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Transitional Selective Linear Phase 1D FIR Filter Function Generated by Christoffel-Darboux Formula for Chebyshev Polynomials

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***Abstract*—In this paper we propose a new economical selective low-pass finite impulse response filter function. First, we describe the two types of already developed filters, and then we obtain three new different types of filters, by combining the previous ones. Also, the general form of the proposed filter is given. In order to verify the effectiveness of the proposed filter function, the cut-off frequencies of the stop-band and pass-band of the filter, for equal constant group delay, are analysed and compared with classical first and second type of filters. It is shown that proposed filter is efficient related to high selectivity and high values of attenuation in the stop-band of the filter.**

***Index Terms*—Difference equations, digital filters, filtering theory, finite impulse response filter.**

I. INTRODUCTION

In recent time the classical orthogonal polynomials have be In recent time the classical orthogonal polynomials have been intensively used for designing low-pass orthogonal filters with numerous applications in mathematics, science and engineering such as in designing orthogonal signal generators, least square approximations, process modelling, identification and practical realizations of optimal and adaptive systems [1]–[5]. During the past several years, we have developed some new concepts of orthogonality (almost and quasi orthogonal filters) using mathematical transformations in complex domain [6], [7]. These filters can be used as generators of functions sequence, suitable for approximations, modelling and analysis of imperfect systems [1], [3], [4].

Moreover, analog and digital filter design is of great importance throughout engineering, applied mathematics, and computer science [8], [9]. In [10], we proposed a method for the design of filter transfer function in an explicit form having the decreasing envelope of the summed sensitivity function in the pass-band. This type of filter function is known for better behaviour of the amplitude response when the filter is realized with real components [3], [4], [10], [11]. Also, in [12]–[15], we proposed a form of a prototype class of low frequency selective polynomial analogue filter functions, derived in compact explicit form on a simple way by direct application of the modified Christoffel-Darboux formula for classical continual orthonormal Jacobi polynomials [16], [17].

Numerical iterative methods for design of FIR filters have already been in the spotlight of many papers. An efficient procedure for the weighted Chebyshev design on non-recursive linear phase FIR digital filters was presented in [18]. In [19], the authors gave a general-purpose attractive computer program capable of designing a large class of optimum linear phase FIR digital filters. An efficient technique which allows using of an iterative Remez exchange algorithm to compute the best approximation to the desired frequency response was described in [20]. In [21], one approach which can be efficiently used for 1-D and 2-D approximation problems was given.

Analytic methods for analysis and synthesis of 1D and 2D filters functions were given in our earlier papers [22]–[24]. In these papers we gave the general structure of the selective low-pass FIR filter. A new version of efficient digital filter design method, based on the most efficient low-order EMQF filter, was presented in [25].

The theory and applications of Cascaded-integrated-comb (CIC) filters was researched in the following papers. A new sharpening technique for improvement of CIC filter characteristics, by connecting an additional filter called CIC compensator, was described in [26], [27]. Furthermore, many methods for the design of CIC compensators had been developed and presented in [28].

In this paper we propose a transitional linear phase 1D FIR filter which is based on linear phase 1D FIR filters described in [22], [23]. It is already shown that physical realization of these filters does not require multipliers. The proposed filter function has three integer parameters and it is efficient in terms of high selectivity and high values of delay in the stop-band of the filter. Also, all the transitional filter functions of even and odd order, which can be generated, are described in this paper. The main advantage of these filters is reflected in new possibilities between initial particular solutions [29], [30].

The paper is organized as follows. In Section II, the five different types of 1D FIR filter functions are described and their realization structures are also given. In Section III, the

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general form of proposed transitional linear phase FIR filter, for different values of length of first type of filter and length of second type of filter, is described for both odd and even order of the filter. Also, the particular solutions of FIR filter function are also given. It is shown that general form of FIR filter function with K blocks cascade-connected [31] significantly suppresses the Gibb's phenomenon. The detailed analysis of all described filter functions, with equal constant delay, is given in Section IV. The examples are illustrated and the cut-off frequencies of the pass-band and stop-band of the filter are given in tables. It is shown that proposed filter has great results in regard to high selectivity and economical realization structure.

II. Linear Phase FIR Filter Functions with Equal Value of Constant Group Delay

In [22], the difference equation of 1D FIR filter function is already developed and it has the following form

$$y[n]=y[n-2]+x[n]-x[n-2N]+ \\ +x[n-2N-2]-x[n-4N-2], \quad (1)$$

where N is an integer parameter. This equation can be represented in the z domain by a causal low-pass FIR filter function as

$$H_1(N,\ z)=H_a(N,\ z)=\frac{1+z^{-(2N+2)}}{1-z^{-2}}\left(1-z^{-2N}\right). \quad (2)$$

The amplitude characteristic of previously described filter function is described by

$$\left|H_1\left(N,e^{j\mu}\right)\right|=2\frac{\cos\left((N+1)\mu\right)}{\sin\mu}\sin(N\mu). \quad (3)$$

For the sake of simplicity, let us label this type of selective 1D FIR filter described by (2) as FIR1 filter. The structure of the FIR1 filter, for N = 10, is shown in Fig. 1.

