

Determination of Sensor Network Coverage using Probabilistic Approach

J. Gorecki, B. Miedzinski

Wroclaw University of Technology,

Wybrzeże Wyspianskiego 27, 50-370 Wroclaw, Poland, phones: +48603848411, +48713203693;

e-mails: jan.gorecki@onet.eu, bogdan.miedzinski@pwr.wroc.pl

H. Nouri

University of the West of England,

Bristol, UK, phone: +44 117 3282631, e-mail: hassan.nouri@uwe.ac.uk

Introduction

Coverage is an important issue when it is necessary to obtain a certain level of quality building any kind of telecommunication network. Placing network nodes in particular area does not guarantee that the network will be operating according to defined performance indicators. The network nodes must be planned accordingly in such way that coverage of this area meets expectations [1–7].

Meaning of Coverage is always linked with the services offered by the network. In case of Public Land Mobile Network coverage means the area where voice and data services are accessible with certain quality of service. Wireless Sensor Networks are offering usually 2 services – sensing service and transport service. Transport service is about transmitting of measured information from source to the sink using sensor infrastructure. A sensor node can act as a component of a transport network receiving data packets from another node and transmitting to the next one.

Coverage of sensing service is about information coverage, i.e. determination of area which is covered by sensors being nodes of sensor network.

This paper deals only with information coverage of sensing service identifying quality of services parameters which is coverage probability and error range. It is shown how coverage probability can be calculated providing real measurements.

Paper elaborates the method to identify near and far coverage area for each sensor estimating quality of service parameters in order to assess the areas of sensor redundancy and shortage. Coverage in near area is used to identify whether single sensor is enough to cover this area (sensor shortage indicating denser sensor layout). Coverage in far area is used to identify which sensors can be removed from the field in order to have lower cost of deployment of sensor network (sensor redundancy).

Background

Typical sensor grids are described using N , d , r , k parameters which are explained below [2]: N – number of sensors is equal to $N = m \times n$, where n – number of grid rows; m – number of grid columns; d – distance between sensors; r – sensing radius. Each sensor has a sensing radius of r assuming disc-based sensing model; k – coverage. The grid is k -covered, when every point in the field is covered by at least k sensors.

Parameters N , d , r and k are visualized in Fig. 1. Physically the sensor nodes can be built as it is designed in [3].

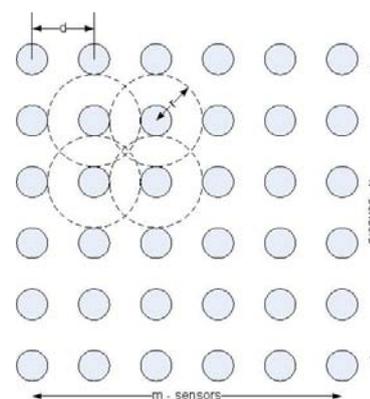


Fig. 1. Grid of sensors

Coverage Probability in Sensor Grid

Setting up a sensor grid, the coverage problem of measured field is an important issue which received considerably high research attention. Majority of papers is using the model that sensing accuracy and sensing range of a sensor is fixed [1]. Therefore the sensor's coverage is

assumed to be a sensing disk A_1 with radius r . Within the sensing disk each point is assumed to be covered with probability ξ . It means that estimated measurement error θ between the pair of points: any point within the sensing radius and the exact point measured by the sensor, is lower than predefined value a , i.e.

$$P(|\theta| \leq a) \geq \xi. \quad (1)$$

In this paper it is assumed that there may exist an area A_2 outside A_1 where equation (1) is fulfilled as well what means that sensor coverage can be bigger than A_1 .

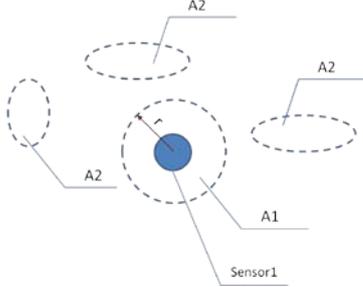


Fig. 2. Illustration of A_1 and A_2 areas

To classify whether particular point Sx belongs to the union of A_1 and A_2 area, there is introduced a definition of point-to-point coverage probability ($P2P\xi$).

$P2P\xi$ is defined as a probability that sensor $S1$ (located in the grid) indicates a value which is within a predefined range with respect to actual value in point Sx (not necessarily covered by any sensor

$$P2P\xi (S1, Sx, Range) |_{Range=\{a,b\}} = P(Sx \in (S1 - a, S1 + b)), \quad (2)$$

where $S1$ and Sx in equation 2 represent the value measured by Sensor1 and actual value in point Sx .

