2 - d_3^2) \cdot (x_1^2 + y_1^2 - d_1^2) \\ \vdots \\ (x_n^2 + y_n^2 - d_n^2) \cdot (x_1^2 + y_1^2 - d_1^2) \end{bmatrix}. \quad (5)$$

In practice, distances are affected by errors. These errors cause that the system of equations is incompatible and  $\bar{\mathbf{x}} \notin \mathcal{R}(\mathbf{H})$ . The circles may not intersect or the intersection is at multiple points, so this simple calculation cannot be used. This problem is known as the distance noise and it can be solved using some approximation methods. We used the method of least squares in this work. The general equation for calculating the multilateration is following [6]

$$\bar{\mathbf{x}} = (\mathbf{H}^T \cdot \mathbf{H})^{-1} \mathbf{H}^T \cdot \bar{\mathbf{b}}. \quad (6)$$

### III. NETWORK NODES LOCALIZATION ALGORITHM

The Monte Carlo method represents the probability distribution of the position with a set of particles (samples). This representation of the distribution is quite often used in many different fields. It is also known as a particle filter, condensation algorithm as well as the bootstrap filter or survival of the fittest [6].

#### A. Monte Carlo Localization

The basic idea of the MCL is a suitable approximation of the believe  $Bel(x)$  with weighted samples, so discrete distribution defined by the samples correspond to the initial continuous distribution. The distribution of the believe is represented by a uniform distribution of the samples set  $m$ , where  $M$  is a count of samples with weight  $M^{-1}$ .

MCL algorithm in principle updates the state by creating a new set of samples from the current set in response to incoming information about the movement of the robot  $u_{t-1}$  (prediction) and the sensory data  $z_{t-1}$  (correction) [7], [8]:

1. Select a random sample  $x_{t-1}$  from current description of the believe  $Bel_{t-1}(x_{t-1})$ .
2. Estimate the new possible position  $x_t$  by the probability distribution  $P(z_t | x_t, m)$  for the selected sample  $x_{t-1}$ .
3. Assign the default value weight to the sample  $x_t$  according to distribution of sensor model and add the sample  $x_t$  to the set of samples representing posterior believe  $Bel_t(x_t)$ .

Steps 1–3 iterate over all the state space samples  $Bel_t(x_t)$  and finally their weights are normalized so their sum is equal to one. Summing up the above facts, it is possible to describe the activities of MCL in the following steps:

- *Prediction*: the translation of all samples based on the information about the change in node position.
- *Correction*: the modification of each sample from set and their weighting by consensus with expectation and the measurements.

#### B. Wireless Nodes Monte Carlo Localization (WNMCL)

The modified MCL – WNMCL is utilized to locate the nodes of wireless network. Each node is represented by the MCL cluster of samples in the state space. The resulting position is derived from the cluster centroids  $\|x\|$ .

The WNMCL algorithm can be described as follows:

1. RSS is measured and filtered using Gaussian filter.
2. Distances  $d_t$  of unknown nodes to the RSS-collector position  $p_t$  are estimated by the path-loss model to  $D_t$ .
3. MCL correction: Euclidean distance of each sample from  $Bel_{t-1}(x_{t-1})$  to the RSS-collector  $\|p_t - x_t^{[m]}\|$  is calculated. From  $D_t$  such  $d_t$  it is selected, that  $\tau_t = (\|p_t - x_t^{[m]}\| - d_t)^2 \min$ . Weight  $w_t^{[m]}$  of sample  $x_t^{[m]}$  is updated according to the minimal difference  $w_t^{[m]} = 1/(1 - \tau_t)$ .
4. MCL resampling and prediction: The samples are resampled by low variance sampler algorithm [7] with position noise injection.
5. The samples are divided into clusters by the weighted variance of the cluster centroid. Each cluster then potentially represents the position of the localized node.