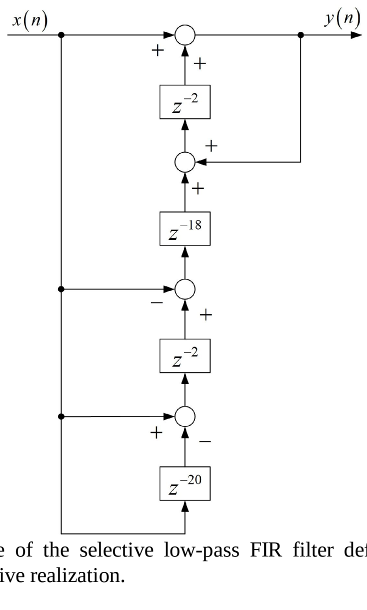


Fig. 1. Structure of the selective low-pass FIR filter defined by (2) for N = 10 in recursive realization.

In [23], we proposed another difference equation for 1D FIR filter function as

$$y[n]=y[n-2]+x[n]+x[n-2N-2]- \\ -x[n-2N]-x[n-4N-2], \quad (4)$$

where N is also an integer number. The last equation can be represented in the z domain as

$$H_2(N,z)=H_b(N,\ z)=\frac{1-z^{-(2N+2)}}{1-z^{-2}}\left(1+z^{-2N}\right). \quad (5)$$

Consequently, the amplitude characteristic of (5) is defined as

$$\left|H_2\left(N,e^{j\mu}\right)\right|=2\frac{\sin\left((N+1)\mu\right)}{\sin\mu}\cos(N\mu). \quad (6)$$

Similarly to the previous filter function, let us label this one as FIR 2. Its realization structure for N/2 = 5, is presented in Fig. 2.

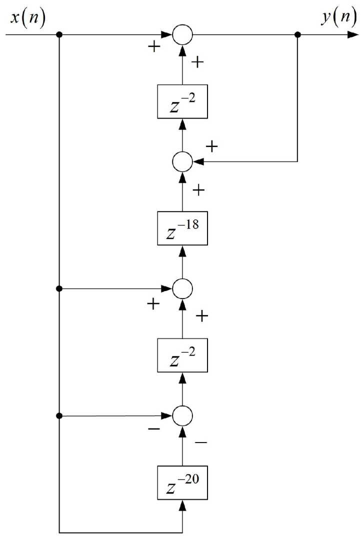


Fig. 2. Structure of the selective low-pass FIR filter defined by (5) for N = 10 in recursive realization.

The main advantages of these filters, over the classical ones, are high selectivity and economical realization structure. The main drawback is that for chosen parameter N we have limited possibility for filter specification.

To overcome this problem, the equal value of constant group delay can be obtained by using two cascade-connected sections of the filter presented in [22] or [23]. In this way, we can provide other features of selectivity of filters.

In order to illustrate this effect, we chose parameter N to be two times smaller than in the case with one section. Now, the third type of the low-pass 1D FIR filter function (FIR3) is expressed as

$$H_3(N,z)=H_a(N/2,z)H_a(N/2,z)= \\ =\frac{1+z^{-(N+2)}}{1-z^{-2}}\left(1-z^{-N}\right)\frac{1+z^{-(N+2)}}{1-z^{-2}}\left(1-z^{-N}\right). \quad (7)$$

The amplitude characteristic of filter function (7) has the following form

$$\left|H_3\left(N,e^{j\mu}\right)\right| = \left(2\frac{\cos\left(\left(N/2+1\right)\mu\right)}{\sin\mu}\sin\left(N\mu/2\right)\right)^2. \quad (8)$$

Figure 3 shows the practical realization of FIR3, for $N/2 = 5$.

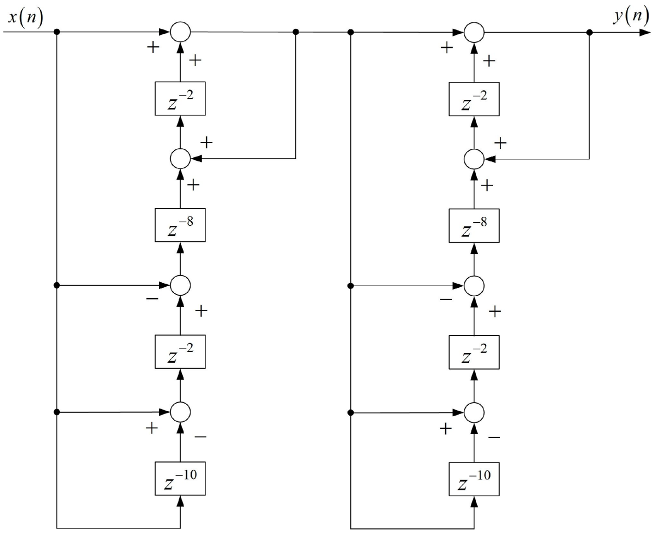


Fig. 3. Structure of the selective low-pass FIR filter defined by (7) for $N/2 = 5$ in recursive realization.

