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Joint Fountain Codes and Network Coding for Wireless Butterfly Networks

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***Abstract*—The wireless butterfly network is a representative model for multi-source multi-destination relay communication scenario, where opportunistic network coding (ONC) can be deployed to improve the system's throughput. However, ONC requires feedback information and this may deteriorate the throughput performance and counteract the advantage of ONC. To this end, in this paper, we firstly propose a new transmission protocol by combing fountain codes and network coding into the transmission strategy design for wireless butterfly network. Then, we formulate and analyse the throughput performance for the proposed protocol. Finally, numerical and simulation results are presented, which shows that our proposed protocol outperforms traditional ONC scheme.**

***Index Terms*—Fountain codes, network coding, cooperative relay networks.**

I. INTRODUCTION

Recently, as an advanced and promising coding solution, fountain codes (FCs) [1] have attracted much attention and been applied into many communication scenarios to improve the system's performance. FCs have the rateless property, which means that the transmitter can encode and generate infinite coded packets, so they are also called as rateless codes. Because each coded packet contains independent mutual information, the receiver only collects (accumulates) sufficient coded packets from the data stream of coded packets, and thus it can successfully decode and recover the original information.

Generally, the number of required collected coded packets is marginally superior to the number of original data packets for successful decoding. In addition, by using FC, the receiver only sends one bit information to the transmitter to start to transmit new data. Since only one bit information is required, feedback is generally ignored. Thus, FCs can be applied to wireless butterfly network to improve its throughput performance.

As for the wireless butterfly network, as shown in Fig. 1, it is a representative scenario for multi-source multi-destination relay networks, where network coding (NC) can be deployed to improve throughput performance compared to traditional routing.

The butterfly network is one of the basic coding configurations exploited in a practical NC protocol COPE [2]. In [3], authors proposed to adopt NC mechanism to process the data streams in intermediate node from different nodes and analyzed the throughput performance, and their results show the system performance can be greatly enhanced. Due to fading of wireless channel, the link transmission is always not successful. So, in [4] the authors presented an opportunistic scheduling for the multiple unicasts scenarios to further improve throughput performance, which is called as the opportunistic NC (ONC). In ONC protocol, intermediate nodes perform optimal scheduling based on the state of neighbouring nodes.

As is known, when ONC is deployed in wireless butterfly network, it will bring some additional system overheads. For example, the relay requires deciding whether it performs NC or not according to the feedback information on the state of neighbouring nodes. Obviously, this may worsen ONC's throughput performance, because sending feedback information consumes a certain amount of time. FCs can be deployed in wireless butterfly network to solve the problem caused by feedback owing to their characteristics mentioned previously. That is, the system employing FCs hardly requires feedback information.

To the best of our knowledge, some works on the joint FCs and NC design for wireless cooperative relay networks can be found in the literature, see e.g., [5]–[8]. Authors in [5] combined FCs and NC for the single source relaying network, where one source transmits its information to two destinations via two relays. The block error rate performance was evaluated and simulation results showed that compared to traditional simple detect and forward scheme, the joint FCs and NC design can obtain the lower block error rate. Authors in [6] studied the problem of using LT codes (a kind of practical FCs) and NC in a randomly deployed wireless sensor network and showed that, by doing so, network lifetime can be clearly increases. In [7], FCs and NC were jointly investigated and discussed for single-source single-destination relay networks. In [8], the delay and throughput were analyzed for the scenario with M source nodes, M relay nodes and M destination nodes. It was shown that rateless codes can minimize the transmission delay, as well as maximizing the system throughput, with the

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combination of NC. In their work, amplify-and-forward (AF) relaying strategy and analogue NC were considered rather than decode-and forward (DF) relaying strategy and digital NC.

In this paper, we also focus on joint FCs and NC design for rely networks. The main difference between our work and the related works is that we propose a new transmission protocol through combing FCs with *digital NC* for the *multi-source multi-destination* relay network, as is shown in Fig. 1, where two sources transmit their information to two destinations via a DF-based relay node, respectively. Our goal is to design efficient transmission strategy to improve the system throughput. In our proposed protocol, firstly the two sources encode original data packets with FCs, and then the relay adopts XOR-based NC (eXclusive OR) to process received packets from two sources. After this, the relay forwards network-coded packet to the destination nodes. Finally, by using some methods similar to superposition decoding, the two destinations can decode and recover the original data packets transmitted from the two sources, respectively.

