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# Generalized Quasi-Orthogonal Functional Networks Applied in Parameter Sensitivity Analysis of Complex Dynamical Systems

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***Abstract*—This paper presents one possible application of generalized quasi-orthogonal functional networks in the sensitivity analysis of complex dynamical systems. First, a new type of first order ($k$ = 1) generalized quasi-orthogonal polynomials of Legendre type via classical quasi-orthogonal polynomials was introduced. The short principle to design generalized quasi-orthogonal polynomials and filters was also shown. A generalized quasi-orthogonal functional network represents an extension of classical orthogonal functional networks and neural networks, which deal with general functional models. A sequence of the first order ($k$ = 1) generalized quasi-orthogonal polynomials was used as a new basis in the proposed generalized quasi-orthogonal functional networks. The proposed method for determining the parameter sensitivity of complex dynamical systems is also given, and an example of a complex industrial system in the form of a tower crane was considered. The results obtained have been compared with different methods for parameter sensitivity analysis.**

***Index Terms*—Orthogonal polynomials; Sensitivity analysis; Functional networks; Tower crane.**

## I. INTRODUCTION

The development of the application of orthogonal functions is very long and dates back to the end of the 18th century [1]–[3]. However, only since the 1990s, the possibility of their application in the field of control systems, and especially in identification, modelling and control of dynamical systems, has been considered. Recently, great progress has been made in the field of orthogonal rational functions, orthogonal algebraic and trigonometric polynomials, as well as orthogonal systems as a whole [4]–[8].

The use of orthogonal functions and filters in various control problems of dynamical systems is mainly motivated by their simplicity, easy practical design, fast calculations and response. The favourable properties of orthogonal filters, such as easy implementation, precision, and reaction speed, have influenced their application in adaptive automatic control systems, as well as controller optimal adjustment [7]–[9].

In particular, a large number of papers deals with the application of orthogonal systems in electronics [9], circuit theory, digital signal processing [10], and telecommunications. One of the most important applications of orthogonal functions is the design of orthogonal filters that can be successfully applied to design orthogonal signal generators [11], [12], for modelling and identification of dynamical systems, as well as for the practical implementation of optimal and adaptive systems [13], [14].

The authors of this paper developed and designed a new type of orthogonal polynomials and filters. In [4], a new type of almost orthogonal polynomials and filters of the Legendre type, i.e., improved almost orthogonal polynomials/filters, was introduced. In practice, there is also often a need to form filters in which the degree difference in the polynomials of the numerator and denominator of the transfer function is greater than one [5], [12]. Quasi-orthogonal polynomials whose Laplace transforms are rational functions with an arbitrary difference in the degree of the polynomials are suitable for the formation of these transfer functions [9], [13]. The terms which are presented as a combination of quasi-orthogonality and almost orthogonality is introduced in [11], [12]. They represent certain generalizations of classical orthogonality suitable for application in control systems, i.e., technology in general. Also, the new mathematical background derived for these types of orthogonality and polynomials based on them is also presented in [3].

In this paper, we propose a new approach to analysing the sensitivity parameters of dynamic systems. During the system sensitivity analysis, various influences that can change the coordinates of the state of the system [15] are taken into account. The impact of changing the system's parameters on sensitivity is achieved through a series of experiments when the values of the parameters are changed following [16]–[18]. The proposed analysis became a useful

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tool in modelling, designing, validating and testing system performance.

Orthogonal functional networks (OFNs) first appeared in scientific works in the late 20th century [19]. These networks represent an update of the standard neural networks (NNs). OFNs have been shown to solve the same problems as standard NNs. On the other hand, there are many examples where NNs cannot send a request for problem solving and are naturally formulated using OFNs [19], [20]. In general, the orthogonal functional network is a useful mathematical tool to solve a wide range of problems in different technical areas of expertise [20]–[24], etc.

In this paper, a generalized quasi-orthogonal functional network (GQOFN) model is presented where neuron functions are approximated by Legendre generalized quasi-orthogonal basis functions. GQOFN is used for analysis of parameter sensitivity of complex dynamical systems, i.e., the tower crane system. The tower crane system (TCS) is a strong nonlinear electromechanical system controlled from a PC [25].

