

# Coding Algorithm Based on the Simplified Semilogarithmic Compression Law for Discrete Input Samples with Laplacian Distribution

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**Abstract**—In this paper, a novel simple algorithm based on the simplified semilogarithmic quantizer is proposed for Laplacian source coding. An analysis of the signal to quantization noise ratio (SQNR) of the simplified semilogarithmic quantizer is provided in the wide variance range for the cases of discrete input samples and continual input samples with Laplacian distribution. For the assumed discrete input samples and continual input samples it is shown that the higher SQNR is achieved by the proposed semilogarithmic quantizer when compared to the uniform quantizer. Another major contribution of this paper is the proof that in the area of smaller variance values the simplified semilogarithmic quantizer provides significantly higher SQNR values than the uniform quantizer, which is of great importance, especially in image coding, where the range of smaller variances is more probable than the range of higher variance values. Due to the simple realization structure and higher achieved SQNR than the uniform quantizer, one can expect that the proposed algorithm will be very applicable in coding of signals which, as well as speech signals and signals of difference between adjacent pixel values of image, follow Laplacian distribution.

**Index Terms**—Coding algorithm, uniform quantizer, simplified semilogarithmic quantizer, discrete input samples.

## I. INTRODUCTION

The A/D conversion is the process of converting a continuous analog signal into a digital signal [1]. Quantization is one of the main steps in A/D conversion and it is usually performed in two phases [2]. In the first phase, quantizers with a large number of quantization levels are usually used [3]. After that, with the aim to compress the quantized samples, which have discrete amplitudes, in the second phase, quantizers with much lower number of levels are used. Quantizers in the first phase have continual input samples (i.e. samples with continual amplitudes) while the quantizers in the second phase have discrete input samples (i.e. samples with discrete amplitudes). The goal of this paper is to design the quantizer for the second phase, i.e. the quantizer for discrete input samples. The uniform quantizer for discrete input samples has already been considered in [4]. Although the semilogarithmic quantizer for continual

input samples has been discussed in [5]–[11], however, to the best of the authors' knowledge, the semilogarithmic quantization of discrete input samples has not been reported in the literature yet. Accordingly, in this paper, the simplified semilogarithmic quantizer is proposed for the quantization of discrete input samples with the Laplacian probability density function (PDF). The simplified semilogarithmic quantizer we propose in this paper is simpler for practical realization compared to the classic semilogarithmic quantizer. In general, a semilogarithmic quantizer is a kind of hybrid quantizer consisting of a uniform and a nonuniform logarithmic quantizer [1]. The main advantage of the simplified semilogarithmic quantizer over the classic semilogarithmic quantizer is based on the fact that his uniform quantizer has the unit gain. This fact enables independent design of the uniform and the nonuniform logarithmic quantizer constituting the simplified semilogarithmic quantizer. However, in the case of the classic semilogarithmic quantizer, his uniform quantizer generally has an arbitrary defined gain and its design is dependent on the design of his nonuniform logarithmic quantizer [1]. The simplified semilogarithmic compressor function need not be continual, which enables independent design of the uniform and the logarithmic quantizer. On the other side, the compressor function of the classic semilogarithmic quantizer is continual, i.e. it has a unique value at the point that separates the region where the uniform quantizer is defined from the region where the logarithmic quantizer is defined [1]. Accordingly, the design of the uniform quantizer and the logarithmic quantizer constituting the classic semilogarithmic quantizer are mutually dependent. Moreover, the realization of the uniform quantizer with an arbitrary defined gain, which is the part of the classic semilogarithmic quantizer, is a more complex than realization of the uniform quantizer with the unit gain, which is the part of the simplified semilogarithmic quantizer, because in the first case, it is necessary to additionally use amplifiers. These are the reasons that point out the advantages of using the simplified semilogarithmic quantizer rather than the classic semilogarithmic quantizer.

This paper is organized as follows. Section II describes performances of the simplified semilogarithmic quantizer and the uniform quantizer, both designed for the continual input and Laplacian PDF. Section III derives performances

of the simplified semilogarithmic quantizer and the uniform quantizer designed for the discrete input and Laplacian PDF. In addition, the design procedure of the simplified semilogarithmic quantizer and novel coding algorithm description is given. Numerical results are given in Section IV. In Section V, experimental results are presented. Finally, Section VI is devoted to the conclusions which summarize the contribution achieved in the paper.