Assuming that exactly in the Sx point there has been located Sensor2 ($S2$) indicating without any noises actual value in point Sx and the range is symmetrical, i.e. $a=b$, the $P2P\xi$ value is estimated as follows

$$\widehat{P2P\xi} (S1, S2, a) = P(S2 \in (S1 - a, S1 + a)), \quad (3)$$

To verify whether Sx point estimated by Sensor2 lies within the union of area A_1 and A_2 , it is enough if below inequity is fulfilled

$$\widehat{P2P\xi} (S1, S2, a) \geq \xi. \quad (4)$$

It should be noted that fulfilling above inequity does not answer whether point Sx belongs to A_1 or A_2 area specifically.

To verify how the model of coverage probability can be applied in practice, respective measurements are performed for a selected grid having inhomogenic properties. The grid is characterized by:

1. Measured physical quantity: temperature;
2. Environment: floor equipped with heating (an under floor system of pipes with water acting as a heat exchanger). The temperature of the water is set to 35°C. The temperature on the floor surface is measured at various points under steady state conditions;

3. Number of sensors: 132 ($n = 11, m = 12$ – see Fig. 1). Sensors are calibrated;
4. Distance between sensors d : 23 cm (see Fig. 1);
5. Area of the grid: 5.82 m²;
6. Number of measurement series: 30;
7. Total number of measurements: 3960 (30 series x 132 sensors).

The difference between values measured by the sensors in single measurement series was up to 11°C what shows field inhomogeneity.

It is estimated that the difference between measured values for particular 2 sensors will be in accordance with Gaussian distribution. However, please note, this is a biased estimate based on the actual measurements. An improved, unbiased estimate could be obtained from study explaining the probability distribution function of dependences between particular pairs of sensors.

To calculate estimators of value difference between particular 2 sensors the following formulas are used:

$$\hat{\mu}_{(i,j)(u,w)} = \frac{1}{series} \sum_{L=1}^{series} (S_{L,(i,j)} - S_{L,(u,w)}), \quad (5)$$

$$\hat{\sigma}_{(i,j)(u,w)} = \frac{1}{series} \sqrt{\sum_{L=1}^{series} (S_{L,(u,w)} - \hat{\mu}_{(i,j)(u,w)})^2}, \quad (6)$$

where $\hat{\mu}_{(i,j)(u,w)}$ - estimator of mean value of difference between 2 sensors. The first sensor is located in i -row, and j -column, whereas a second sensor is located in u -row and w -column; series – number of series which is 30; $S_{L,(x,y)}$ – measured value by sensor located in x -row and y -column in measurement series number L ; $\hat{\sigma}_{(i,j)(u,w)}$ - estimator of standard deviation value difference between 2 sensors. The first sensor is located in i -row; j -column, whereas a second sensor is located in u -row and w -column.

Having calculated the estimators, the probability density function of value indifference between 2 sensors can be calculated as follows

$$f_{(i,j)(u,w)}(x) = \frac{1}{\sqrt{2\sigma_{(i,j)(u,w)}^2}} e^{-\frac{(x - \mu_{(i,j)(u,w)})^2}{2\sigma_{(i,j)(u,w)}^2}}. \quad (7)$$

Finally the $P2P\xi$ is derived from cumulative distribution function for a specified range, i. e.

$$P2P\xi_{(i,j)(u,w), Range} |_{Range=(a,b)} = \int_a^b f_{(i,j)(u,w)}(x) dx. \quad (8)$$

$P2P\xi$ calculated for range $\pm 0.5^\circ\text{C}$ has shown that the closest geographical neighbours do not obtain the highest $P2P\xi$ values as it is intuitively expected. This proves that field under measurements has inhomogenic properties.

Estimating Sensor Grid Coverage

Calculating $P2P\xi$ value indicates whether particular point is within a coverage area of particular sensor, however does not identify whether this area

belongs to A_1 or A_2 . To achieve 1-covered grid it is required to assume the model that sensing disk radius r of each sensor must be $\geq \frac{d\sqrt{2}}{2}$ what is illustrated in Fig. 1. Having assumed a constant sensing area for each sensor, it is needed to estimate what is the coverage probability ξ for area A_1 within measurement range $\pm a$.

Each sensor in the grid has 4 of the closest neighbours which are located with distance d from this sensor. Sensors located on the edge or in the corner have 3 and 2 of closest neighbours respectively.

Proposed coverage probability estimation method is introducing 4 virtual sensors SVx which are located on the line linking the reference sensor (SI) with each neighbour $S2x$. The distance of each virtual sensor from the reference sensor equals x (Fig. 3).