Combining first order ($k = 1$) generalized quasi-orthogonal polynomials and orthogonal functional networks presents a powerful mathematical tool. It can be used successfully in the analysis of some properties of dynamically complex systems, control of systems, modelling, identification of parameter systems, sensitivity analysis, robustness, etc.

This paper is organized as follows. Section I represents a detailed introduction. In Section II, the generalized quasi-orthogonal polynomials (GQOP) are presented. Section III describes a process of determining parameter sensitivity using newly designed GQOP of Legendre type. The development and design processes of GQOFN processes are explained in a mathematical tool overview in Section IV. The tower crane is described and explained in detail in Section V. Experimental results that verify the given method are presented in Section VI.

## II. GENERALIZED QUASI-ORTHOGONAL POLYNOMIALS

In this section, orthogonal polynomials in a strictly defined interval [-1, 1] were considered. All orthogonal polynomials over finite range (Legendre, Chebyshev I kind, Chebyshev II kind, Jacobi, and Gegenbauer) are defined over interval [-1, 1]. For the purpose of analysing and processing real signals in control systems, which can have values over arbitrary intervals, classical polynomials can be redefined, i.e., shifted on desired interval [0, 1] in complex domain, i.e., on interval [0, ∞] in time domain.

Now, quasi-orthogonal polynomials as sequence of polynomials $\left\{P_n^k(x)\right\}$ were introduced [5]

$$\left(P_n^k(x), P_m^k(x)\right) = \int_a^b P_n^k(x) P_m^k(x) w(x) dx = \\ = \begin{cases} 0, & 0 \le m \le n-k-1, \\ N_{n,m}^k \ne 0, & n \ge k+1, \end{cases} \quad (1)$$

where $k$ represents the order of quasi-orthogonality, $a$ and $b$ are the boundaries of the quasi-orthogonality interval, and $w(x)$ is the weight function.

Quasi-orthogonality was introduced by the Hungarian mathematician Riesz in [26]. Quasi-orthogonal functions were later explored in several papers in [3], [5]–[7], [27], and [28]. In order to analyze the properties and relations of quasi-orthogonal polynomials, the boundaries of the interval of quasi-orthogonality are $a = 0$, $b = 1$ and the weight function is $w(x) = x.$ Now, quasi-orthogonal polynomials have the following inner product

$$\int_0^1 P_n^k(x) P_m^k(x) x dx = \begin{cases} 0, & 0 \le m \le n-k-1, \\ N_{n,m}^k \ne 0, & n \ge k+1, \end{cases} \quad (2)$$

where the norms $N_{n,m}^k$ can be easily obtained in closed form [6], [9].

To introduce a new sequence of polynomials, as an orthogonal basis to use in GQOFN, an arbitrary function $f(x)$ was developed into the class of quasi-orthogonal polynomials of the $k$-th order in the following way (determining quasi-Fourier coefficients)

$$f(x) = \sum_{i=0}^{n} a_i P_i^k(x). \quad (3)$$

For example, for the polynomials of first order ($k = 1$) quasi-orthogonality, after multiplying by $P_i^1$ and integrating from 0 to 1 (boundaries of the quasi-orthogonality interval), it was obtained

$$\int_0^1 f(x) P_i^1(x) dx = a_{i-1} \int_0^1 P_{i-1}^1(x) P_i^1(x) dx + \\ + a_i \int_0^1 P_i^1(x) P_i^1(x) dx + a_{i+1} \int_0^1 P_{i+1}^1 P_i^1(x) dx. \quad (4)$$

If introduce label: $F_i^1 = \int_0^1 f(x) P_i^1(x) dx,$ it obtains

$$a_{i-1}^1 N_{i,i-1}^1 + a_i^1 N_{i,i}^1 + a_{i+1}^1 N_{i,i+1}^1 = F_i^1, \quad (i = 1, \ldots, n), \quad (5)$$

i.e., system of equations (where $n = 0, 1, \ldots, n$)

$$\begin{bmatrix} a_0^1 N_{0,0}^1 + a_1^1 N_{0,1}^1 = F_1^1, \\ a_0^1 N_{0,1}^1 + a_1^1 N_{1,1}^1 + a_2^1 N_{1,2}^1 = F_2^1, \\ \vdots \\ a_{n-2}^1 N_{n-2,n}^1 + a_{n-1}^1 N_{n-1,n-1}^1 + a_n^1 N_{n-1,n}^1 = F_n^1. \end{bmatrix} \quad (6)$$