The same principle can be applied for the 1D FIR filter function proposed in [23] and fourth type of low-pass 1D FIR filter function (FIR4) is described with

$$H_4\left(N,z\right) = H_b\left(N/2,z\right)H_b\left(N/2,z\right) =$$

$$= \frac{1-z^{-(N+2)}}{1-z^{-2}}\left(1+z^{-N}\right)\frac{1-z^{-(N+2)}}{1-z^{-2}}\left(1+z^{-N}\right). \quad (9)$$

The corresponding amplitude characteristic is

$$\left|H_4\left(N,e^{j\mu}\right)\right| = \left(2\frac{\sin\left(\left(N/2+1\right)\mu\right)}{\sin\mu}\cos\left(N\mu/2\right)\right)^2. \quad (10)$$

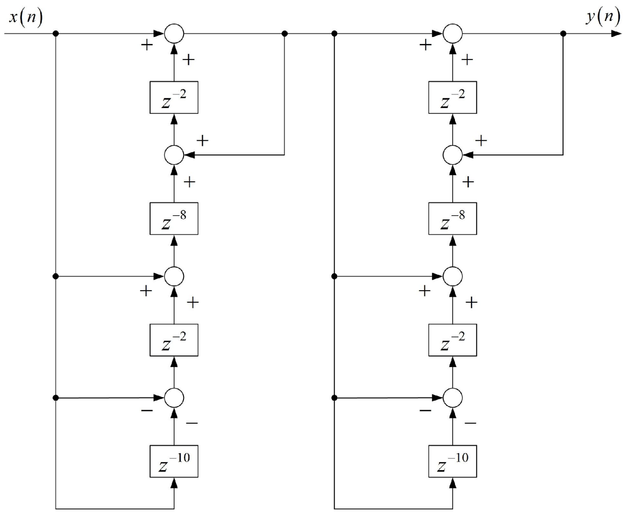


Fig. 4. Structure of the selective low-pass FIR filter defined by (9) for N=5 in recursive realization.

Herein, we propose a new transitional 1D FIR filter (FIR5). It consists of cascade-connected FIR1 and FIR2 filters. The proposed filter has equal value of constant group delay, and for $N/2 = 5$ it is shown in Fig. 5.

The proposed low-pass 1D FIR filter function is now determined by

$$H_5\left(N,z\right) = H_a\left(N/2,z\right)H_b\left(N/2,z\right) =$$

$$= \frac{1+z^{-(N+2)}}{1-z^{-2}}\left(1-z^{-N}\right)\frac{1-z^{-(N+2)}}{1-z^{-2}}\left(1+z^{-N}\right). \quad (11)$$

The corresponding amplitude characteristic is then fully described with

$$\left|H_5\left(N,\ e^{j\mu}\right)\right| = \left(2\frac{\cos\left(\left(N/2+1\right)\mu\right)}{\sin\mu}\sin\left(N\mu/2\right)\right)\times$$

$$\times\left(2\frac{\sin\left(\left(N/2+1\right)\mu\right)}{\sin\mu}\cos\left(N\mu/2\mu\right)\right). \quad (12)$$

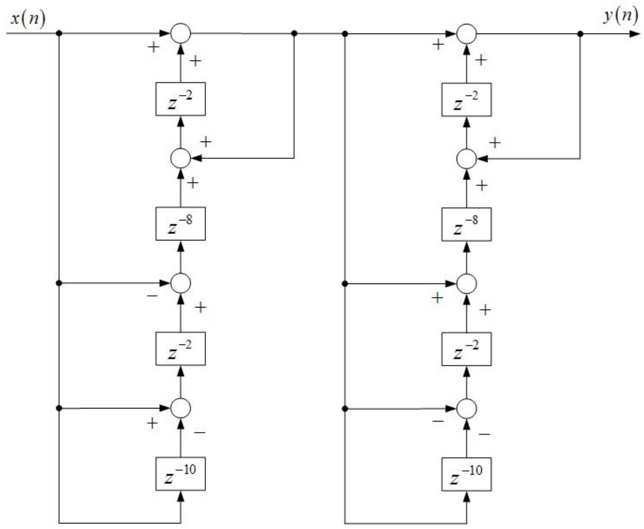


Fig. 5. Structure of the selective low-pass FIR filter defined by (11) for $N/2 = 5$ in recursive realization.