## II. PERFORMANCES OF THE UNIFORM QUANTIZER AND THE SIMPLIFIED SEMILOGARITHMIC QUANTIZER DESIGNED FOR CONTINUAL INPUT SIGNAL WITH LAPLACIAN PDF

Samples of a signal, which come onto the input of quantizers, are usually continual in amplitude, i.e. they can take any real value from the interval  $(-\infty, +\infty)$ . In this paper we consider Laplacian PDF of the input samples, which is one of the most commonly used distributions [1]. Without any loss of generality, we assume that the information source is Laplacian source with memoryless property and zero mean value. The probability density function for a random variable with a Laplacian distribution, zero mean and variance  $\sigma^2$  is given by

$$p(x) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{\sqrt{2}|x|}{\sigma}\right). \quad (1)$$

During quantization, an error is made, which can be measured by distortion. In general, the total distortion  $D$  is equal to the sum of the granular  $D_g$  and the overload  $D_o$  distortion, i.e.  $D = D_g + D_o$ . The quality of the quantized signal along with distortion is usually measured by signal to quantization noise ratio (SQNR) [1]

$$\text{SQNR} = 10 \log_{10} \left( \frac{\sigma^2}{D} \right), \quad (2)$$

where  $\sigma^2$  is a variance of an input signal, and  $D$  is the total distortion. In this paper, the analysis of numerical results is conducted using SQNR rather than distortion.

### A. Performances of the Uniform Quantizer Designed for Continual Laplacian Source

A uniform quantizer  $Q_u$  is defined by the following parameters:  $N$  - the number of quantization levels,  $x_{\max}$  - the support region threshold and  $\Delta = 2x_{\max} / N$  - the quantizer step size [4]. The granular  $D_g$  and the overload  $D_o$  distortion of a uniform quantizer for the assumed Laplacian PDF of variance  $\sigma^2$  are respectively given by [4]:

$$D_g(Q_u) = \frac{k^2 t^2 \sigma^2}{3N^2} (1 - \exp(-\sqrt{2}kt)), \quad (3)$$

$$D_o(Q_u) = \exp(-\sqrt{2}kt) \sigma^2 (1 + \sqrt{2} \frac{kt}{N} + (\frac{kt}{N})^2), \quad (4)$$

where  $t = \sigma / \Delta$  and  $k = x_{\max} / \sigma$ .  $\sigma_0^2$  denotes a referent variance for which the uniform quantizer is designed.

### B. Performances of the Simplified Semilogarithmic Quantizer Designed for Continual Laplacian Source

The simplified semilogarithmic quantizer  $Q_s$  is a kind of a hybrid quantizer consisting of a uniform and a nonuniform logarithmic quantizer [9]. Its support region  $[-x_{\max}, x_{\max}]$  is composed of the uniform part  $[-x_{\min}, x_{\min}]$  and the logarithmic part  $[-x_{\max}, -x_{\min}) \cup (x_{\min}, x_{\max}]$ .  $x_{\min}$  is the

threshold between these two parts and  $x_{\max}$  is the support region threshold of the simplified semilogarithmic quantizer. The simplified semilogarithmic quantizer is defined according to the compressor function proposed in [9]

$$c(x) = \begin{cases} x, & |x| \leq x_{\min}, \\ \frac{x_{\max}}{B} (1 + \log(\frac{B}{x_{\max}} |x|)) \operatorname{sgn}(x), & x_{\min} < |x| \leq x_{\max}. \end{cases} \quad (5)$$