This is the system of $n$ equations and $n + 1$ unknown variable. To fully determine unknown coefficients $a_0, a_1, \ldots, a_n$, we need one more additional equation. By using the property of quasi-orthogonality of Legendre polynomials $P_i(0) = (-1)^i$ and (1) we obtain an additional equation

$$a_0^1 - a_1^1 + a_2^1 + \cdots + (-1)^n a_n^1 = -f(0). \quad (7)$$

Determinant ($dimD = (n + 1) \times (n + 1)$) of the system is

$$D=\begin{vmatrix} N_{0,0}^1 & N_{0,1}^1 & 0 & 0 & \cdots & 0 \\ N_{0,1}^1 & N_{1,1}^1 & N_{1,2}^1 & 0 & \cdots & 0 \\ 0 & N_{1,2}^1 & N_{2,2}^1 & N_{2,3}^1 & \cdots & 0 \\ 0 & 0 & N_{2,3}^1 & N_{3,3}^1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & N_{n-1,n}^1 \\ 1 & -1 & 1 & -1 & \cdots & (-1)^n \end{vmatrix}. \quad (8)$$

It can be noticed that for the case $k = 1$ it obtains a system of (6), with three terms on the left side of the equality sign (with the exception of the first and the last). The consequence is the three-diagonality of determinant (8). For the general case of quasi-orthogonal polynomials of $k$-th order, the procedure is the same, except that system (6) on the left side has $2k + 1$ terms, and the determinant (8) is $2k + 1$ diagonal.

The orthogonal Legendre polynomials $P_n^{(\delta)}(x)$ were defined and improved in [3], [4] as

$$P_n^{(\delta)}(x)=\sum_{i=0}^{n} A_{n,i}^{\delta} x^i, \quad (9)$$

where $A_{n,i}^{\delta}$ represents the coefficients defined by

$$A_{n,i}^{\delta}=(-1)^{n+i}\frac{\Gamma(n\delta+i+1)}{\Gamma(n\delta+1)\,i!\,(n-i)!}, \quad (10)$$

here $\delta$ is a constant close to one: $\delta = 1 + \varepsilon \approx 1$, constant $\varepsilon$ close to zero: $\varepsilon \approx 0$, and $\Gamma$ is a symbol for the gamma function [2]. Now, if applied (2) on the improved almost orthogonal polynomials given by (9) and (10), the GQOP of Legendre type is obtained over interval (0, 1) with the weight function $w(x)=1$:

$$P_n^{(k,\delta)}(x)=\sum_{i=0}^{n} A_{n,i}^{k,\delta} x^i, \quad (11)$$

$$A_{n,i}^{k,\delta}=(-1)^{n+i+k}\frac{\prod_{j=1}^{n-k}(i+j\delta)}{i!\,(n-i)!}. \quad (12)$$

These polynomials represent a combination of improved almost and quasi-orthogonal polynomials of Legendre type. In further consideration, we use the first order ($k = 1$) of GQOP. First members of the sequence of proposed polynomials are given at the following system equations

$$\begin{bmatrix} P_1^{(1,\delta)}(x)=-x+1, \\ P_2^{(1,\delta)}(x)=-\frac{(\delta+2)}{2}x^2+(\delta+1)x-\frac{\delta}{2}, \\ P_3^{(1,\delta)}(x)=-\frac{(\delta+3)(2\delta+3)}{6}x^3+(\delta+1)(\delta+2)x^2-\frac{(\delta+1)(2\delta+1)}{2}x+\frac{\delta^2}{3}, \\ P_4^{(1,\delta)}(x)=-\frac{(\delta+2)(\delta+4)(3\delta+4)}{12}x^4+\frac{(\delta+1)(\delta+3)(2\delta+3)}{2}x^3-\frac{(\delta+1)(\delta+2)(3\delta+2)}{2}x^2+ \\ \qquad +\frac{(\delta+1)(2\delta+1)(3\delta+4)}{6}x-\frac{\delta^3}{4}. \end{bmatrix} \quad (13)$$