As we mentioned before, all the filter functions (2), (5), (7), (9), (11) have equal constant group delay described by

$$^2\left(\mu\right) = -2N. \quad (13)$$

III. THE GENERAL FORM OF PROPOSED TRANSITIONAL LINEAR PHASE FIR FILTERS

In this Section, we propose the general solution of the transitional linear phase FIR filters in cascade form

$$H\left(N,z\right) = H_a\left(N_a,z\right)H_b\left(N_b,z\right). \quad (14)$$

For odd order N, $N = 10$, the possible particular solutions can be obtained for different values of parameters N_a and $N_b = N - N_a$. These transfer functions are given by:

$$H\left(10,z\right) = H_a\left(10,z\right)H_b\left(0,z\right), \quad (15)$$

$$H\left(10,z\right) = H_a\left(9,z\right)H_b\left(1,z\right), \quad (16)$$

$$H(10,z)=H_a(8,z)H_b(2,z), \quad (17)$$

$$H(10,z)=H_a(7,z)H_b(3,z), \quad (18)$$

$$H(10,z)=H_a(6,z)H_b(4,z), \quad (19)$$

$$H(10,z)=H_a(5,z)H_b(5,z), \quad (20)$$

$$H(10,z)=H_a(4,z)H_b(6,z), \quad (21)$$

$$H(10,z)=H_a(3,z)H_b(7,z), \quad (22)$$

$$H(10,z)=H_a(2,z)H_b(8,z), \quad (23)$$

$$H(10,z)=H_a(1,z)H_b(9,z), \quad (24)$$

$$H(10,z)=H_a(0,z)H_b(10,z). \quad (25)$$

It is obvious that for $N_b = 0$ and for $N_a = 0$, the relations (15) and (25) correspond to particular solutions given in [22] and [23], respectively.

For even order N, N = 9, we can generate the new transitional filter functions described by the following equations:

$$H(9,z)=H_a(9,z)H_b(0,z), \quad (26)$$

$$H(9,z)=H_a(8,z)H_b(1,z), \quad (27)$$

$$H(9,z)=H_a(7,z)H_b(2,z), \quad (28)$$

$$H(9,z)=H_a(6,z)H_b(3,z), \quad (29)$$

$$H(9,z)=H_a(5,z)H_b(4,z), \quad (30)$$

$$H(9,z)=H_a(4,z)H_b(5,z), \quad (31)$$

$$H(9,z)=H_a(3,z)H_b(6,z), \quad (32)$$

$$H(9,z)=H_a(2,z)H_b(7,z), \quad (33)$$

$$H(9,z)=H_a(1,z)H_b(8,z), \quad (34)$$

$$H(9,z)=H_a(0,z)H_b(9,z). \quad (35)$$

In this case, the particular solutions are given by (26) and (35).

An example of a filter is demonstrated in Fig. 6, for particular values $N_a = 3$ and $N_b = 6$.

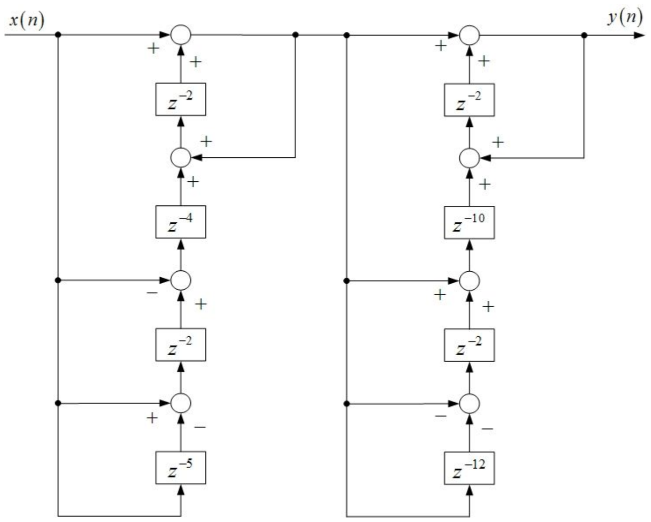


Fig. 6. Structure of the selective low-pass FIR filter defined by (32) and (2) and (5), for N_a = 3 and N_b = 6 in recursive realization.

IV. EXPERIMENTAL RESULTS

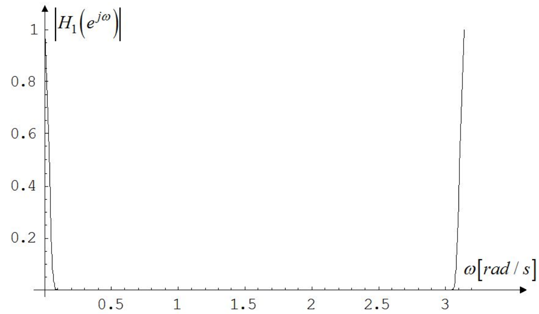
In this paper we consider the transfer function described in [22], which represents the generalized solution given in [31] and it fully suppresses the Gibb's phenomenon

$$H(M-1,z)H(M,z)H(M+1,z). \quad (36)$$

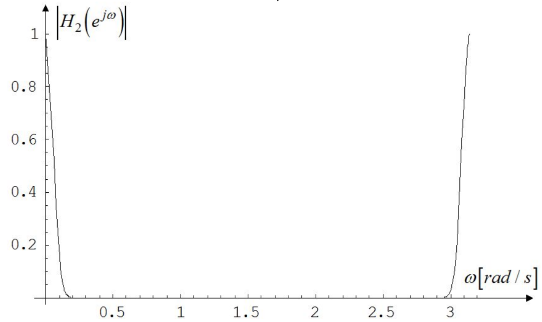
So, general form of the proposed filter function, using free real parameter K, is given by the following expression

$$\left[H(M-1,z)H(M,z)H(M+1,z)\right]^K. \quad (37)$$

The detailed analysis of all five types of FIR filters, for M = 10 and K = 3, is illustrated in this paper. Group delay of considered functions is equal, $^{2}(\mu) = -180s$.



