

Performance Analysis of Optimum Combiner in Power Limited Cognitive Radios with Multiple Primary Interferers

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Abstract—This paper investigates the performance of power limited Cognitive Radio system with optimum combining at the cognitive user receiver under the influence of interference from multiple primary users' transmitters (L_t) in flat Rayleigh fading channels. An approximate analytical result of the probability density function of maximum signal-to-interference ratio at the output of CR-OC receiver is derived. Using this derived PDF, the closed form expressions for the performance metrics viz. Average post processed SIR, Ergodic capacity, Average bit error rate and outage probability of CR-OC system are derived by taking into account peak interference power constraint denoted by 'Q' at PU-Rx. Based on the achieved result, it is concluded that the performance of the proposed system degrades when number of primary interferers exceeds from $L_t = 3$. Analytical results for CR-OC system are validated through Monte Carlo simulations also.

Index Terms—Cognitive Radio; Diversity methods; Radio spectrum management; Spatial diversity.

I. INTRODUCTION

Current spectrum allocation policies have made a larger part of the licensed spectrum to remain idle at a given time and location, leading to inefficient use of overall radio spectrum [1]. Cognitive Radio (CR) is one of the promising technique for future communication to utilize the scarce spectrum resources in a more efficient and flexible way. In a CR network, there are different spectrum access techniques such as underlay, overlay and interweave. Underlay spectrum sharing is the access policy for secondary users' whereas in overlay strategy, the CR-Tx is allowed to share the spectrum of primary user (PU) [2] simultaneously with PU. Nevertheless, such simultaneous transmissions may result in degradation of the performance of PU. Therefore, the interference power received at the PU-Rx due to CR base station transmissions must be managed to be within some predefined margins [2], [3]. In order to satisfy the constraint, the transmitted power of CR-Tx has to be regulated using different transmit power policies either by considering peak interference received power (PIP) or

outage constraint at PU-Rx [4], [5].

Diversity reception with array processing is a powerful technique which suppresses the detrimental effects of interference and fading [6]. Antenna arrays can provide diversity paths to combat multipath fading of the desired signal as well as the interference at the intended receiver. Spatial diversity is an efficient solution, using multiple antennas at one or both sides of the transmission link, to alleviate the effects of multipath fading and enhance system throughput. Hence, to improve system capacity of CR networks different combining techniques such as selection combining (SC), maximal ratio combining (MRC), optimum combining (OC) etc. are studied in the literature. MRC, a combining technique applied in presence of noise and independent fading, is thoroughly studied in MIMO as well in cognitive radios [7]–[15]. The weight vector at each antenna element compensates the effect of phase shift which is proportional to the received signal strength and maximizes the SNR. In [9], the authors analysed MRC in the presence of co-channel interference. The result shows that it maximizes the signal-to-noise-ratio (SNR) at the output and is the most effective choice in noise limited scenario. However, it becomes sub-optimal option in the presence of interference. Whereas in OC [10]–[15], the received signal at different antenna elements are properly weighted and combined to maximize the SINR at the receiver output. In [15], the authors studied and analysed the performance comparison of MRC, EGC and OC in the presence of interference. The study considered MRC with arbitrary number of interferers, whereas in OC the number of interferer sources was larger than the number of antenna elements, such that the array degrees of freedom are not sufficient enough to completely null the interference. However, even a moderate increase in SNR at the Rx output may result in significant improvement in system capacity. MRC is also analysed in the presence of multiple equal power interferers in a Nakagami fading scenario [10]. The authors showed that the MRC is beneficial even in interference limited environment and increasing the order of diversity further improves the system performance. In [16], the authors have examined the analytical performance evaluation of generalized selection combining (GSC) in

interference environment in terms of SINR and SNR. This paper considers the two extreme cases i.e. when number of best branches to be combined is $L_c = 1$ (SC) and $L_c = N_r$ (MRC). They also provide the new outage analysis, which gives insight to the GSC reception in the interference limited environment.