#### Algorithm 1 WNMCL Particles Correction and Resampling

**Require:**  $x_{t-1}, D_t, p_t$

```

 $\bar{x}_t \leftarrow W, \bar{w}_t \leftarrow W$ 
for  $m \leftarrow 1$  to  $M$  do
   $x_t^{[m]} \leftarrow x_{t-1}^{[m]}, d_t^{[1..n]} \leftarrow D_t$ 
   $w_t^{[m]} \leftarrow \frac{1}{1 + \min\left(\left(\|p_t - x_t^{[m]}\| - d_t^{[1..n]}\right)^2\right)}$ 
   $\bar{x}_t \leftarrow \bar{x}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$ 
end for
for  $m \leftarrow 1$  to  $M$ 
do //resampling
  draw sample from  $\bar{x}_t$  with probability  $w_t^{[m]}$ 
   $\bar{x}_t \leftarrow \bar{x}_t + \langle x_t^{[m]}, 1 \rangle$ 
end for
return  $\bar{x}_t$ 

```

where  $X_t$  stands for the set of samples (representing posterior probability distribution).

#### C. Clustering Algorithm

The methods suitable for clustering are those that generate clusters with a low dispersion of samples around the centroid. Experimentally proved methods are *ward* [9] and *k-means* [10]. The *ward* method minimizes the total variance within clusters. It can be summarized in the following steps:

1. *Initialization*: each sample is a separate cluster (clusters are singletons).
2. Find the pair of clusters that have the minimum Euclidean distance of its centroid.
3. Merge pair.
4. Repeat steps 2 to 3 until the number of clusters is greater than  $K$ .

#### Algorithm 2 Clustering algorithm

```

Require:  $\bar{x}_t$ 
 $done \leftarrow 1$ 
for  $i \leftarrow 1$  to  $M$  and  $done = 0$  do
   $C^{[i]} \leftarrow \text{ward}(\bar{x}_t, i)$ 
   $done \leftarrow 1$ 
  for  $c$  in  $C^{[i]}$  do
     $centroid \leftarrow [\text{mean}(c_x), \text{mean}(c_y)]$ 
     $\dagger \leftarrow \text{var}(\|c - centroid\|)$ 
    if  $\dagger > \dagger_{\max}$  then
       $C \leftarrow C^{[i]}$ 
       $done \leftarrow 0$ 
      break
    end if
  end for
end for
return  $C$ 

```

where  $\dagger_{\max}$  is threshold cluster variance.

Many heuristic methods are used to determine the optimal number of clusters [10], [10]. The WNMCL uses the method that chooses such  $K$ , so that the total inner variance of weighted samples variance from the centroid of each cluster is less than a stated threshold. The clustering algorithm is

described in **Algorithm 2**.

#### IV. TESTING

The WNMCL algorithm verification was performed on a model situation on one floor of a building with thin walls (their influence is insignificant). RSS-collector position is known in advance. The odometry and laser sensor are used for RSS-collector own localization. The number of samples dedicated to the localization of the WiFi nodes (AP) is 500.

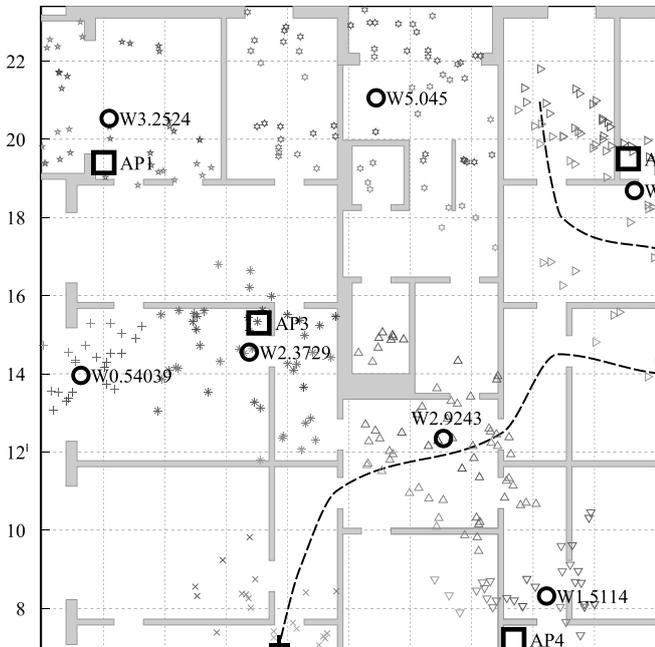


Fig. 4. Localization of Wi-Fi nodes, 3 iteration.

The samples are randomly spread out at the beginning, but after the first few iterations the samples tend to converge on the circles, where APs are located, see Fig. 4.

In the 12th iteration all the APs are located, even if the number of clusters is greater than the number of APs. The same number of clusters and APs from the 18th iteration.