Our contributions are summarized as follows. i) We propose the transmission protocol with joint FCs and NC for wireless butterfly network, which is referred to as FNC protocol in this paper. ii) We formulate and analyze the throughput performance of the proposed protocol. iii) Moreover, we compare the throughput performance of our proposed FNC with traditional 4-timeslot protocol (FSMH) and ONC protocol. Numerical and simulation results show that our designed FNC protocol outperforms traditional FSMH and ONC protocols in terms of system high throughput.

The rest of this paper is organized as follows. In section II, the system model and transmission protocol are introduced. In section III, the throughput performance is analyzed. Section IV gives some numerical and simulation results and compares transmission protocols' performance. Finally, section V concludes this paper.

II. SYSTEM MODEL AND PROTOCOL

A. System Model

We consider the wireless butterfly network, as shown in Fig. 1. The system model consists of two source nodes S_1 and S_2, two destination nodes D_1 and D_2, and a relay node R. Each node is equipped with one antenna. The two sources wish to communicate with the corresponding destinations respectively, that is, the source S_1 transmits information to D_1, and S_2 transmits information to D_2. It is assumed that there is no direct link between each pair of source node and destination node, and relay node R helps them to forward data to their corresponding destination nodes. Simultaneously, the sources' information can be overheard by the near destinations by the direct link between the source and the near destination, for example, the destination D_1 can overhear the information from the source S_2. All nodes are assumed to be the half-duplex, so the system is working in a time division mode.

The data is transmitted in the form of data packet and each node transmits one packet in one timeslot. The length of one timeslot is denoted as ^{2} . The packets transmitted from S_1 to D_1 are represented as $\{a_1, a_2, ...\}$, and the packets transmitted from S_2 to D_2 are $\{b_1, b_2, ...\}$. The transmission process

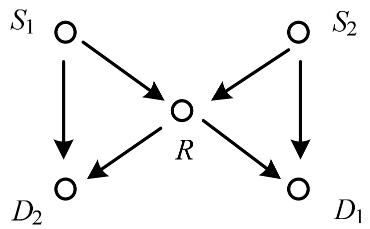

Fig. 1. System model (wireless butterfly network).

completes through several rounds of transmissions. Each round transmission is assume to be independent and can be described as follows. Firstly, the two source nodes respectively use one timeslot to broadcast their own data packets. The broadcast packets can be received by relay R. It is assumed that relay R only can store the packets received during current round. Secondly, relay R forwards the received data packets to the destinations by adopting certain transmission protocol, which is introduced in subsection C. Finally, the destination nodes will be able to recover and obtain their own data packets.

B. Channel Model

The channel between any two nodes is assumed to be flat block fading. The received signal of destination q from p is $y_q = h_{p,q} x_p + n_q$ and $p \in \{S_1, S_2, R\}$, $q \in \{R, D_1, D_2\}$, $h_{p,q}$ denotes the channel gain between node p and node q, which is zero mean complex Gaussian random variables with variance of $\lambda_{p,q}^{-1}$, and n_q is additive white Gaussian noise (AWGN) with variance N_0, which is assumed to be the same at different nodes. $\lambda_{p,q}$ is determined by the propagation distance $D_{p,q}$ between nodes p and q and the path loss exponent α, i.e., $\lambda_{p,q} = D_{p,q}^{\alpha}$. The transmit power of all the transmit nodes is equal, which is denoted as P, and thus the transmit signal-to-noise ratio (SNR) is $SNR=P/N_0$.

One can easily find that effect of channel fading and noise makes that the receiver cannot successfully receive all the data packets. If the receiver fails to successfully receive some packet, the packet is declared to be erased; otherwise, the packet is successfully received and can be stored by the receiver. Here we think if the transmission data rate $r \le \log(1 + |h_{p,q}|^2 SNR)$, the packet from node p can successfully be received by node q. Since the channel is assumed to be complex Gaussian distributed, it corresponds to Rayleigh fading amplitudes. So the probability that a packet is successfully received, which is called as success probability, can be computed as

$$p_{success} = \Pr\{r \le \log(1 + |h_{p,q}|^2 SNR)\} = \exp(-{}^{\mathrm{a}}{}_{p,q} \frac{2^r - 1}{SNR}). \quad (1)$$

C. Transmission Protocol

In this subsection, first two traditional transmission protocols are given simply as comparisons, and then a new protocol based on FCs and NC is proposed.