Parameter  $B$  denotes the ratio  $B = x_{\max} / x_{\min}$ .  $N = N_1 + N_2$  and  $N$  is the total number of quantization levels, whereas  $N_1$  and  $N_2$  are the number of quantization levels in the uniform and in the logarithmic part, respectively. It is obvious that the uniform part has a unit gain. Accordingly, there are certain advantages in the realization of the uniform quantizer, which is an integral part of the simplified semilogarithmic quantizer, and, accordingly, in the realization of the whole simplified semilogarithmic quantizer. One of the simplified semilogarithmic quantizer's advantages is the fact that his uniform quantizer can be designed apart from his nonuniform logarithmic quantizer. That is the main reason we have named this semilogarithmic quantizer the simplified semilogarithmic quantizer [9]. In the case of the simplified semilogarithmic quantizer  $Q_s$ , the granular distortion consists of the distortions from the uniform  $D_{g1}(Q_s)$  and logarithmic part  $D_{g2}(Q_s)$ , i.e.  $D_g = D_{g1}(Q_s) + D_{g2}(Q_s)$ . Accordingly, the total distortion of the simplified semilogarithmic quantizer  $D(Q_s)$  consists of three components, two granular distortions  $D_{g1}(Q_s)$  and  $D_{g2}(Q_s)$ , and one overload distortion  $D_o(Q_s)$ . For the assumed Laplacian source of variance  $\sigma^2$  the following expressions for these distortions are obtained in [9]:

$$D_{g1}(Q_s) = \frac{x_{\min}^2}{3N_1^2} \left( 1 - \exp\left(-\frac{\sqrt{2}x_{\min}}{\sigma}\right) \right), \quad (6)$$

$$D_{g2}(Q_s) = \frac{(\log B)^2}{3N_2^2} \times \left( \exp\left(-\frac{\sqrt{2}x_{\min}}{\sigma}\right) (x_{\min}^2 + \sqrt{2}\sigma x_{\min} + \sigma^2) - \exp\left(-\frac{\sqrt{2}x_{\max}}{\sigma}\right) (x_{\max}^2 + \sqrt{2}\sigma x_{\max} + \sigma^2) \right), \quad (7)$$

$$D_o(Q_s) = \sigma^2 \exp\left(-\frac{\sqrt{2}x_{\max}}{\sigma}\right). \quad (8)$$

## III. PERFORMANCES OF THE UNIFORM QUANTIZER AND THE SIMPLIFIED SEMILOGARITHMIC QUANTIZER DESIGNED FOR THE DISCRETE INPUT SAMPLES WITH LAPLACIAN PDF

Assume that continual amplitude samples having Laplacian PDF are quantized by the uniform quantizer  $Q_u$  having  $N_u$  output levels  $X = \{x_1, \dots, x_{N_u}\}$ . Let us denote the maximal amplitude of the uniform quantizer  $Q_u$  with  $x_{\max\text{cont}}$ . This amplitude depends on the input signal amplitude range. The design of the quantizer  $Q_u$  can be described in another way: firstly, the uniform quantizer for the unit standard deviation ( $\sigma = 1$ ) is designed with the maximal amplitude  $x_{\max\text{cont}}^{\sigma=1}$  [1] and after that, the denormalization is done, by dividing all thresholds and representation levels with  $\delta = \frac{x_{\max\text{cont}}^{\sigma=1}}{x_{\max}}$  ( $x_{\max}$  is maximal amplitude of discrete input samples). The denormalization is performed with the aim to adjust the support region  $[-x_{\max\text{cont}}^{\sigma=1}, x_{\max\text{cont}}^{\sigma=1}]$  to the support region  $[-x_{\max}, x_{\max}]$ . In the second step, samples having discrete amplitudes that are the output of the quantizer  $Q_u$  are further quantized by the second quantizer  $Q$

having  $N$  levels,  $N < N_u$ . The input samples of the quantizer  $Q$  can take  $N_u$  discrete values from the set  $X$  of the output levels of the quantizer  $Q_u$ . The probabilities of these discrete levels for Laplacian PDF are  $P(x_i) = p(x_i)\Delta_u = \left(\frac{1}{\sqrt{2}\sigma}\right) \exp\left(-\frac{\sqrt{2}|x_i|}{\sigma}\right)\Delta_u$ ,  $i = 2, \dots, N_u-1$ ,  $\Delta_u = \frac{2x_{\max}}{N_u}$ , and  $P(x_1) = P(x_{N_u}) = \frac{1}{2} \exp\left(-\frac{\sqrt{2}x_{\max}}{\sigma}\right)$  [10]. Since discrete samples are limited in amplitude by  $x_{\max}$  quantizer  $Q$ , having the support region  $[-x_{\max}, x_{\max}]$ , produces only the granular distortion  $D_g$ , i.e. the total distortion is equal to the granular distortion  $D = D_g$ . The aim of this paper is to design the quantizer  $Q$ .