a)



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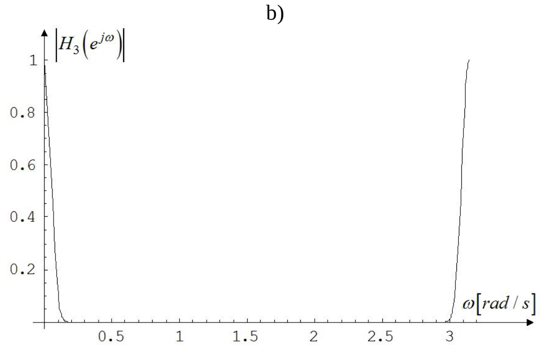

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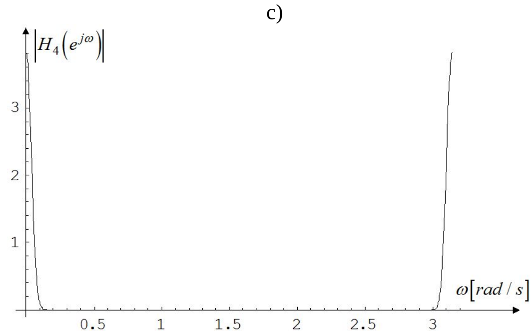

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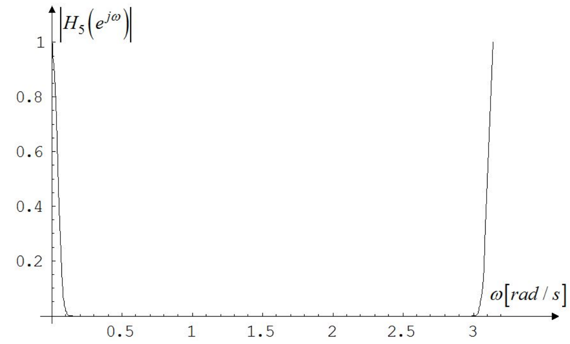
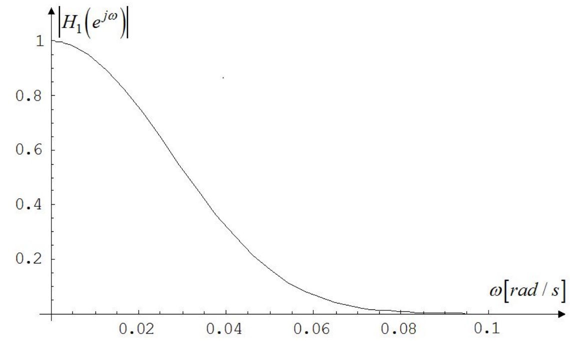
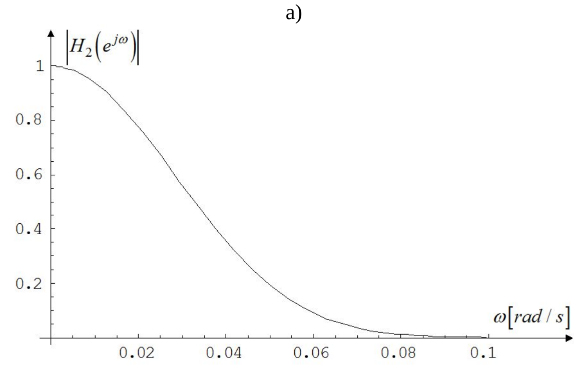


e)

Fig. 7. The normalized magnitude characteristics of linear phase 1D FIR filter, for M = 10 and K = 3: a) FIR1, b) FIR2, c) FIR3, d) FIR4, e) FIR5.



a)



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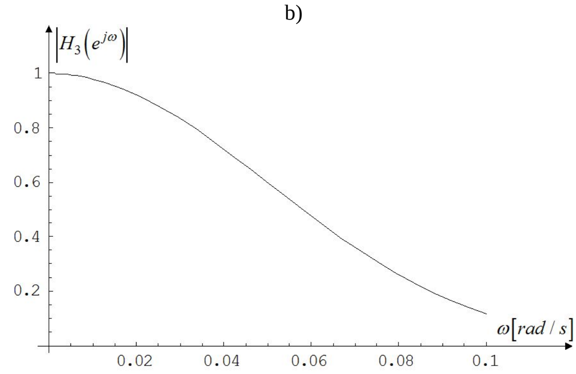