## III. PARAMETER SENSITIVITY

It is widely known that every complex industrial system is in some way imperfect [2], [4], [9], [13], [15]. Namely, every real system is made of technical components that can never be realized with a 100 % accuracy of the nominal value. The consequence is that the entire system will not operate as designed. The measure to which the system deviates from the desired state with certain deviations or changes in parameters represents the sensitivity of the system to these changes in parameters. If the behaviour of the system deviates more than desired, for a certain change of parameters, it is all more sensitive. The sensitivity of a system defined in this way represents parametric sensitivity. In the analysis and synthesis of technical systems, and especially automatic control systems, the sensitivity functions related to the output system are important, because they are the quantities that are controlled.

On the other hand, after introducing substitution $x=e^{-t}$ into (11) and applying the Laplace transform, we obtain a transfer function (the first order $k = 1$) suitable for further consideration [5], [12]

$$W_n^{(1,\delta)}(s)=\frac{\prod_{i=1}^{n-1}(s-i\delta)}{\prod_{i=0}^{n}(s+i)}=\frac{(s-\delta)\cdots(s-(n-1)\delta)}{s(s+1)\cdots(s+n)}. \quad (14)$$

The transfer function given above by (14), can be rewritten in the following form

$$W_n^{(1,\delta)}(s)=\frac{\prod_{i=1}^{n-1}(s-i\delta)}{\prod_{i=0}^{n}(s+i)}=\frac{Y(s)}{X(s)}=$$
$$=\frac{s^{n-1}-A_1\delta s^{n-2}+A_2\delta^2 s^{n-3}-\cdots-A_{n-1}\delta s+(n-1)!\,\delta^{n-1}}{s^{n+1}+B_1 s^n+B_2 s^{n-1}+\cdots+B_n s}, \quad (15)$$

where $A_i$ and $B_j$ represent real constants, $X(s)$ represents the input of the system, and $Y(s)$ represents the output of the system.

The transfer function given by (15) represents a linear system, which has ($2n - 1$) parameters, $A_i$ ($i = 1,\ldots, n - 1$) and $B_j$ ($j = 1, \ldots, n$). Therefore, it is possible to define the ($2n - 1$) sensitivity functions in the $s$-domain as follows

$$\begin{bmatrix} u_{A_i}(s) = \dfrac{\partial Y(s)}{\partial A_i} \quad i = 1,\ldots,n-1, \\ u_{B_j}(s) = \dfrac{\partial Y(s)}{\partial B_j} \quad i = 1,\ldots,n. \end{bmatrix} \tag{16}$$

For parameters $A_i$ and $B_j$, sensitivity functions are given by the following expressions:

$$u_{A_i}(s) = \frac{s^{n-1} - A_1\delta s^{n-2} + A_2\delta^2 s^{n-3} - \cdots\cdots - A_{n-1}\delta s + (n-1)!\,\delta^{n-1}}{s^{n+1} + B_1 s^n + B_2 s^{n-1} + \cdots + B_n s} \times \frac{s^i}{s^{n+1} + B_1 s^n + B_2 s^{n-1} + \cdots + B_n s} X(s), \tag{17}$$

$$u_{B_j}(s) = \frac{s^i}{s^{n+1} + B_1 s^n + B_2 s^{n-1} + \cdots + B_n s} X(s). \tag{18}$$

In the case of the system sensitivity in steady state, it can be used [15], [23]:

$$Y_i(\infty) = \lim_{s\to 0} sW_s(s)X(s), \tag{19}$$

$$\begin{bmatrix} u_{A_i} = \dfrac{\Delta Y_i(\infty)}{\left(A_i - (\pm mA_i)\right)}, \\ u_{Bj} = \dfrac{\Delta Y_i(\infty)}{\left(B_j - (\pm mB_j)\right)}, \end{bmatrix} \tag{20}$$

where $m$ represents limits ±20 %, ±10 %, ±5 %, and ±1 % from the optimal value of the parameter $A_i$ and $B_j$, respectively.

## IV. GENERALIZED QUASI-ORTHOGONAL FUNCTIONAL NETWORKS

The typical architecture of a classical orthogonal functional network [15], [19]–[22] is shown in Fig. 1.