The graph on Fig. 6 shows the accuracy and localization speed assessment according to the number of samples. The number of clusters depends on the number of iterations and the number of samples is on Fig. 7.

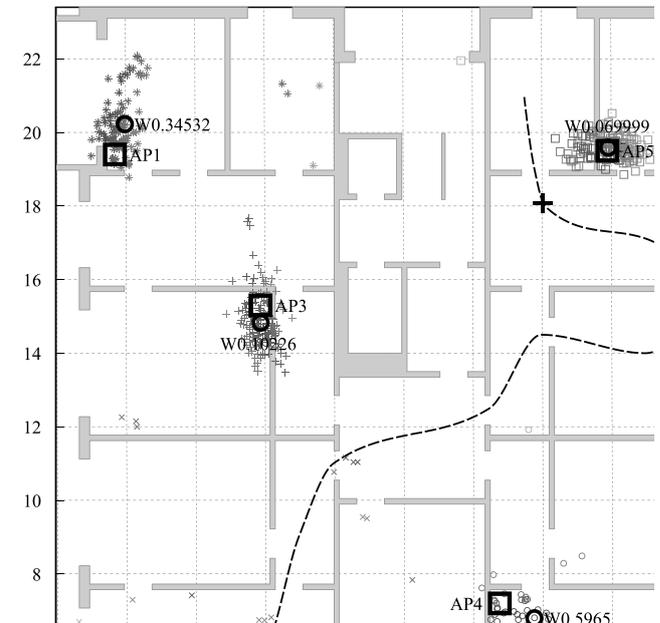


Fig. 5. Localization of Wi-Fi nodes, 18 iteration.

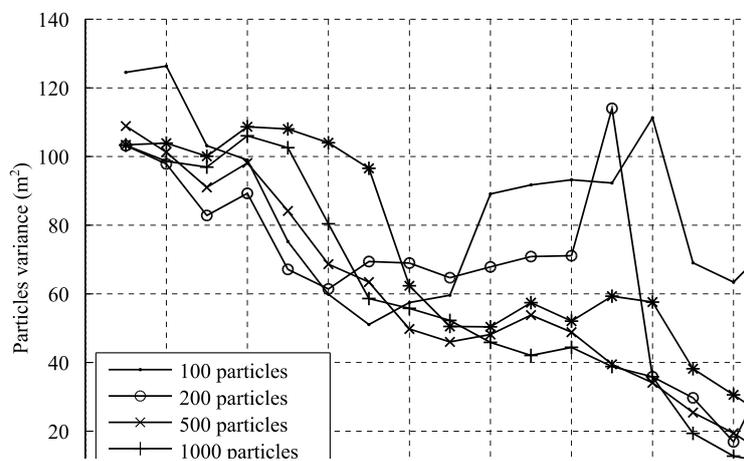


Fig. 6. Weighted variation distance of samples from the correct position in relation to the number of samples.

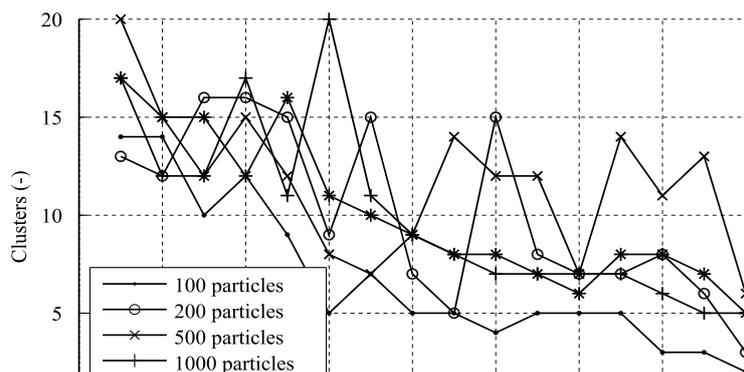


Fig. 7. The number of clusters in iterations.

## V. CONCLUSIONS

We have presented the algorithm for the location estimation of unknown wireless network nodes. The algorithm is based on MCL and probabilities of distribution modelled by a set of samples. The computational complexity is relatively low due to the fact that the localization uses only the unified set of samples for modelling all visible APs.

The further work is to verify the algorithm in a real environment. The possible continuation is to extend the algorithm for situations where there is no known (or initial) position of RSS-collector.

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