#### A. Design of the Uniform Quantizer for Discrete Input Samples

Let us recall shortly explanation of the uniform quantizer for discrete input samples that is reported in [4]. Output levels of the  $N$ -level quantizer  $Q$  are denoted by  $y_j$ ,  $j = 1, \dots, N$  and it is valid that  $N_u = N \cdot L$ , where  $L$  is an integer. This means that  $L$  discrete input levels  $X_j = \{x_{j1}, \dots, x_{jL}\} \in X$  are mapped to one output level  $y_j$ ,  $j = 1, \dots, N$ . Since the quantizer  $Q$  is symmetric during the design we take into account only the positive range. The quantization step size of the considered quantizer designed for  $\sigma = 1$  is  $\Delta = 2x_{\max}^{\sigma=1}/N$  and, accordingly, the thresholds and representation levels are given by [4]:

$$x_i^{\sigma=1} = i \times \Delta, \quad i = 0, \dots, N/2, \quad (9)$$

$$y_i^{\sigma=1} = (i - 1/2) \times \Delta, \quad i = 1, \dots, N/2. \quad (10)$$

The support region of the uniform quantizer obtained in this way  $[-x_{\max}^{\sigma=1}, x_{\max}^{\sigma=1}]$  is different from the support region  $[-x_{\max}, x_{\max}]$ . In order to adjust the support region of the uniform quantizer, the thresholds and representation levels should be divided by  $\delta_n = x_{\max}^{\sigma=1}/x_{\max}$ :

$$x_i = x_i^{\sigma=1}/\delta_n, \quad i = 0, \dots, N/2, \quad (11)$$

$$y_i = y_i^{\sigma=1}/\delta_n, \quad i = 1, \dots, N/2. \quad (12)$$

The granular distortion of the uniform quantizer for discrete input samples is given by [4]

$$D_g = \sum_{j=1}^N \sum_{k=1}^L (x_{jk} - y_j)^2 P(x_{jk}). \quad (13)$$

#### B. Design of the Simplified Semilogarithmic Quantizer for Discrete Input Samples

The design of the simplified semilogarithmic quantizer is based on the companding technique application, whose scheme is shown on Fig. 1.

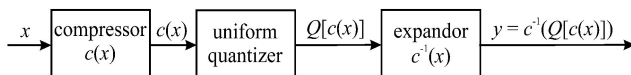


Fig. 1. Companding quantizer.

From Fig. 1 one can observe that nonuniform quantization can be achieved in the following way: first, by compressing the input signal  $x$  using a nonuniform compressor characteristic  $c(x)$ ; next by quantizing the compressed signal  $c(x)$  by employing a uniform quantizer  $Q(c(x))$ ; and finally by expanding the quantized version of the compressed

signal using a nonuniform transfer characteristic  $c^{-1}(Q(c(x)))$ , which is inverse to the characteristic of the compressor. The overall structure, which consists of a compressor, a uniform quantizer and an expander in cascade, is called a compandor [1], or a companding quantizer.

Figure 2 shows the positive range of the simplified semilogarithmic compressor function  $c(x)$ . The representation levels  $x_i$  of the simplified semilogarithmic quantizer are denoted on the  $x$  axis. Since the considered simplified semilogarithmic quantizer is symmetric while designing we take into account only the positive range.

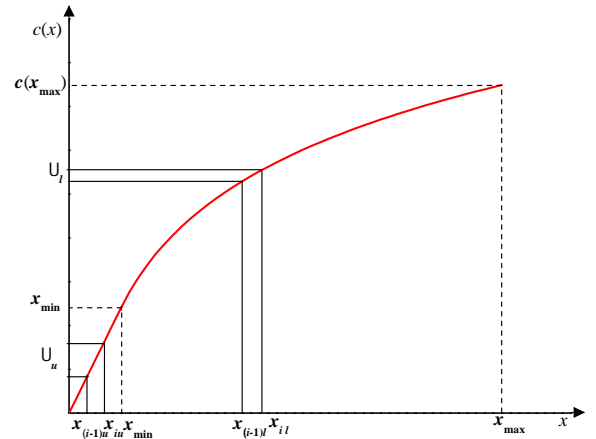


Fig. 2. The simplified semilogarithmic compressor function.