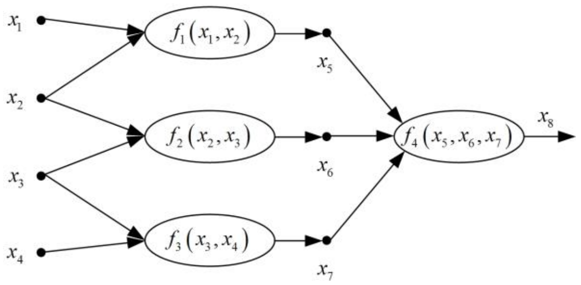


Fig. 1. Block diagram of the orthogonal functional network model.

The orthogonal functional network (Fig. 1) consists of four inputs ($x_1$, $x_2$, $x_3$, and $x_4$), one output ($x_8$) layer, one or more layers of intermediate storage units ($x_5$, $x_6$, and $x_7$), and one or multiple layers of processing units ($f_1$, $f_2$, $f_3$, and $f_4$). In the case where the input values are known, the output is determined by the function and type of neurons. If we use generalized quasi-orthogonal basis functions for neurons, we can get GQOFN (see Fig. 2)

$$y = \sum_{i=1}^{n} w_i f_i(x). \tag{21}$$

If we use Legendre polynomials of the first order ($k = 1$) of the GQOP basis, we can design a GQOFN of Legendre type. The function $f_i$ in this case will represent the first $i$ function in the first layer, i.e., $P_i^{(1,\delta)}(x)$. The output signals of these layers are orthogonal in the time domain. The proposed type of network can be used to approximate and model each system with optimal accuracy, adjust the weight of the function $w_i$, and minimize the criterion function. A genetic algorithm will settle optimal weight values [3]–[13].

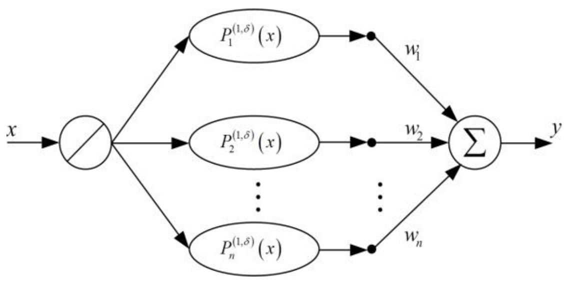


Fig. 2. The generalized quasi-orthogonal functional network.

## V. TOWER CRANE-MATHEMATICAL MODEL

For the purpose of sensitivity analysis of dynamical systems model, we will use the tower crane system shown in Fig. 3.

The tower crane is a powerful nonlinear electromechanical system with complex dynamic behaviour [29]–[33]. Previous works have shown that it is challenging to design a control for such a system. The operation of this system is done via PC. Therefore, it comes with hardware and software that can be easily assembled and installed under laboratory conditions.

The tower crane setup (Fig. 3) is made of a payload hanging on a pendulum-like lift-line wound by a motor mounted on a cart. The payload can be lifted and lowered in the $z$-direction. Horizontal movement is possible for both the arm and the cart.

The angle $\theta$ represents the angular position of the arm. The payload can move in all three dimensions. Three independent DC motors power this system. Also, this system has five encoders installed to measure five variable states.

Fig. 3. Tower crane by INTECO.

The encoders measure movements with high resolution equal to up to 4096 pulses per rotation (*ppr*). These encoders, together with the specialised mechanical solution, create a unique measurement unit. The deviation of the load is measured with a high accuracy equal to 0.0015*rad*. The power interface amplifies the control signals that are transmitted from the PC to the DC motors. The schematic diagram related to the mathematical model of the tower crane is shown in Fig. 4 [25].

The length of the tower is 1.63 *m*, the jib is 1.215 *m* long. A constant length of 0.17 *m* of the payload lift-line is considered; the payload mass is 0.333 *kg*.

To understand the work of TCS, we can measure the following quantities: $x_w$, $\theta$, $L$, $\alpha$, and $\beta$, which represent the distance of the cart along the arm from the centre of the construction frame, the angular position of the arm, the length of the lift-line, the angle between the z axis and the projection of the lift-line onto the *x*-*z* plane, and the lift-line, respectively.

