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Modeling of Steady State Operation of Two-Input DC-DC Converter for Combining of Harvested and Reserve Powers

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***Abstract*—Modeling of the designed two input DC-DC converter for combining harvested and reserve powers is presented in the paper. Analytical model of the converter in steady state conditions was derived using differential equations averaging method. The results agreement comparing with the converter circuit simulation in LTSPICE software is shown. Analytical expression between control variables (switching duty cycles) and independent quantities (sources' voltages and load current) is derived. A converter controller algorithm seeking to minimize the power drawn from the reserve source is suggested.**

***Index Terms*—Pulse width modulation converters, modeling, circuit analysis, photovoltaic cells.**

I. INTRODUCTION

The electric energy generated by renewable sources (photovoltaic, wind, hydro, etc.) most often has to be converted to the electrical energy having voltage and current compliant to the requirements of an electrical load. A variety of electronic power converters (DC-DC, DC-AC, AC-DC) are used for this purpose [1]. It became common that these converters are also responsible for combining and shaping energies generated by the several renewable sources and/or energy storage batteries in order to supply the load. In this paper we consider the system configuration shown in Fig. 1.

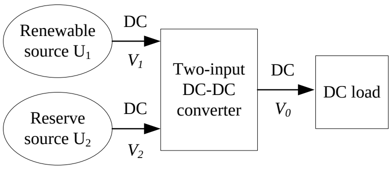

Fig. 1. Power supply from two sources via two-input converter.

The reserve source U_2 supplies the energy from the power grid. This is the energy that a consumer is billed for. We assume that $U_2$is always capable of supplying the power demanded for the normal load operation. The energy generated from the renewable source, for example solar panel, is fluctuating over the day and year season. It is possible that instantaneous power drawn from the source U_1 is not sufficient to fully supply the load. The proposed setup is aimed to minimize energy consumed from the reserve source U_2 as a result of some portion of energy taken from the source U_1. Such a configuration could be suitable for loads having DC plug for power supply and a solar panel mounted individually at the consumer side. The conversion from DC (from renewable source) to AC, connecting AC to the power grid or wiring new AC lines to the load location are the difficulties that may be avoided using the proposed setup.

In this paper we target some design issues of the two-input DC-DC converter. The input voltages V_1 and V_2 in general are not equal. Even more voltage V_1 is not fixed due to the varying availability of the renewable energy. We also assume that the load voltage V_0 is equal to V_2, since V_2 is the output of the original DC power supply of the load (laptop computer, TV set-top-box, charger, antenna amplifier, etc.). We will present a steady state analysis of the suggested converter. It is expected that the results of this analysis will support the possibility of implementing the DC-DC converter for combining power generated by the renewable and reserve sources. In the future definition of the converter's dynamic characteristics will also be needed for the design of closed-loop control of the converter. Meanwhile, we focus on electronic schematics without a feedback controller.

II. STRUCTURE OF THE CONVERTER

Many publications report investigations of the so called current sharing techniques and structures [2], [3]. However, the most often authors deal with the problem of ensuring current balance as in the implementation of redundant power supplies [4]. In our case it is the opposite, i.e. consumption of the energy from the renewable source is the priority, while the remaining energy should flow from the reserve source. Some authors have already presented similar approach using conversion to AC and/or using energy storage batteries [5], [6]. In order to minimize the cost of the converter we attempt to avoid batteries and conversion to AC. In addition to operating two power supplies in parallel the converter must ensure voltage level conversion. Therefore, we expect that switching DC-DC converter with two inputs should serve for the purpose (Fig. 2). PWM control signals of switches S_1, S_2 and S_3 are shown in Fig. 3. It is essential that

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the switches do not appear in ON state simultaneously. Otherwise, two power sources may become connected in parallel or shorted to the ground terminal, and the threat of damage may arise.

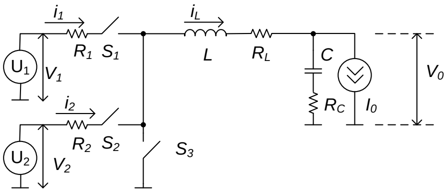


Fig. 2. Structure of the converter.

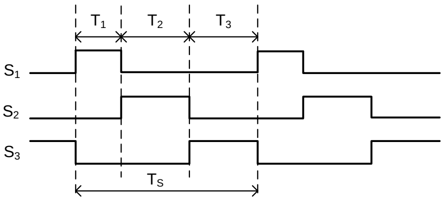


Fig. 3. PWM control signals of switches.

III. MODELING OF STEADY STATE CONDITIONS

A. Analytical averaged model of the converter

To obtain the steady state values of the output quantities $\mathbf{y} = (i_1, i_2, V_0)^T$, we will follow the well established methodology of averaging the system of differential equations that describe circuit of the converter dependent on the status of control switches [7]–[9]. The application of the method was already demonstrated for multiple-input [10] and multiphase [11] converters.