The thresholds and representation levels in the uniform part are defined by the following expressions:

$$x_{iu}^{\sigma=1} = i \times \Delta_u, \quad i = 0, \dots, N_1/2, \quad (14)$$

$$y_{iu}^{\sigma=1} = (i - 1/2) \times \Delta_u, \quad i = 1, \dots, N_1/2, \quad (15)$$

where  $\Delta_u = 2x_{\min}^{\sigma=1}/N_1$  and  $\Delta_l = 2x_{\min}^{\sigma=1} \log B/N_2$ ,  $N = N_1 + N_2$ . The thresholds and representation levels in the logarithmic part of the simplified semilogarithmic quantizer are defined by:

$$c(x_{il}^{\sigma=1}) = x_{\min}^{\sigma=1} \left(1 + \log \frac{x_{il}^{\sigma=1}}{x_{\min}^{\sigma=1}}\right) = x_{\min}^{\sigma=1} + i\Delta_l, \quad i = 0, \dots, N_2/2, \quad (16)$$

$$c(y_{il}^{\sigma=1}) = x_{\min}^{\sigma=1} \left(1 + \log \frac{y_{il}^{\sigma=1}}{x_{\min}^{\sigma=1}}\right) = x_{\min}^{\sigma=1} + (i - 1/2)\Delta_l, \quad i = 1, \dots, N_2/2. \quad (17)$$

By solving (16) and (17), the following expressions for the thresholds and representation levels in the logarithmic part are obtained:

$$x_{il}^{\sigma=1} = x_{\min}^{\sigma=1} B^{2i/N_2}, \quad i = 0, \dots, N_2/2, \quad (18)$$

$$y_{il}^{\sigma=1} = x_{\min}^{\sigma=1} B^{(2i-1)/N_2}, \quad i = 1, \dots, N_2/2, \quad (19)$$

where  $B$  denotes ratio  $B = x_{\max}^{\sigma=1}/x_{\min}^{\sigma=1}$ ,  $x_{\max}^{\sigma=1}$  is the support region threshold of the simplified semilogarithmic quantizer and  $x_{\min}^{\sigma=1}$  is the threshold between the uniform and the logarithmic part of the simplified semilogarithmic quantizer. The support region of the simplified semilogarithmic quantizer obtained in this way is  $[-x_{\max}^{\sigma=1}, x_{\max}^{\sigma=1}]$  and it has to be adjusted to the range  $[-x_{\max}, x_{\max}]$ . This means that the thresholds and

representation levels should be divided by  $\delta_n = x_{\max}^{\sigma=1}/x_{\max}$ :

$$x_{iu} = x_{iu}^{\sigma=1}/\delta_n, \quad i = 0, \dots, N_1/2, \quad (20)$$

$$y_{iu} = y_{iu}^{\sigma=1}/\delta_n, \quad i = 1, \dots, N_1/2, \quad (21)$$

$$x_{il} = x_{il}^{\sigma=1}/\delta_n, \quad i = 0, \dots, N_2/2, \quad (22)$$

$$y_{il} = y_{il}^{\sigma=1}/\delta_n, \quad i = 1, \dots, N_2/2. \quad (23)$$

Granular distortion of the simplified semilogarithmic quantizer is given by

$$D_g = \sum_{i=1}^{N_1/2} \sum_{j=1}^{\mu_i} (x_{ij} - y_{iu})^2 P(x_{ij}) + 2 \sum_{i=1}^{N_2/2} \sum_{j=1}^{\mu_{N_1/2+i}} (x_{(N_1/2+i)j} - y_{il})^2 P(x_{(N_1/2+i)j}), \quad (24)$$

where  $\mu_i, i = 1, \dots, N$  is the parameter which stands for the number of the input levels that are mapped to one output level. Also, it is valid that  $N_u = \sum_{i=1}^N \mu_i$ . Finally,  $x_{ij} \in X$  are input samples of the simplified semilogarithmic quantizer.