1) Four Slot Multi-hoping Protocol (FSMH)

In the traditional protocol, each round transmission generally consists of four timeslots. In timeslot 1, S_1 sends its packet a_n ; In timeslot 2, S_2 sends its packet b_m ; If R successfully receives the two packets from S_1 and S_2, it forwards a_n to D_1 in timeslot 3, and b_m to D_2 in timeslot 4; If R successfully receives only one of the two packets, it forwards the packet to its corresponding destination in timeslot 3; If R fails to receive any packet, it keeps silent.

2) Opportunistic NC Protocol (ONC)

In the protocol, relay R can adopt ONC to improve the system throughput over FSMH protocol. In timeslot 1, S_1 broadcasts its packet a_n ; In timeslot 2, S_2 broadcasts its packet b_m; If R successfully receives a_n and b_m, and D_1 receives b_m, D_2 receives a_n , then R broadcasts the network-coded packet $a_n \oplus b_m$ in timeslot 3, where $\oplus$ denote XOR operation; If R successfully receives a_n and b_m, but D_1 doesn't receive b_m or D_2 doesn't receive a_n , R forwards the two packets in the following two timeslots; If R only successfully receives one packet from two source nodes, R forwards the received packet in the following one timeslot; If R fails to receive any packet, it keeps silent.

Note that in the protocol, relay R requires performing opportunistic scheduling according to the situation whether each destination node successfully receives the packet from neighboring source node or not. In addition, the source nodes also need to decide whether to retransmit these packets which are not successfully received. So the feedback information from the receiver to the transmitter is necessary. It is assumed that required feedback time in each round is denoted as $\Delta²$, and feedback time is less than or equal to the length of one timeslot, that is, $\Delta² \le ²$. One can find that in FSMH protocol the source nodes also need to decide whether to retransmit, so the feedback is also required. Here we assume the feedback time of the two protocols is identical, i.e., $\Delta²$.

3) Fountain NC Protocol (FNC)

In proposed fountain NC protocol, its transmission process is different from the two protocols. One difference is that before the transmission starts, fountain coding operations are performed by the two source nodes.

For performing FCs, the two source nodes firstly divide their data packets into segments of length k, and each segment consists of k data packets. Then the k data packets are encodes with FCs to generate infinitely fountain-coded packets. After fountain coding operations, the original data packets of two source nodes are transform to $\{a_1', a_2', ...\}$ and $\{b_1', b_2', ...\}$ respectively. According to the property of FCs, when the receiver collects $k' = k + £, £ > 0$ coded packets, it can successfully decode the original data packets, where £ is fountain decoding overhead, and it is given in [9] as

$$£ = 2\ln(S / ¢)S , \tag{2}$$

where $S = c\ln(k / ¢)\sqrt{k}$, c is an appropriate constant, ¢ is the allowable decoder failure probability. Note because k' must be an integer, so actually k' needs to round up to the nearest integer.

After fountain coding operations, the transmission process starts. Each round transmission process is described as follows. In timeslot 1, S_1 broadcasts its packet a_n' ; In timeslot 2, S_2 broadcasts its packet b_n' ; If R successfully received a_n' and b_n' , then R broadcasts the network-coded packet $a_n' \oplus b_n'$ in timeslot 3, otherwise if only one packet is received, R forwards this packet to corresponding destination in timeslot 3; If R fails to receive any packet, it keeps silent.

Through several rounds of transmissions, the destination node will try decoding. When some destination node has successfully decoded and recovered its original data packets, the corresponding source node stops transmitting and only the other source node continues transmitting. Until both destination nodes successfully decode and obtain their original data, the current transmission process completes and can start new data transmission.

We can see since the two source nodes don't need to retransmit coded packet on the basis of FCs' property, the feedback information from the destination to the source is not necessary. At the same time, the relay doesn't need to perform opportunistic scheduling, so the feedback information from the destination node to the relay isn't also necessary. Therefor FNC protocol doesn't require feedback information and feedback time.