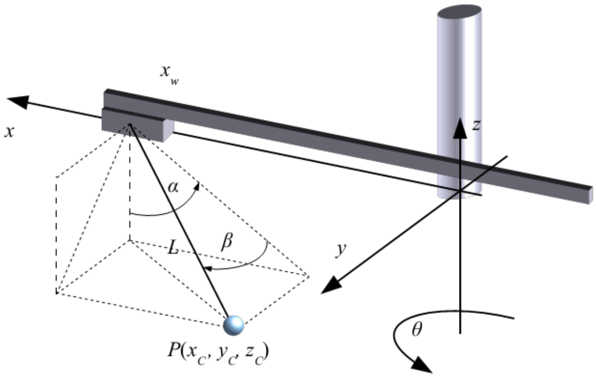


Fig. 4. TCS model coordinates.

To obtain an appropriate mathematical model of the observed system, it is necessary to choose the coordinates of the state. The main point (0, 0, 0) of the Cartesian system is in the centre of TCS. In Fig. 4, we can see the payload position. Now, the position of the payload from Fig. 4 can be express [25]

$$\begin{bmatrix} x_c = x_w - L\cos\beta\sin\alpha, \\ y_c = L\sin\beta, \\ z_c = -L\cos\alpha\cos\beta. \end{bmatrix} \tag{22}$$

The first derivatives of the previous equations and the kinetic and potential energy of the payload are calculated. Based on the Lagrange equations [25], the following equation that describes the dynamics of TCS is given in [29]

$$\begin{bmatrix} \dot{x}_1 = x_5, \\ \dot{x}_2 = x_6, \\ \dot{x}_3 = x_7, \\ \dot{x}_4 = x_8, \\ \dot{x}_5 = -\dfrac{1}{2L}\begin{pmatrix} 2g\cos x_2 \sin\dot{x}_1 + 4x_7x_8\cos x_1 - Lx_8^2\sin(2x_1)\cos^2 x_2 - 2x_3x_8^2\sin x_1\sin x_2 - \\ -4Lx_6x_8\cos x_2\cos^2 x_1 + Lx_6^2\sin(2x_1) + 2\sin x_1\sin x_2 u + 2x_3\cos x_1 u_2 - 2L\sin x_2 u_2 \end{pmatrix}, \\ \dot{x}_6 = -\dfrac{1}{2L\cos x_1}\begin{pmatrix} g\sin x_2 + x_3x_8^2\cos x_2 - Lx_8^2\sin x_2\cos x_1\cos x_2 + 2Lx_5x_8\cos x_1\cos x_2 - \\ -2Lx_5x_6\sin x_1 - \cos x_2 u_1 + L\sin x_1\cos x_2 u_2 \end{pmatrix}, \\ \dot{x}_7 = u_1, \\ \dot{x}_8 = u_2, \end{bmatrix} \tag{23}$$

where: $\dot{x}_1 = \beta$, $\dot{x}_2 = \alpha$, $\dot{x}_3 = x_w$, $\dot{x}_4 = \theta$, $\dot{x}_5 = \dot{\beta}$, $\dot{x}_6 = \dot{\alpha}$, $\dot{x}_7 = x_w$, $\dot{x}_8 = \dot{\theta}$, $u_1 = x_w$, $u_2 = \ddot{\theta}$, $L = const$. The complete mathematical background of complex mathematical tools can be found in [29].

## VI. EXPERIMENTAL RESULTS

Through a series of experiments, the minimization of the criterion function (in our case, the mean square error (MSE)) was performed, with the goal of obtaining the transfer function of the tower crane system.

To obtain the model of the tower crane system, we will use the GQOFN first order ($k = 1$) of Legendre type, which has four weight $w_i$. The data used in the experiment is the trolley position in the *x*-direction. Applying the method of identification of the linear TCS system [2], [3], we obtained the transfer function in the form

$$W(s) = \frac{0.00002064s + 1.128}{0.0005s^2 + 0.0446s + 1}. \tag{24}$$

Now, using the previous transfer function, we can use GQOFN to obtain the transfer function of the TCS. This transfer function depends on parameter $\delta$, which represents the system's imperfection. The values of $\delta$ contain different impacts of all imperfect components from which the system is made. This parameter directly influences the output system response. After applying the series of tests and experiments with different ranges of the parameter $\delta$ [4], [12], it was determined that the optimal value for $\delta$ is equal to 1.000345.