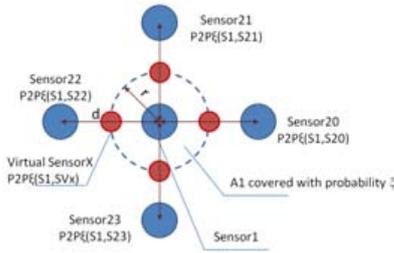


Fig. 3. Virtual sensors

In each measurement series, each virtual sensor measures value derived from below equation

$$\widehat{SVx}_L(x) = S1_L + f(x)(S2_L - S1_L), \quad (9)$$

where $\widehat{SVx}_L(x)$ – estimated value in point SVx in single measurement series; x - distance between sensor SI and $S2x$. For neighbouring sensors, $x=0$ represents sensor SI and $x=d$ represents sensor $S2x$; $S1_L, S2_L$ - measured value by SI and $S2x$ in single measurement series; $f(x)$ – continuous function determining monotonicity of measured value. This function must have following properties: $f(0)=0$ and $f(d)=1$.

Placing the virtual sensor at the border of sensing disk the argument x equals r , i.e. radius of Area₁. For grid under consideration $f(x)$ is modelled as in equation, similarly as it is assumed in [2]

$$f(x) = \left(\frac{x}{d}\right)^\alpha \text{ where } \alpha \geq 1. \quad (10)$$

In order to model function $f(x)$ as linear, $\alpha=1$ is taken for further calculations.

Calculating estimators of value difference μ and σ for reference sensor and virtual sensor as shown in equations 6 and 7, the following inequities are true

$$P2P\xi(S1, SVx, a) \geq P2P\xi(S1, S2x, a). \quad (11)$$

To estimate finally probability ξ for area A_1 with known disk radius r , the minimum $P2P\xi(S1, SVx, r)$ value of all 4 neighbouring sensors must be taken, i.e.

$$\hat{\xi} = \min \{P2P\xi(S1, SV1, a), P2P\xi(S1, SV2, a), P2P\xi(S1, SV3, a), P2P\xi(S1, SV4, a)\}. \quad (12)$$

There are 2 reasons which makes ξ a biased estimator. This is an error introduced by sensing disk model and assumption that each neighbouring sensor belongs to A_1 area.

Having calculated the coverage probability ξ for A_1 area for each sensor in the grid, it is relatively easy to estimate coverage probability for A_2 area. Let's take A_2 area of Sensor1 which is A_1 area of Sensor2. The coverage probability of Sensor1 will be multiplication of coverage probability ξ for A_1 of Sensor2 and $P2P\xi$ between Sensor1 and Sensor2

$$\hat{\xi}_{S1 \rightarrow S2} = \hat{\xi}_{S2} \times P2P\xi(S1, S2, a). \quad (13)$$

Having calculated coverage probability for A_1 and A_2 area it is possible to identify areas where the sensor grid shall be denser (basing on A_1) and to identify which sensors can be removed from the grid.

Verification of presented model

As indicated in previous chapter there has been conducted measurements to verify practical case against presented model. There were performed calculations how to guarantee 0.68 confidence level of grid coverage with precision range $\pm 0.5^0C$.

Table 1 shows sensor grid coverage identifying well covered own area (with white background), badly covered own area (with black background) which require denser grid locally. Third group of sensors (marked with gray background) is the most interesting, since they do not provide proper coverage of own area, but can cover area of belonging to different sensors. It means that A_1 area is rudimental, however A_2 is large.

Table 1. Sensor grid coverage

	1	2	3	4	5	6	7	8	9	10	11	12
1	3 0	6 0	6 0	2 1	2 0	1 4	1 14	1 24	1 30	1 24	4 10	4 0
2	6 0	4 0	4 0	2 12	2 22	1 31	1 33	1 35	1 31	1 18	3 17	3 0
3	6 0	4 0	2 12	1 29	1 31	1 25	1 30	1 36	1 32	1 28	4 15	4 0
4	2 0	1 14	1 15	1 23	1 32	1 33	1 31	1 23	1 30	1 18	4 7	4 0
5	2 0	2 16	1 9	1 32	1 34	1 36	1 34	1 27	1 30	1 33	3 7	3 0
6	2 30	2 18	1 15	1 21	1 27	1 7	1 33	1 24	1 25	1 27	4 20	4 0
7	2 0	2 2	1 4	1 9	1 14	1 22	1 1	1 34	1 34	3 23	4 0	3 0
8	2 0	2 0	1 5	1 9	1 13	1 15	1 25	2 32	3 18	3 0	4 0	3 0
9	2 0	2 0	2 6	1 17	2 1	1 28	1 28	2 9	3 28	3 0	3 0	3 0
10	4 1	3 11	4 26	4 31	3 29	3 28	3 28	2 2	3 0	3 0	2 0	2 0
11	4 0	3 0	4 0	4 0	3 0	3 0	3 0	2 0	4 0	4 0	2 1	1 1

Legend: X|Y – A_1 area is X-times smaller than required. Y sensors are well covered by this sensor

Conclusions

The usefulness of applied model has been confirmed in a real example. The area which is under-dimensioned is located always at the edges of the field, where there is a border of the heating system. It was identified that grid under measurements requires on average 3 times denser grid in this area. Basing on equation 13, it is denoted that those sensors cannot be replaced by other sensors in the grid.