In the following, we give the decoding process of the destination nodes of FNC protocol. Since two destination nodes' decoding operations is similar, here we consider destination node D_1's decoding. We can find that during each round transmission, D_1 can receive and store three types of packets. If $a_n' \oplus b_n'$ packet from relay R and b_n' packet from source S_2 are received, then it can recover a_n' by operating $(a_n' \oplus b_n') \oplus b_n'$. We call this packet a_n' as type-I packet and let this case's probability be P_1 . If only b_n' from S_2 is received, then we call this packet as type-II packet and let this case's probability be P_2 . If only $a_n' \oplus b_n'$ from relay R is received, then we call this packet as type-III packet and let this case's probability be P_3. The three types of packets are stored in corresponding data buffers respectively, that is, buffer-I, buffer-II, and buffer-III. It is worth noting in the case that type-I packet is received, the type-II packet is also necessarily received, so in this case, type-I packet is stored in buffer-I, and simultaneously type-II is stored in buffer-II. But type-III packet is not stored, because the packet has been transformed to type-I packet.

After several rounds of transmissions, D_1 will try to decode the original data packets by using packets in data buffer. If the number of type-I packets in buffer-I has been equal to k', success decoding is achieved, and D_1 can recover the original data packets by fountain decoding. Otherwise, If the number of type-II packets in buffer-II is equal to k' , then D_1 can decode and recover the information of S_2. At the moment type-III packets in buffer-III will transform to type-I packets and can be transfer to buffer-I. This transformation process is similar to superposition decoding, that is, the information of S_2 of the type-III packet can be eliminated and the information of S_1 of the type-III packet can be recovered. Then D_1 continues to try decoding by using packets in buffer-I. Until the number of type-I packets in buffer-I is equal to k', D_1 can recover the original data packets.

III. Performance Analysis

In this section, the throughput performance of the three mentioned protocols is analyzed. It is tedious for analysis of general channel, so we consider symmetric channel model in [10], [11], where the channel gains of two direct links (from S_1 to D_2 and from S_2 to D_1) are identical, denoted by h_d, and the channel gains of four links associated with the relay (from S_1 to R, from S_2 to R, from R to D_1 and from R to D_2), which are called as relay links, are also identical, denoted by h_r. The direct link's success probability is expressed as

$$p_d = \Pr\{r \le \log(1+|h_d|^2 SNR)\} = \exp(-\alpha_d \frac{2^r-1}{SNR}) \quad (3)$$

and the relay link's success probability is expressed as

$$p_r = \Pr\{r \le \log(1+|h_r|^2 SNR)\} = \exp(-\alpha_r \frac{2^r-1}{SNR}), \quad (4)$$

where α_d^{-1} and α_r^{-1} are the variance of channel gains h_d and h_r respectively.

Trough N_r rounds of transmission, the total timeslots is K, and the destination receives M packet, then the average throughput (T_h) is $T_h=M/K$. We define the long-term throughput (T_l) as

$$T_l \doteq \lim_{N_r\to\infty} M/K. \quad (5)$$

We denote the number of timeslots and the number of received packets of each round transmission as $K_i(1\le K_i \le N_r)$ and $M_i(1\le i\le N_r)$ respectively. Then the long-term throughput can be rewritten as

$$T_l = \lim_{N_r\to\infty}\frac{\sum_{i=1}^{N_r} M_i}{\sum_{i=1}^{N_r} K_i} = \lim_{N_r\to\infty}\frac{\sum_{i=1}^{N_r} M_i/N_r}{\sum_{i=1}^{N_r} K_i/N_r}. \quad (6)$$

Since each round transmission is independent, according to the law of large numbers, $\lim_{N_r\to\infty}\sum_{i=1}^{N_r} M_i/N_r = E\{P\}$ and $\lim_{N_r\to\infty}\sum_{i=1}^{N_r} K_i/N_r = E\{S\}$, where $E\{\cdot\}$ is expectation operation and then $E\{P\}$ and $E\{S\}$ are respectively average number of required timeslots and average number of received packets of each round transmission. It is worth noting here = in these two equations actually represents convergence with probability 1. So the long-term throughput can also be defined as

$$T_l \doteq E\{P\}/E\{S\}. \quad (7)$$

Similar definition can be found in [8], [12]. In the following, we use (3) to analyze the throughput of FSMH protocol and ONC protocol, and use (2) to analyze the throughput of FNC protocol.