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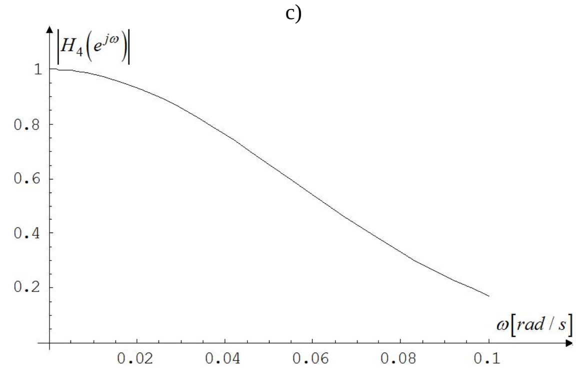

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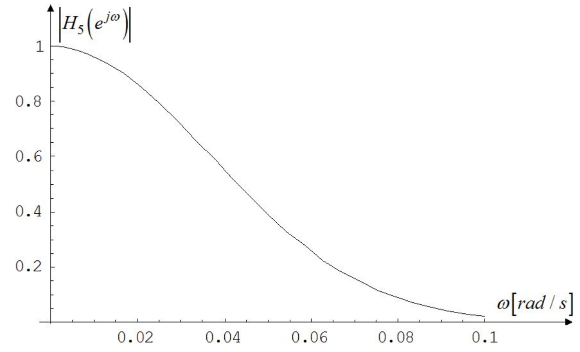


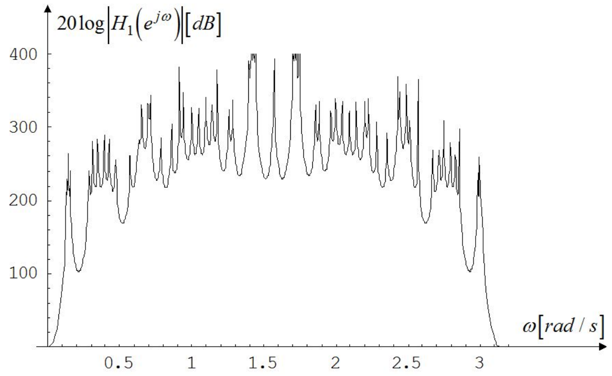
e)

Fig. 8. The normalized zoomed magnitude characteristics of linear phase 1D FIR filter, for M = 10 and K = 3: a) FIR1, b) FIR2, c) FIR3, d) FIR4, e) FIR5.

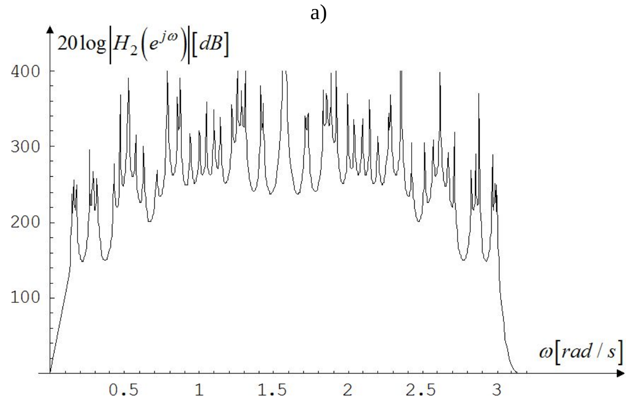
The magnitude characteristics of all the five types of linear phase 1D FIR filters are shown in Fig. 7(a)–Fig. 7(e), respectively.

The zoomed characteristics of previous figures are presented in Fig. 8(a)–Fig. 8(e), respectively.

The magnitude characteristics of all the five types of 1D FIR filter functions in dBs, for M = 10 and K = 3, are shown in Fig. 9(a)–Fig. 9(e).



a)



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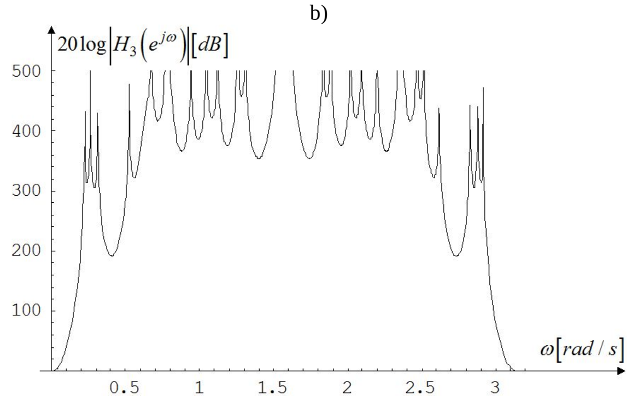