### C. Novel Coding Algorithm

Every quantizer can be viewed as the combination of an encoder and a decoder. In the case of a compandor, the encoder consists of a compressor and a uniform quantizer while the decoder is composed of an expander [1]. Assume that onto the input of an encoder come  $M$  samples  $x_i, i = 1, \dots, M$ , which belong to the set of  $N_0$  different samples from the set  $X$  ( $x_i \in X$ ). Therefore, at the output of the uniform quantizer there are  $M$  quantization levels  $q_i, i = 1, \dots, M$  which can take the values from the set of  $N$  different values that are defined by the representation levels of the uniform quantizer.

The novel coding algorithm for the  $N$ -level quantizer defined by the considered simplified semilogarithmic compressor function  $c(x)$  (5) consists of the following steps:

1. The compression of samples  $x_i$  by applying the simplified semilogarithmic compressor function  $c(x)$  in order to obtain the compressed samples  $c(x_i), i = 1, \dots, M$ ;
2. The uniform quantization of the compressed samples  $c(x_i), i = 1, \dots, M$ , where quantization levels  $q_i, i = 1, \dots, M$  are obtained;

3. The encoding of the obtained quantization levels  $q_i, i = 1, \dots, M$  with binary words  $I_i, i = 1, \dots, M$ , ( $q_i \in I_i, i = 1, \dots, M$ ) and further transmission to the decoder, via the communication channel;

4. The decoding of the transmitted binary words  $I_i, i = 1, \dots, M$ , whereby corresponding quantization levels  $q_i, i = 1, \dots, M$  ( $I_i \in q_i, i = 1, \dots, M$ ) are obtained;

5. The expanding of the obtained quantization levels, by applying the inverse simplified semilogarithmic compressor function  $c^{-1}(x)$  whereby representation levels  $y_i, i = 1, \dots, M$  are obtained.

## IV. NUMERICAL RESULTS

This section discusses the performances that we have ascertained by applying the simplified semilogarithmic quantizer and the uniform quantizer in the quantization of signals having Laplacian PDF and a wide variance range. A variance  $\sigma^2$  can be expressed in the logarithmic domain as

[dB] =  $20 \log_{10}(\sigma^2 / \sigma_0^2)$ , where  $\sigma_0^2$  is some referent variance. By assuming  $\sigma_0 = 255$  as in [10], [dB] in our analysis ranges within [-48.13 dB, 0 dB], ( $20 \log_{10}(1/255) = -48.13$  dB,  $20 \log_{10}(255/255) = 0$  dB). The comparison of the SQNR characteristics for the uniform and the simplified semilogarithmic quantizer with  $N = 32$  and 64 quantization levels for the case of the continual and discrete input with Laplacian PDF are shown on Fig. 3 and Fig. 4. From the Fig. 3 and Fig. 4 one can notice that in the case of the continual input samples SQNR characteristic of the uniform quantizer, at first increases to its maximum and then rapidly declines. However, the simplified semilogarithmic quantizer provides a more constant and, in almost all of the observed variance range, higher level of SQNR, which can be considered via the gain in the average SQNR of about 8 dB, 10.6 dB, for the case of  $N = 32$  and 64 quantization levels, respectively. Accordingly, in the case of the continual input samples, a better robustness of SQNR is obtained by the simplified semilogarithmic quantizer compared to the uniform quantizer. In the Fig. 3 and Fig. 4, it is interesting to notice that SQNR characteristics differ depending on whether the input signal is discrete or continuous, regardless of the same assumed variance range of the input signal.