A. FSMH Protocol

According to symmetry of channels, the two destination nodes will simultaneously successfully or unsuccessfully receive corresponding packets. So when data packets are successfully received, that is, relay link transmission is successful and its probability is p_r, the destination nodes will receive 2 data packets within 4 timeslots. When unsuccessfully received and its probability is 1-p_r, the two destination nodes cannot obtain any data packets in 2 timeslots. So $E\{P\}$ and $E\{S\}$ of this protocol can be computed respectively as:

$$E\{P\} = 2\cdot p_r + 0\cdot(1-p_r) = 2p_r, \quad (8)$$

$$E\{S\} = (4+\Delta^2)p_r + (2+\Delta^2)(1-p_r), \quad (9)$$

where each round transmission needs to contain Δ^2 feedback time. So the long-term throughput of FSMH protocol is given by

$$T_{l_1} = \frac{2p_r}{(4+\Delta^2)p_r + (2+\Delta^2)(1-p_r)}. \quad (10)$$

One can find when $\Delta^2 = 0$ and $p_r = 1$, the average throughput is 0.5, and this is also the maximum throughput.

B. ONC Protocol

According to ONC protocol mentioned before, if transmissions in both direct links and relay links are successful, then the NC are adopted, and the destination nodes will receive 2 data packets within 3 timeslots, and then the probability that the case occurs is computed as

$$\Pr\{\log(r \le 1+|h_r|^2 \mathrm{SNR})\}\cdot\Pr\{r\le\log(1+|h_d|^2\mathrm{SNR})\} = p_r p_d. \quad (11)$$

If transmissions of direct links are unsuccessful and relay links are successful, then the destination nodes will receive 2 data packets within 4 timeslots, and then the probability that the case occurs is similarly computed as $p_r(1-p_d)$.

Otherwise, that is to say, if transmissions of relay links are unsuccessful, the destination nodes cannot receive their corresponding packets within 2 timeslots, and then the probability that the case occurs can be similarly computed as 1-p_r. So $E\{P\}$ and $E\{S\}$ of this protocol can respectively be computed as:

$$E\{P\} = 2p_r p_d + 2p_r(1-p_d) = 2p_r, \quad (12)$$

$$E\{S\} = (3+\Delta^2)p_r p_d + (4+\Delta^2)p_r(1-p_d) + \\ +(2+\Delta^2)(1-p_r). \quad (13)$$

So the long-term throughput of ONC protocol is easily expressed as

$$T_{l_2} = \frac{2p_r}{(3+\Delta^2)p_r p_d + (4+\Delta^2)p_r(1-p_d) + (2+\Delta^2)(1-p_r)}. \quad (14)$$

One can find when $\Delta^2 = 0$, $p_r = 1$ and $p_d = 1$, the average throughput is 2/3 and this is also the maximum throughput; When p_d = 0, that is to say, the direct links are always unsuccessful, then the average throughput of ONC protocol is equal to that of FSMH protocol, this is because when direct links are always unsuccessful, relay R will work in the mode identical to FSMH protocol.

C. FNC Protocol

Analysis of FNC protocol is different from that of the two protocols above. In FSMH and ONC protocols, we only need to consider each round transmission, while in FNC protocol, through several rounds of transmission and until the destination nodes collect sufficient fountain-coded packets, they can achieve successful decoding and recover original data packets. Here we define one successful decoding as one period. For achieve L successful decoding, we denote the number of required timeslots of each successful decoding as T_i (i=1, …, L) respectively. Provided that after L periods of transmissions the destination nodes can achieve L successful decoding and can acquire $2k\times L$ original data packets, the average throughput of FNC protocol can be expressed as

$$T_{h_3}=\frac{2k\times L}{\sum_{i=1}^{L}T_i}=\frac{2k}{\sum_{i=1}^{L}T_i/L}. \tag{15}$$

When $L\to\infty$, the long-term throughput is expressed as

$$T_{l_3}=\lim_{L\to\infty}\frac{2k\times L}{\sum_{i=1}^{L}T_i}=\lim_{L\to\infty}\frac{2k}{\sum_{i=1}^{L}T_i/L}=\frac{2k}{\lim\limits_{L\to\infty}\sum_{i=1}^{L}T_i/L}. \tag{16}$$