Following the Kirchhoff's laws the systems of differential equations are composed for variables for each circuit shown in Fig. 4. For example, for the first circuit in Fig. 4:

$$\begin{cases}\dfrac{di_L}{dt} = -\dfrac{R_1 + R_{s1} + R_L + R_C}{L} i_L + \dfrac{1}{L} V_1 - \dfrac{1}{L} u_C + \dfrac{R_C}{L} I_0,\\[2mm] \dfrac{du_C}{dt} = \dfrac{1}{C} i_L - \dfrac{1}{C} I_0.\end{cases} \tag{1}$$

In a matrix form the system (1) can be presented [8], [10] by

$$\frac{d\mathbf{x}}{dt} = \mathbf{A_1 x} + \mathbf{B_1 u}, \tag{2}$$

where $\mathbf{x} = (i_L, u_C)^T$ is the vector state space quantities, $\mathbf{u} = (V_1, V_2, I_0)^T$ is the vector of independent quantities, and matrixes $\mathbf{A_1}$ and $\mathbf{B_1}$ are:

$$\mathbf{A_1} = \begin{bmatrix} -\dfrac{R_{ch1} + R_C}{L}, & -\dfrac{1}{L} \\ \dfrac{1}{C}, & 0 \end{bmatrix}, \tag{3}$$

$$\mathbf{B_1} = \begin{bmatrix} \dfrac{1}{L}, & 0, & \dfrac{R_C}{L} \\ 0, & 0, & -\dfrac{1}{C} \end{bmatrix}, \tag{4}$$

where $R_{ch1} = R_1 + R_{s1} + R_L$. Correspondingly for the circuits 2 and 3 in Fig. 4 the matrixes $\mathbf{A_2}$, $\mathbf{B_2}$ and $\mathbf{A_3}$, $\mathbf{B_3}$ can be expressed as follows:

$$\mathbf{A_2} = \begin{bmatrix} -\dfrac{R_{ch2} + R_C}{L}, & -\dfrac{1}{L} \\ \dfrac{1}{C}, & 0 \end{bmatrix}, \tag{5}$$

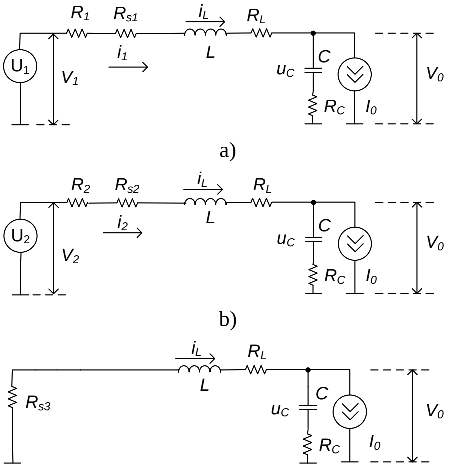
$$\mathbf{B_2} = \begin{bmatrix} 0, & \dfrac{1}{L}, & \dfrac{R_C}{L} \\ 0, & 0, & -\dfrac{1}{C} \end{bmatrix}. \tag{6}$$

where $R_{ch2} = R_2 + R_{s2} + R_L$, and

$$\mathbf{A_3} = \begin{bmatrix} -\dfrac{R_{ch3} + R_C}{L}, & -\dfrac{1}{L} \\ \dfrac{1}{C}, & 0 \end{bmatrix}, \tag{7}$$

$$\mathbf{B_3} = \begin{bmatrix} 0, & 0, & \dfrac{R_C}{L} \\ 0, & 0, & -\dfrac{1}{C} \end{bmatrix}, \tag{8}$$

where $R_{ch3} = R_{s3} + R_L$.



a)

b)

c)

Fig. 4. Converter equivalent circuits dependent on the status of switches: a – S1 – on, S2, S3 – off; b – S2 – on, S1, S3 – off; c – S3 – on, S1, S2 – off.