The optimal values of the adjustable parameters $w_1$, $w_2$, $w_3$, and $w_4$, needed for the best model of the unknown system, are determined using the genetic algorithm [3], [4], [9]. Genetic algorithm used in simulation has the following parameters: a population of 300, a number of generations of 600, the selection was stochastic uniform and reproduction with 10 elite individuals. The chromosome has a structure that consists of four adjustable parameters. After the previous procedure was completed, we obtained the following values: $w_1$ = 0.0000247, $w_2$ = -0.03256, $w_3$ = 0.11478, and $w_4$ = 0.06454. The quality of identification is evident from the comparison of output time responses of the real device and the model (24) shown in Fig. 5.

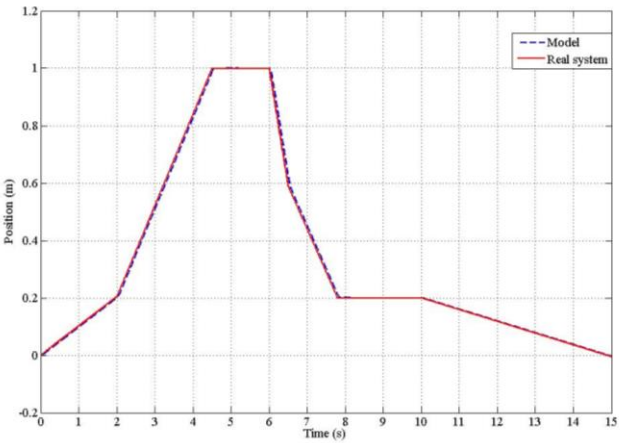


Fig. 5. Trolley position in the *x*-direction.

Next, we tuned the first parameter, $w_1$, in the range of ±20 %, ±10 %, ±5 %, and ±1 % from the optimal value, while the other parameters kept their values.

After each change in parameter $w_1$, we measured the output in a steady state through experiments. Based on (19) and (20), the sensitivity value related to parameter $w_1$ was calculated and the results are shown in Table I-IV.

TABLE I. SENSITIVITY VALUE RELATED TO PARAMETER $W_1$.

| Tol. *m* | $w_1$ | $\Delta y(\infty)$ | $\lvert u_{w1}\rvert$ |
|---|---|---|---|
| +20 % | 0.000029 | -0.00000016796 | 0.034002 |
| +10 % | 0.000027 | -0.00000006754 | 0.028144 |
| +5 % | 0.000026 | -0.00000003401 | 0.026158 |
| +1 % | 0.000025 | -0.00000000660 | 0.022007 |
| -1 % | 0.000024 | 0.000000015480 | 0.022112 |
| -5 % | 0.000023 | 0.000000044804 | 0.026355 |
| -10 % | 0.000022 | 0.00000007614 | 0.028200 |
| -20 % | 0.000019 | 0.000000194513 | 0.034125 |

TABLE II. SENSITIVITY VALUE RELATED TO PARAMETER $W_2$.

| Tol. *m* | $w_2$ | $\Delta y(\infty)$ | $\lvert u_{w2}\rvert$ |
|---|---|---|---|
| +20 % | -0.042312 | 0.002340274104 | 0.031257 |
| +10 % | -0.035816 | 0.000091223352 | 0.028017 |
| +5 % | -0.034188 | 0.000042331256 | 0.026002 |
| +1 % | -0.032886 | 0.000007124404 | 0.021854 |
| -1 % | -0.032234 | -0.000007168088 | 0.021988 |
| -5 % | -0.030932 | -0.000042743141 | 0.026225 |
| -10 % | -0.029304 | -0.000090839144 | 0.027899 |
| -20 % | -0.026048 | -0.000202692512 | 0.031126 |

TABLE III. SENSITIVITY VALUE RELATED TO PARAMETER $W_3$.

| Tol. *m* | $w_3$ | $\Delta y(\infty)$ | $\lvert u_{w3}\rvert$ |
|---|---|---|---|
| +20 % | 0.137736 | -0.000694901076 | 0.030271 |
| +10 % | 0.126258 | -0.000316276290 | 0.027555 |
| +5 % | 0.120519 | -0.000147102048 | 0.025632 |
| +1 % | 0.115928 | -0.000024112592 | 0.021004 |
| -1 % | 0.113632 | 0.000024197544 | 0.021078 |
| -5 % | 0.109041 | 0.000147934203 | 0.025777 |
| -10 % | 0.103302 | 0.000311501442 | 0.027139 |
| -20 % | 0.091824 | 0.000696324348 | 0.030333 |