Obviously, according to the law of large numbers, $\lim\limits_{L\to\infty}\sum_{i=1}^{L}T_i/L=E\{T\}$, where T is the number of required timeslots of one period, and $E\{T\}$ is average number of required timeslots of one period. So the long-term throughput of FNC protocol can be re-expressed as

$$T_{l_3}=2k/E\{T\}. \tag{17}$$

To compute $E\{T\}$, using total probability theorem, $E\{T\}$can be computed as

$$E\{T\}=\sum_{n=k'}^{\infty}E\{T\,|\,N=n\}\Pr\{N=n\}, \tag{18}$$

where N is the number of required rounds of transmissions of one successful decoding. When N=n, this means that n round transmissions can ensure successful decoding, but n-1 round transmissions cannot. Evidently, for successful decoding, $n\geq k'$.

Firstly, we calculate the probability distribution of random variance N, that is, Pr{N=n}. For computing the probability, here we consider the decoding process of D_1, since the channels are symmetric and then the two destination nodes work in the same mode.

As mentioned before, D_1 can collect three types of packets, if all links' transmissions are successful, the type-I packet is received, and this case's probability is computed as

$$\begin{aligned}P_1=\Pr\{\log(1+|h_r|^2\,\mathrm{SNR})\geq r\}\cdot\Pr\{\log(1+\\+|h_d|^2\,\mathrm{SNR})\geq r\}=p_r p_d.\end{aligned} \tag{19}$$

Similarly, the probability of the received type-II and type-III packets can respectively be computed and expressed as $P_2=(1-p_r)p_d$, $P_3=p_r(1-p_d)$, and the probability of no received packet can be computed and expressed as $P_0=(1-p_r)(1-p_d)$.

When the number of the rounds of transmission achieves N=n, the following three cases may occur:

Case 1: at n-th round, type-III packet is received. According to the decoding process of FNC protocol mentioned before, this case means that preceding n-1 round transmissions, D_1 has successfully decoded D_2's information and has just collected $k'-1$ type-I or type-III packets. The probability of this case is computed as

$$\Pr\{\text{Case 1}\}=\sum_{m=0}^{k'-1}\mathrm{C}_{n-1}^{m}P_3{}^{m}\mathrm{C}_{n-1-m}^{k'-1-m}P_1^{k'-1-m}\sum_{i=m+1}^{n-k'}\mathrm{C}_{n-k'}^{i}P_2^{i}P_0{}^{n-k'-i}\times P_3. \tag{20}$$

Note in the computation we have considered when $k'-1-m$ type-I packets are received and stored, $k'-1-m$ type-II packets can also be received and stored.

Case 2: at n-th round, type-I packet is received. This case means that preceding n-1 round, D_1 is possible to have decoded D_2's information, which is called as the Former event, or D_1 is possible to have not decoded D_2's information, which is called as the Latter event. If the Former event happens, it is similar to Case 1, and the probability Pr{Former} can be computed as

$$\Pr\{\text{Former}\}=\sum_{m=0}^{k'-1}\mathrm{C}_{n-1}^{m}P_3{}^{m}\mathrm{C}_{n-1-m}^{k'-1-m}P_1^{k'-1-m}\sum_{i=m+1}^{n-k'}\mathrm{C}_{n-k'}^{i}P_2^{i}P_0{}^{n-k'-i}\times P_1. \tag{21}$$

If the Latter event happens, by using similar method, the probability Pr{Latter} can be computed as

$$\Pr\{\text{Latter}\}=\sum_{m=0}^{k'-1}\mathrm{C}_{n-1}^{m}P_2{}^{m}\mathrm{C}_{n-1-m}^{k'-1-m}P_1^{k'-1-m}\sum_{i=m}^{n-k'}\mathrm{C}_{n-k'}^{i}P_3^{i}P_0{}^{n-k'-i}\times P_1. \tag{22}$$