Then the averaged equations system can be written [8]

$$\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bu}, \tag{9}$$

where averaged matrixes are:

$$\mathbf{A} = d_1\mathbf{A_1} + d_2\mathbf{A_2} + (1 - d_1 - d_2)\mathbf{A_3}, \tag{10}$$

$$\mathbf{B} = d_1\mathbf{B_1} + d_2\mathbf{B_2} + (1 - d_1 - d_2)\mathbf{B_3}, \tag{11}$$

and the duty cycles of PWM control signals of the switches S_1 and S_2 are:

$$d_1 = T_1 / T_S, \tag{12}$$

$$d_2 = T_2 / T_S. \tag{13}$$

In the steady state conditions all derivatives are assumed to be equal to zero, and the solution of (9) is obtained by solving the system of linear equations [8]

$$\mathbf{x_{st}} = -\mathbf{A}^{-1}\mathbf{Bu}. \tag{14}$$

As described in [8] the steady state solution of the output quantities $\mathbf{y} = (i_1, i_2, V_0)^T$ can be expressed in the following

$$\mathbf{y_{st}} = \mathbf{C}\mathbf{x}_{st} + \mathbf{Eu}, \tag{15}$$

where:

$$\mathbf{C} = d_1\mathbf{C_1} + d_2\mathbf{C_2} + (1 - d_1 - d_2)\mathbf{C_3}, \tag{16}$$

$$\mathbf{E} = d_1\mathbf{E_1} + d_2\mathbf{E_2} + (1 - d_1 - d_2)\mathbf{E_3}, \tag{17}$$

and

$$\mathbf{C_1} = \begin{bmatrix} 1, & 0 \\ 0, & 0 \\ R_C, & 1 \end{bmatrix}, \tag{18}$$

$$\mathbf{C_2} = \begin{bmatrix} 0, & 0 \\ 0, & 1 \\ R_C, & 1 \end{bmatrix}, \tag{19}$$

$$\mathbf{C_3} = \begin{bmatrix} 0, & 0 \\ 0, & 0 \\ R_C, & 1 \end{bmatrix}, \tag{20}$$

$$\mathbf{E}_1 = \mathbf{E_2} = \mathbf{E}_3 = \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & -R_C \end{bmatrix}. \tag{21}$$

Equation (15) was used to calculate steady state currents i_{1st} and i_{2st} of two supply sources, and converter output voltage V_{0st}.

B. Verification using Spice modeling

Solution of (15) and its implementation with Matlab were verified by simulating schematics shown in Fig. 5 using LTSPICE simulation software. The period of PWM signal was accepted $T_S = 10\mu s$.

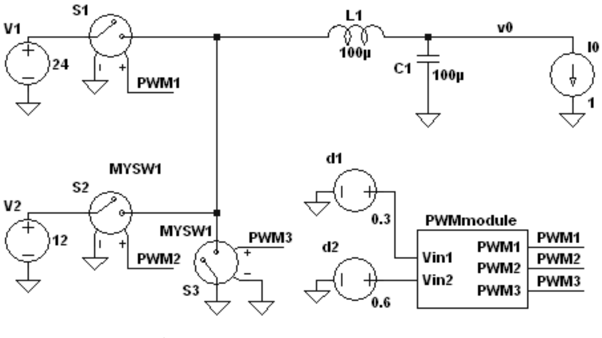


Fig. 5. Schematics of the simulated two-input converter.

From the waveforms given in Fig. 6 it was obvious that the converter was operating in the continuous current mode (CCM). The used method of differential equations averaging is applicable for converter modeling in CCM [8]. Steady state output voltage V_{0st} was estimated at the time moment t = 1.5 ms (see Fig. 6). Comparison of V_{0st} values obtained using LTSPICE simulation and those calculated from (15) have shown very good agreement at several combinations of duty cycles d_1 and d_2, and load current I_0. The mismatch between the values was only at the milivolt level.

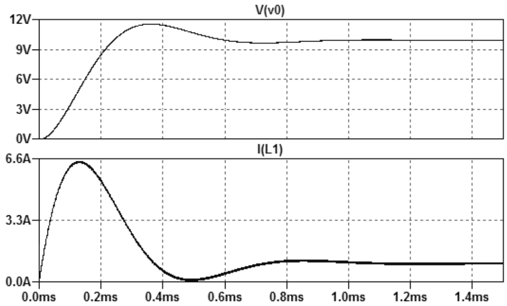


Fig. 6. Output voltage and inductor current transient waveforms. $d_1 = d_2 = 0.3$.

C. Relationship between control and independent quantities

In order to approve control of the output voltage, it is of interest to derive the expression relating duty cycles d_1 and d_2 (control variables), the independent quantities V_1, V_2 and I_0, and the output voltage V_0. To achieve this we use the third equation from (15)

$$V_{0st} = c_{31}i_{Lst} + c_{32}u_{Cst} + e_{13}V_1 + e_{23}V_2 + e_{33}I_0, \tag{22}$$

where c_{13} and c_{23} are corresponding elements of matrix **C**, and e_{13}, e_{23} and e_{33} are corresponding elements of matrix **E**. Expressions for i_{Lst} and u_{Cst} are obtained substituting (7) and (8) to (11). Then from (22) the duty cycle can be expressed

$$d_2 = \frac{V_{0\ st} - d_1V_1 + I_0(d_1R_{ch1} + (1-d_1)R_{ch3})}{V_2 + I_0(R_{ch3} - R_{ch2})}. \tag{23}$$

The correctness of the expression was verified by numerically solving (15) using Matlab *fsolve* function. No mismatch was observed. The duty cycle d_2 calculated according to (23) must also be constrained by the following inequality

$$0 < d_1 + d_2 \leq 1. \tag{24}$$

If calculated d_2 does not comply to the (24) that means the converter is not able to ensure the requested output voltage.