TABLE IV. SENSITIVITY VALUE RELATED TO PARAMETER $W_4$.

| Tol. *m* | $w_4$ | $\Delta y(\infty)$ | $\lvert u_{w4}\rvert$ |
|---|---|---|---|
| +20 % | 0.077448 | -0.000369852924 | 0.028653 |
| +10 % | 0.070994 | -0.000155922186 | 0.024159 |
| +5 % | 0.067767 | -0.000068983579 | 0.021377 |
| +1 % | 0.065185 | -0.00001239819 | 0.019222 |
| -1 % | 0.063895 | 0.000012241455 | 0.018979 |
| -5 % | 0.061313 | 0.000068625382 | 0.021266 |
| -10 % | 0.058086 | 0.000154902454 | 0.024001 |
| -20 % | 0.051632 | 0.000368407228 | 0.028541 |

Experiments have shown that the TCS system is the most sensitive to parameter $w_1$ and the least susceptible to parameter $w_4$. This indicates that if we need to adjust the system's output in steady state, we need to change the values of the parameter $w_1$. If it is not possible to adjust the system output with one parameter, it is necessary to change two parameters ($w_1$ and $w_2$) in specified ranges.

To compare the results obtained using GQOFN, a series of experiments were performed using the nominal range sensitivity method from [15], [34]. This method is based on the principle that only one of the parameters is tuned until the system output is acceptable while the other parameters retain their optimal value. The results using this method are shown in Table V.

From Table V, we can conclude that the new results are similar to the previous results presented in other tables, with the dependencies shifted to lower sensitivity values. The disadvantage of this method is that it does not include the effect of input correlation. Also, a significant drawback is that a very complex mathematical tool is needed to obtain the parameters [34].

TABLE V. SENSITIVITY VALUE RELATED TO PARAMETER $W_1$–$W_4$ USING NOMINAL RANGE METHOD.

| **Tol. *m*** | $\lvert u_{w1} \rvert$ | $\lvert u_{w2} \rvert$ | $\lvert u_{w3} \rvert$ | $\lvert u_{w4} \rvert$ |
|---|---|---|---|---|
| +20 % | 0.033876 | 0.030003 | 0.029432 | 0.028331 |
| +10 % | 0.028056 | 0.027799 | 0.027122 | 0.023888 |
| +5 % | 0.025551 | 0.025811 | 0.025701 | 0.021233 |
| +1 % | 0.020072 | 0.020991 | 0.020699 | 0.018866 |
| -1 % | 0.020123 | 0.021012 | 0.020812 | 0.018001 |
| -5 % | 0.024898 | 0.026015 | 0.025743 | 0.021011 |
| -10 % | 0.027977 | 0.027671 | 0.026771 | 0.023712 |
| -20 % | 0.033512 | 0.029756 | 0.029117 | 0.028119 |

## VII. CONCLUSIONS

In this paper, the concept of a combination of generalized, quasi- and almost orthogonal functional networks is exploited to consider the influence of system parameters on the sensitivity analysis of dynamic complex systems. First, a type of the first order ($k$ = 1) generalized quasi-orthogonal polynomials of Legendre type was introduced through classical quasi-orthogonal polynomials. The complex mathematical backgrounds for the proposed approach to determine the sensitivity of dynamical systems through GQOFN are also given.

Experiments with the TCS system were performed to validate the theoretical results. The advantages of the application of newly obtained generalized quasi-orthogonal functions in combination with orthogonal functional networks in the analysis of parameter sensitivity of this complex TCS system based on proposed method have been demonstrated in experiments. All results have been compared with known nominal range sensitivity. The combination of orthogonal functional networks and generalized quasi-orthogonal polynomials could be successful in solving problems of some properties of wide spectra dynamical complex systems. On the other hand, the method proposed in this paper can be used in the fields of telecommunications, control of complex industrial systems, modelling, and identification of parameters of unknown dynamical systems.

## CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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