Since the Former event and the Latter event are mutual exclusive, so the probability of Case 2 is expressed as

$$\Pr\{\text{Case 2}\}=\Pr\{\text{Former}\}+\Pr\{\text{Latter}\}. \tag{23}$$

Case 3: at n-th round, only type-II packet is received. By using similar method, the probability of Case 3 can be computed as

$$\Pr\{\text{Case 3}\}=\sum_{m=0}^{k'-1}\mathrm{C}_{n-1}^{m}P_2{}^{m}\mathrm{C}_{n-1-m}^{k'-1-m}P_1^{k'-1-m}\sum_{i=m+1}^{n-k'}\mathrm{C}_{n-k'}^{i}P_3^{i}P_0{}^{n-k'-i}\times P_2. \tag{24}$$

Since these three cases are mutual exclusive, the probability distribution of N is given as

$$\begin{aligned}\Pr\{N=n\}=\Pr\{\text{Case 1}\}+\Pr\{\text{Case 2}\}+\\+\Pr\{\text{Case 3}\},n\geq k'.\end{aligned} \tag{25}$$

Secondly, we compute $E\{T\,|\,N=n\}$. We know during one round transmission, when type-I and type-III packets are received, the number of required timeslots are 3, and when type-II packet or no packet is received, the number of required timeslots are 2.

During n round transmissions, if Case 1 occurs and on the condition that N=n, for given $m(0 \leq m \leq k'-1)$ and $i(m+1 \leq i \leq n-k')$, from (6), the number of the required timeslots can be easily computed as

$$T = 3m + 3(k'-1-m) + 2i + 2(n-k'-i) + 3 \qquad (26)$$

and the corresponding probability is

$$\Pr\{T = 3m + 3(k'-1-m) + 2i + 2(n-k'-i) + \\ +3 \big| N = n\} = \mathrm{C}_{n-1}^{m} P_3^{m} \mathrm{C}_{n-1-m}^{k'-1-m} P_1^{k'-1-m} \mathrm{C}_{n-k'}^{i} P_2^{i} P_0^{n-k'-i} \times \\ \times P_3 / \Pr\{N = n\}. \qquad (27)$$

As for Case 2 and Case 3, the number of the required timeslots and the corresponding probability can also be computed similarly from (7) and (8). And then $E\{T \big| N = n\}$ can be computed. Then by using (5), $E\{T\}$ can be computed. Finally, the long-term throughput of FNC protocol can be obtained by using (4).

IV. Numerical and Simulation Results

In this section numerical and simulation results of three protocols are presented and compared. In the simulation, the transmission data rate r is set to 1 bit/s/Hz. Each segment's length is set as k=50 and FCs' decoding overhead can be computed by (1) with c=0.01 and $¢ = 0.1$.

Fig. 2 and Fig. 3 show numerical and simulation results of the throughput performance of these three protocols in symmetric channel model. We set $\lambda_d = 4$ and $\lambda_r = 5$. Fig. 2 shows the throughput against the signal to noise ratio (SNR) with normalized feedback time $\Delta^{2}/^{2} = 0.2$. It is observed that with the increase of SNR, the throughput of the three protocols increases and for given feedback time, the throughput of FNC protocol is superior to that of ONC protocol and FSMH protocol. Fig. 3 shows the throughput against normalized feedback time with *SNR*=20dB. It is observed that the throughput of FNC is not relevant to $\Delta^{2}/^{2}$, while the throughput of FSMH and ONC will decrease with the increase of $\Delta^{2}/^{2}$. Moreover, we can observe that when no feedback time exists, that is, $\Delta^{2}/^{2} = 0$, the throughput of ONC precedes that of FNC, while with the increase of feedback time, the throughput of FNC will precede that of ONC. In addition, results show analytical results are excellently in line with the simulation results.

When the channels are non-symmetric, the performance is also compared in the simulation. The distances are set as $D_{S_1,R} = 5$, $D_{S_2,R} = 6$, $D_{S_1,D_2} = 3$, $D_{S_2,D_1} = 2$, $D_{R,D_1} = 5$, $D_{R,D_2} = 4$, and the path loss exponent is set as $\alpha = 3$. Fig. 4 compares the throughput of three protocols with respect to SNR for $\Delta^{2}/^{2} = 0.2$. The relation between the throughput and $\Delta^{2}/^{2}$ for *SNR*=30dB is shown in Fig. 5. The similar results with symmetric channels can be found.