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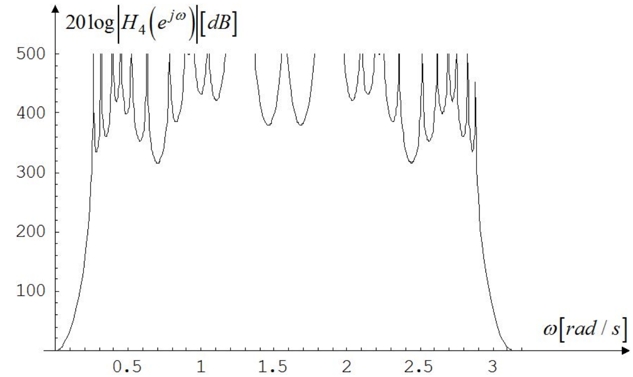

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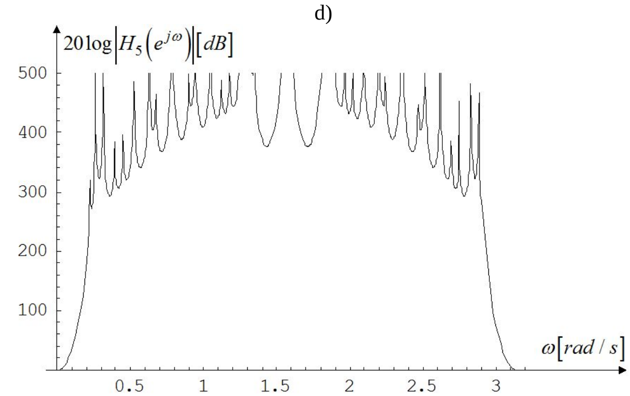
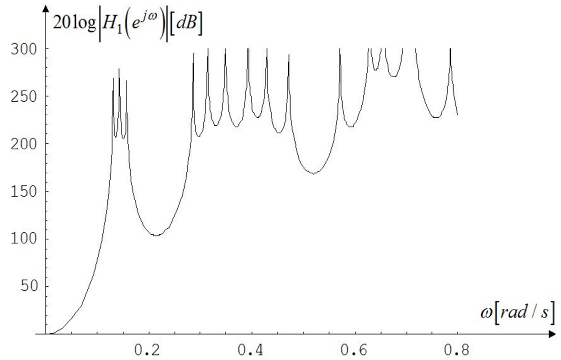


e)

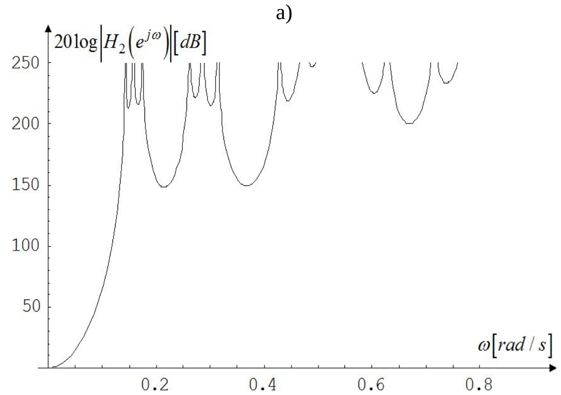
Fig. 9. The normalized magnitude characteristics of linear phase 1D FIR filter in dBs, for M = 10 and K = 3: a) FIR1, b) FIR2, c) FIR3, d) FIR4, e) FIR5.

The normalized zoomed magnitude characteristics of all the five types of 1D FIR filter functions in dBs, for M = 10 and K = 3, are presented in Fig. 10(a)–Fig. 10(e).

Values of cut-off frequency of the pass-band of all the five types of 1D FIR filters for which the attenuation is $a\left(\mu_{cp1}\right)=0.28dB$ and $a\left(\mu_{cp2}\right)=3dB$, are shown in Table I and Table II, respectively.



a)



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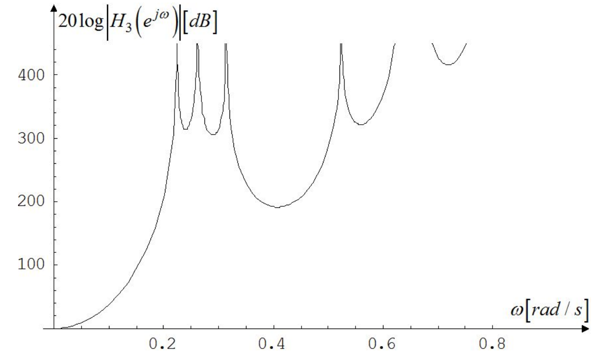

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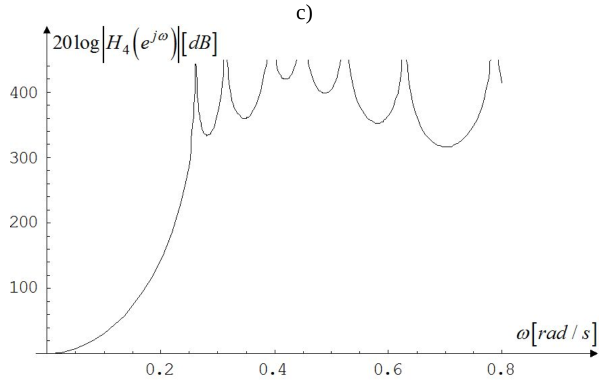