IV. A SUGGESTION FOR THE CONTROL ALGORITHM

Usually a controller of a switching converter is implemented as a PID controller. It takes feedback from the output voltage (and current) and seeks to adjust duty ratio such that to keep V_0 equal to reference voltage V_{ref}. In our case $V_{ref} = V_2$. Influence of load current and input voltage fluctuations on V_0 must be suppressed (first criterion) by the controller operating in disturbance rejection mode. In the analyzed two input converter two control variables are available (d_1 and d_2). It could be many combinations of d_1 and d_2 that ensure the preset output voltage. Therefore, we suggest that the second criterion for the adaptive control algorithm should be the minimization of the power P_2

$$P_2 = i_{2st} V_2, \tag{25}$$

where i_{2st} can be obtained from (15). In Fig. 7(a) the dependences of P_2 vs. d_1 at two load currents are shown, assuming that d_2 is selected using (23) and (24) in order to ensure output voltage equal to V_2. It can be seen that there exists an optimal value of $d_1 = d_1^*$ which corresponds to the minimum of P_2.

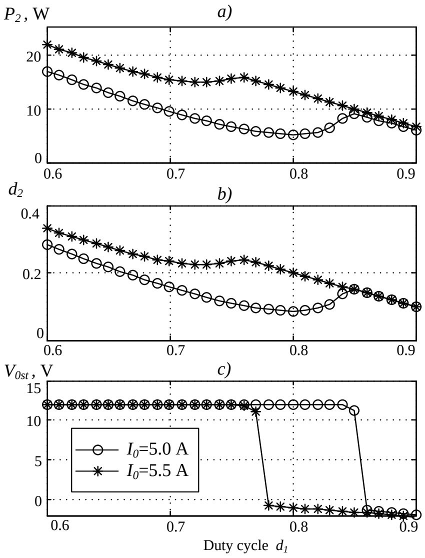


Fig. 7. The reserve source power: a – the duty cycle d_2; b – the output voltage; c – dependence on duty cycle d_1.

It must be commented that the drop of the P_2 curve in the range of $d_1 > 0.85$, is due to the fact that it is no longer possible to find a value for d_2 such that to keep $V_0 = V_2$. It can be seen from Fig. 7(c). Indeed, the d_2 was calculated according to (23) but finally was set to $d_2 = 1 - d_1$ if the condition $d_1 + d_2 \leq 1$ was not satisfied. Therefore, the range of $d_1 > 0.85$ (when I_0 = 5 A) should not be treated as a normal operating region of the converter.

When plotting Fig. 7 the voltage of the source V_1 was modeled by the function presented in [12], which describes generated voltage of a photovoltaic cell (PV) in respect to its load current i_{1PV}. In particular the output voltage of the PV cell PV2-70W can be expressed [12]

$$V_1 = V_{oc} + V_T \ln(1 - i_{1PV} / I_{sc}) - R_s i_{1PV}, \tag{26}$$

where PV cell parameters in the model are $V_{oc} = 22.2\text{V}$, $V_T = 1.06\text{V}$, $I_{sc} = 4.27\text{A}$, and $R_s = 0.44\Omega$. The current of the PV cell was assumed

$$i_{1PV} \approx d_1 I_0. \tag{27}$$

The following parameters of the converter (Fig. 2) components were used to plot Fig. 7: V_2 = 12 V, L = 100 μH, C = 100 μF; R_L = 0.5 Ω, R_C = 0.1 Ω, $R_1 = R_2$ = 0.5 Ω, $R_{s1} = R_{s2} = R_{s3}$ = 0.1 Ω.

The optimal value of $d_1 = d_1^*$ seems to be close to the value $d_{1mp} = I_{mp} / I_0$ corresponding to the maximum power point (MPP) of the PV cell. However, the internal resistances of sources U₁ and U₂ may have an influence on d_1^*, which is yet not investigated.

V. CONCLUSIONS

The results of steady state modeling of the considered two-input DC-DC converter approve its suitability for combining unstable harvested power with reserve source power for DC load supply. The control variables are duty cycles of electronic switches connected in the branches of each power source. By manipulating these duty cycles, the output voltage may be kept constant in the certain range of renewable source power fluctuations and load current demand.

It was demonstrated that there exists an optimal set of control variables (duty cycles) that minimize the power drawn from the reserve source.

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