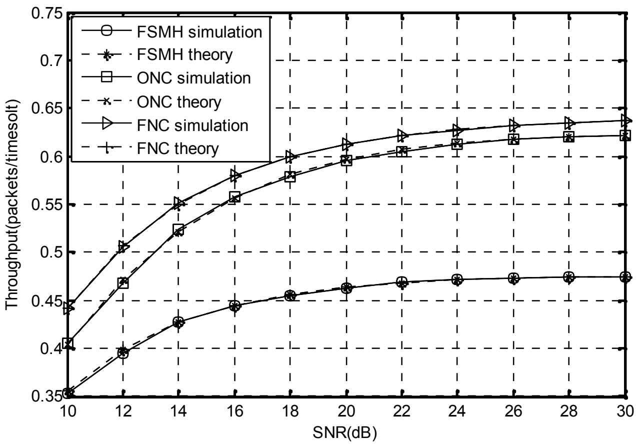

Fig. 2. Throughput vs. SNR for symmetric channel model: $\Delta^{2}/^{2} = 0.2$.

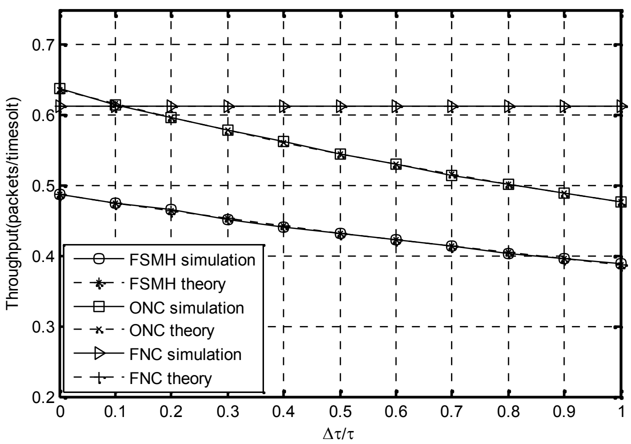

Fig. 3. Throughput vs. $\Delta^{2}/^{2}$ for symmetric channel model: *SNR*=20dB

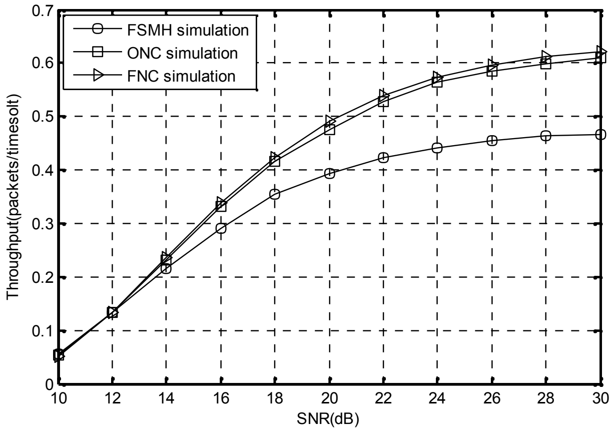

Fig. 4. Throughput vs. SNR for general channel model: $\Delta^2 / ^2 = 0.2$.

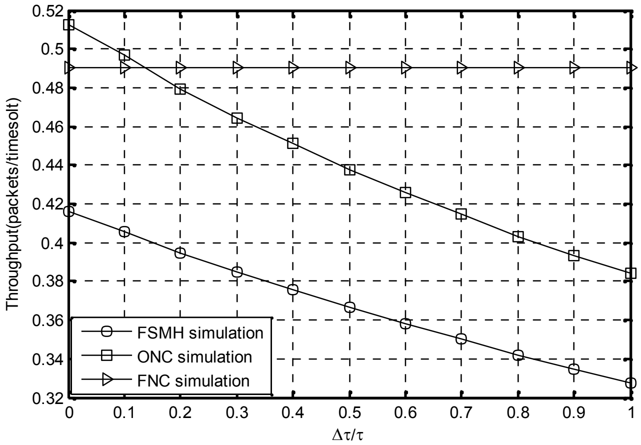

Fig. 5. Throughput vs. $\Delta^2 / ^2$ for general channel model: *SNR*=30dB.

V. CONCLUSIONS

In this paper, we have proposed the FNC protocol based on FCs and NC in wireless butterfly network. We analyze the throughput performance of FNC protocol and traditional FSMH and ONC protocol. Numerical and simulation results show the throughput of FNC surpasses that of FSMH and ONC when there is feedback time in the system of FSMH and ONC protocols.

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