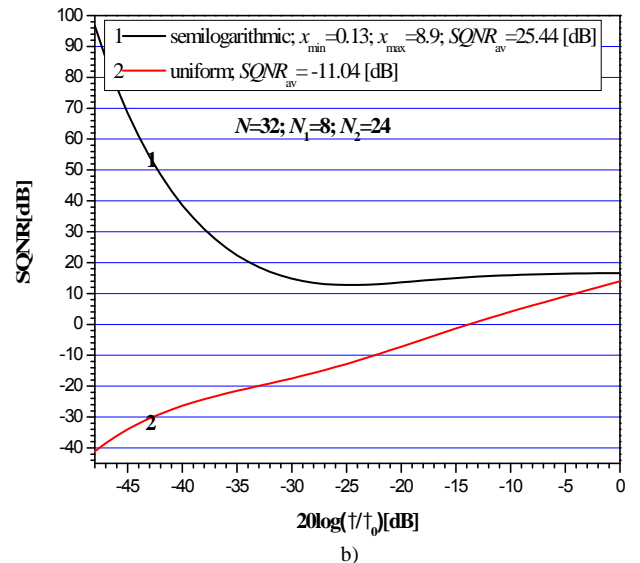
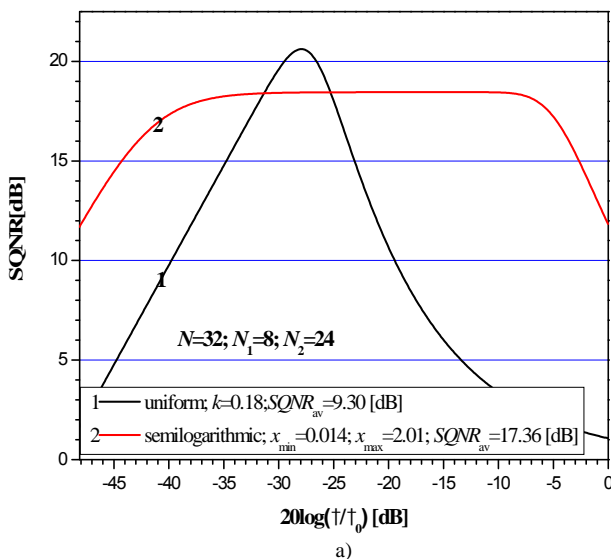


Fig. 3. SQNR characteristics of the uniform and the simplified semilogarithmic quantizer with  $N = 32$  quantization levels for the case of: a) continual input and b) discrete input.