Applied model has proved the usefulness in identification of area which has sensor redundancy and can be further optimized to decrease number of sensors. In the performed measurements this area is exactly in the middle. Between the redundancy and under-dimensioned area there are sensors which are not able to cover own area, however are able to cover the well-covered area as well.

Presented concept can be utilized in the following cases valid for sensor networks:

1. Optimization of the number of sensors, for example omitting sensors which have a high $P2P\zeta$ value with respect to the neighbouring sensor and more efficient battery management [3, 5, 6];
2. Calculations of k-coverage of the sensor grid, i.e. determining how many sensors cover a certain point of the grid;
3. Determining which neighbouring sensor can take over the measurement activities upon sensor failure;
4. Performing remote calibration by selecting the neighbour with a high enough $P2P\zeta$ value;
5. Impacting transport network topology and routing in wireless sensor networks [4].

Acknowledgements

This paper has been written as a result of realization of the project entitled: "Detectors and sensors for measuring factors hazardous to environment – modeling and monitoring of threats".

The project financed by the European Union via the European Regional Development Fund and the Polish state budget, within the framework of the Operational Programme Innovative Economy 2007÷2013.

The contract for refinancing No. POIG.01.03.01-02-002/08-00.

References

1. **Wang B., Chaing Chua K., Srinivasan V., Wang W.**, Information coverage in randomly deployed wireless sensor networks // *IEEE Transactions on Wireless Communications*, 2007. – Vol. 6. – No. 8. – P. 2994–3004
2. **Kumar S.** Foundations of Coverage in Wireless Sensor Networks, PhD thesis. – Ohio State University, 2006
3. **Miedzinski B., Rutecki K., Habrych M.** Autonomous Monitoring System of Environment Conditions // *Electronics and Electrical Engineering*. – Kaunas: Technologija, 2010. – No. 5(101). – P. 63–66.
4. **Cichon J., Gebala M., Kutylowski M.** On Optimal One-dimensional Routing Strategies in Sensor Networks // *4th International Conference on Broadband Communication, Information Technology & Biomedical Applications, BroadBandCom'09*. – Wroclaw, Poland, 2009.
5. **Mosic A., Peric Z., Savic M., Panic S.** Switched Semilogarithmic Quantization of Gaussian Source with Low Delay // *Electronics and Electrical Engineering*. – Kaunas: Technologija, 2011. – No. 2(108). – P. 71–74.
6. **Tiliute Doru E.** Battery Management in Wireless Sensor Networks // *Electronics and Electrical Engineering*. – Kaunas: Technologija, 2007. – No. 4(76). – P. 9–12.
7. **Thai M. T., Wang F., Du D. H., Jia X.**, Coverage problems in wireless sensor networks: designs and analysis // *International Journal of Sensor Networks*. – Inderscience, 2008. – Vol. 3. – No. 3. – P. 191–200.

Received 2011 03 21

J. Gorecki, B. Miedzinski, H. Nouri. Determination of Sensor Network Coverage using Probabilistic Approach // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 5(111). – P. 7–10.

Paper elaborates the method to identify near and far coverage area for each sensor estimating quality of service parameters in order to assess the areas of sensor redundancy and shortage. Coverage in near area is used to identify whether single sensor is enough to cover this area (sensor shortage indicating denser sensor layout). Coverage in far area is used to identify which sensors can be removed from the field in order to have lower cost of deployment of sensor network (sensor redundancy). Ill. 3, bibl. 7, tabl. 1 (in English; abstracts in English and Lithuanian).

J. Gorecki, B. Miedzinski, H. Nouri. Jutikliais-šaltiniais skleidžiamų signalų aprėpimo ploto nustatymas tikimybinu metodu // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 5(111). – P. 7–10.

Siūlomas metodas, padedantis įvertinti jutiklių-šaltinių skleidžiamų signalų aprėpimą plotą prie pačių jutiklių ir nutolusiuose taškuose, kas leistų mažiau naudoti jutiklių ir įvertinti trūkumus. Tiriant zoną prie paties jutiklio siekiama nustatyti, ar vieno jutiklio užtektų reikiamam plotui aprėpti. Tiriant tolimąją zoną siekiama nustatyti, ar galima pašalinti kurį nors iš signalą skleidžiančių jutiklių. Il. 3, bibl. 7, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).