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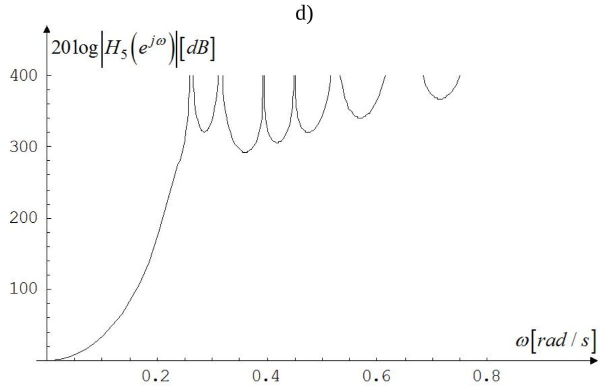

e)

Fig. 10. The normalized zoomed magnitude characteristics of linear phase 1D FIR filter in dBs, for M = 10 and K = 3: a) FIR1, b) FIR2, c) FIR3, d) FIR4, e) FIR5.

TABLE I. VALUES OF CUT-OFF FREQUENCY OF THE PASS-BAND OF THE FILTERS FOR $a\left(\mu_{cp1}\right)=0.28\ dB$.

Filter type	$\mu_{cp1}\left[rad/s\right]$
FIR1	0.0068015349644563500289186971
FIR2	0.0071318247691272073916766534
FIR3	0.0089352748930827449886303128
FIR4	0.0097692348175474606930587763
FIR5	0.0093243659171983084796361394

It can be seen from these tables that proposed solution has the smallest value of cut-off frequencies of the pass-band of the filter. This can be very useful in many applications.

TABLE II. VALUES OF CUT-OFF FREQUENCY OF THE PASS-BAND OF THE FILTERS FOR $a\left(\mu_{cp2}\right)=3\ dB$.

Filter type	$\mu_{cp2}\left[rad/s\right]$
FIR1	0.0222042918447692877924868862
FIR2	0.0232940817749605300286517873
FIR3	0.0292196490155110760481861192
FIR4	0.0319614015774925142368478912
FIR5	0.0304972670848479768581634966

Values of cut-off frequency of the stop-band of all the five types of 1D FIR filters for which the attenuation is $a(\mu_{cs1}) = 40dB$, $a(\mu_{cs2}) = 80dB$ and $a(\mu_{cs3}) = 120dB$, are shown in Table III–Table V, respectively.

TABLE III. VALUES OF CUT-OFF FREQUENCY OF THE STOP-BAND OF THE FILTERS FOR $a(\mu_{cs1}) = 40\ dB$.

Filter type	$\mu_{cs1}[rad/s]$
FIR1	0.0766348786434958382825109686
FIR2	0.0809648629075731582023975048
FIR3	0.1033587206385411687432862691
FIR4	0.1137789146904111539435026303
FIR5	0.0776284402591858941223681712

TABLE IV. VALUES OF CUT-OFF FREQUENCY OF THE STOP-BAND OF THE FILTERS FOR $a(\mu_{cs2}) = 80\ dB$.

Filter type	$\mu_{cs2}[rad/s]$
FIR1	0.1016362461854144644398307805
FIR2	0.1082452028422997216604331001
FIR3	0.1412333642271181910367913223
FIR4	0.1566038055630499720916917349
FIR5	0.1480659306943155102883467931

TABLE V. VALUES OF CUT-OFF FREQUENCY OF THE STOP-BAND OF THE FILTERS FOR $a(\mu_{cs3}) = 120\ dB$.

Filter type	$\mu_{cs3}[rad/s]$
FIR1	0.1161386654979718017306191764
FIR2	0.1246638178896196820422018223
FIR3	0.1667856406143178393970837682
FIR4	0.1934467253150714155601733683
FIR5	0.1751374308684126144430066768

Based on the values of cut-off frequencies given in Table III–Table V it can be concluded that for equal group delay we can have different values of cut-off frequency of the stop-band of the filter. In this way, the specification requirements in the pass-band and stop-band of the filter can be satisfied.

V. CONCLUSIONS

In this paper, the detailed analysis for a new class of transitional 1D FIR filter functions is given. These filters have three free integer parameters for equal constant group delay. The cut-off frequencies of the pass-band and the cut-off frequencies of the stop-band of all the considered filters are also analysed and presented in tables. The examples of the proposed FIR filter function, whose realization does not require the multipliers, are illustrated and discussed. It is shown that the possible solutions of the proposed transitional filters are between two particular solutions. Based on relevant parameters (order of the filter, level of the group delay, attenuation, cut-off frequency of the pass-band and cut-off frequency of the stop-band) and using given tables in this paper, the values of the real integer parameters which satisfy the initial requirements can be found.

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