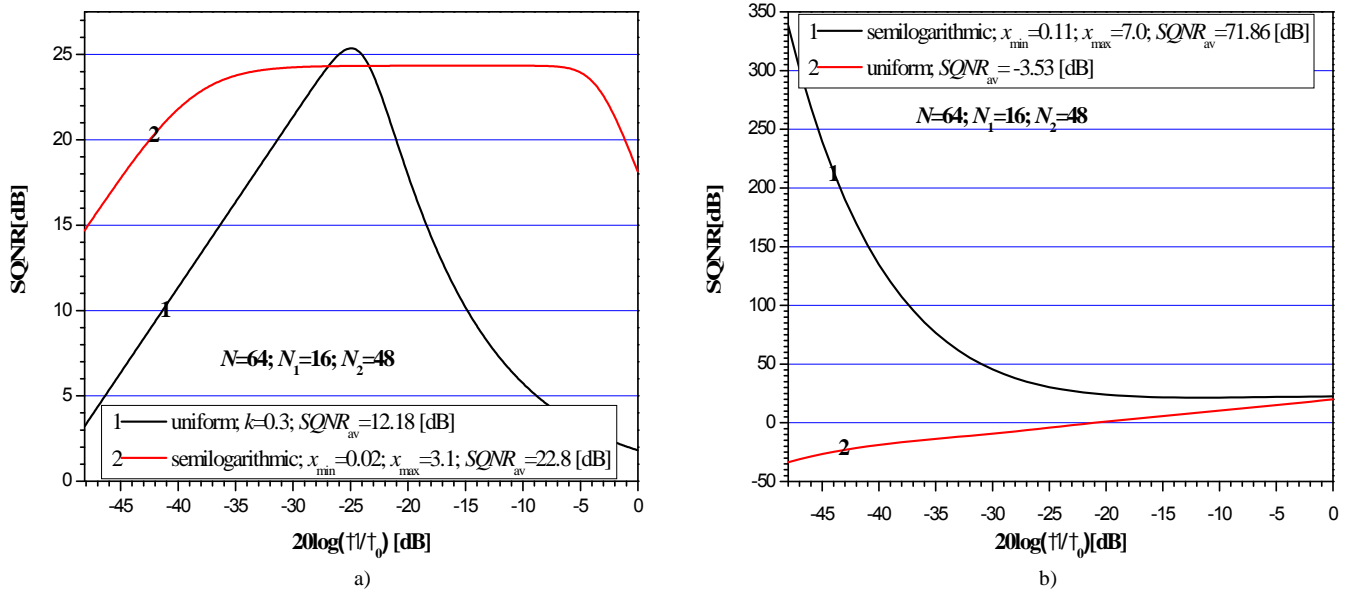


Fig. 4. SQNR characteristics of the uniform and the simplified semilogarithmic quantizer with  $N = 64$  quantization levels for the case of: a) continual input and b) discrete input.

Also, an interesting observation is that in the considered variance range the simplified semilogarithmic quantizer, designed for the discrete input samples, reaches a higher SQNR compared to the uniform quantizer [4] designed for the discrete input samples and the equal number of quantization levels. In addition, it is important to perceive that SQNR characteristics of both quantizers, designed for the discrete input samples, vary a lot in the observed variance range. As in the case of the continual input samples, in the case of discrete input samples we have calculated the gain in the average SQNR that amounts to about 36.5 dB, 75.4 dB, for the case of  $N = 32$  and 64 quantization levels, respectively.

Finally, it is important to highlight that in the area of smaller variance values the simplified semilogarithmic quantizer provides a significantly higher SQNR values than the uniform quantizer. The range of smaller variances is more often present in the practical applications (grayscale image coding) than the range of higher variance values [10]. Therefore, the application of the simplified semilogarithmic quantizer yields results that are significantly better than what can be concluded from the theoretical point of view, i.e. from the comparison of the average SQNR values.

## V. EXPERIMENTAL RESULTS

In Table I, experimental results for the simplified

semilogarithmic quantizer for discrete input samples are given, for different values of  $N_1$  and  $N_2$ . Values in Table I are averaged values for three standard test images (Lena, Street and Boat) shown on Fig. 5.

PSQNR values in Table I are very high, i.e. near lossless compression is achieved using simplified semilogarithmic quantizer designed for discrete input samples [10].

TABLE I. EXPERIMENTAL RESULTS FOR THE SIMPLIFIED SEMILOGARITHMIC QUANTIZER DESIGNED FOR DISCRETE INPUT SAMPLES.

	PSQNR[dB]		
	Lena	Street	Boat
$N = 64; N_1 = 32;$ $N_2 = 32$	54.40	56.61	51.94
$N = 32; N_1 = 16;$ $N_2 = 16$	47.60	48.24	45.28

In Fig. 6, three images from Fig. 5, after compression with the simplified semilogarithmic quantizer with  $N = 32$  levels are shown. We can observe that the reconstructed images are almost visually identical to the original images. Based on these facts, conclusion arises, that proposed model of the simplified semilogarithmic quantizer designed for discrete input samples, can be used with great success, for image compression.



Fig. 5. The grayscale images, size 512 x 512 pixels a) Lena b) Street c) Boat.



Fig. 6. The Grayscale images from Fig. 5 after compression with the simplified semilogarithmic quantizer with  $N = 32$  levels a) Lena b) Street c) Boat.

## VI. CONCLUSION

This paper has demonstrated the significance of the novel coding algorithm development via the gain in the performances that have been ascertained by applying the proposed simplified semilogarithmic quantizer in the quantization of signals having Laplacian PDF and a wide variance range over the uniform quantizer designed for the same PDF. Particularly, by comparing the SQNR characteristics of the proposed simplified semilogarithmic quantizer and the uniform quantizer, both designed for the discrete input samples with Laplacian PDF, it has been revealed that the proposed simplified semilogarithmic quantizer, along with the gain in the average SQNR that ranges up to 75.4 dB, in the whole of the considered variance range provides a higher level of SQNR. Moreover, it has been shown that in the area of smaller variance values the simplified semilogarithmic quantizer provides a significantly higher SQNR values than the uniform quantizer. Experimental results are very well matched with theoretical expectations i.e. obtained PSQNR values indicate a high quality level of reconstructed images. Accordingly, one can believe that the proposed simplified semilogarithmic quantizer will be of great importance for many practical image coder realizations.

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