To enhance the performance of Cognitive radio network, various diversity combining techniques have been employed [17]–[23]. In [17], the authors have analysed the ergodic capacity of spectrum sharing system employing MRC at the secondary user receiver (SU-Rx). In [18], the authors have studied the GSC in terms of ergodic capacity in a cognitive radio environment under the imperfect CSI. In [19], the author has analysed the spectrum sharing system with MRC diversity in terms of ergodic capacity and symbol error rate (SER), when the proposed system is constrained of transmit power constraint. The impact of multiple PU trans-receivers on the single relay spectrum sharing system has been analysed in [20]. Also, the outage performance of spectrum sharing CR system by employing MRC at SU under the influence of interference from multiple PU's has been examined in [21]. The OC is also studied with transmit antenna selection (TAS) for aggregate interference from multiple secondary users in underlay CR [24]. All this prior work on diversity combining improved our insight into the usefulness of diversity combining schemes in cognitive radios. Motivated by these observations, we observed that MRC mitigates the effects of fading, however it fails to combat interference. OC addresses both the problems of multipath fading and the effect of interference. Thus by considering the advantages of OC over MRC, we have studied and analysed the underlay spectrum sharing CR system by employing Optimum Combining at CR Rx under the impact of multiple PU interferers. Our main contributions in this paper are summarized as following: (i) An approximate analytical expression for the probability density function (PDF) of signal to interference ratio (SIR) at the CR-Rx output is derived, considering the effect from L_t equal power PU interferers. (ii) Using the derived PDF, a closed form expression Average post processed SIR, Ergodic capacity, Average bit error rate (ABER) and Outage probability are obtained.

II. SYSTEM OVERVIEW

Consider an underlay CR interference limited scenario which consists of CR base station, a CR Rx, L_t equal power PU interferers and a PU Rx. CR base station and CR Rx are equipped with N_t ($i = 1 \dots N_t$) and N_r ($j = 1 \dots N_r$) antennas, respectively as shown in Fig. 1.

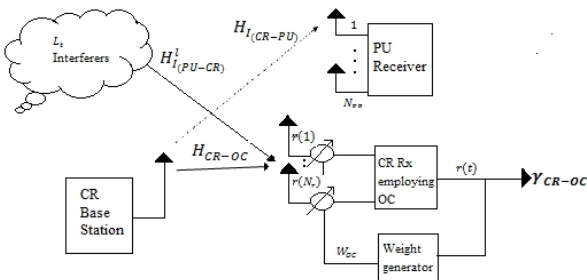


Fig. 1. System model.

The L_t Interferers are equipped with single transmit antenna and PU Receiver has N_{pr} ($k = 1 \dots N_{pr}$) receive antennas. The interference temperature limit is denoted by 'Q', which is the maximum allowable interference received power at the PU-Rx. We also assume that the number of interferers at the CR-Rx are larger than the size of the receive antenna array i.e. ($L_t \geq N_r$). This paper assumes slowly varying Rayleigh flat fading channel. We further assume that the level of interference is sufficiently high for the effect of thermal noise on the system performance is negligible. Let \mathbf{H}_{CR-OC} be an ($N_t \times N_{r_r}$) dimensional channel matrix between CR-Tx and the CR-Rx. Let $\mathbf{H}_{I(PU-CR)}^1$ denote the ($1 \times N_r$) dimensional channel matrix between the i^{th} PU interferer and the CR-Rx. In addition, let $\mathbf{H}_{I(CR-PU)}$ denote ($N_t \times N_{pr_r}$) dimensional channel matrix between the CR-Tx and the PU-Rx. The entries $g_{N_t-N_r}$, \mathbf{h}_{PU-CR}^1 and \mathbf{h}_{CR-PU} of \mathbf{H}_{CR-OC} , $\mathbf{H}_{I(PU-CR)}^1$ and $\mathbf{H}_{I(CR-PU)}$ are independent and identically distributed (i.i.d) exponential random variables. We denote signal bandwidth as 'B' and the variance of additive white Gaussian noise as 'N₀'. The system employs Binary Phase Shift Keying (BPSK) and all channels path gains are assumed to be an i.i.d random variables. The transmit power of all L_t interferers is presumed to be equal and is denoted by P. Considering the interference from all the interferers, the combined received signal $r(t)$ [15] at the output of receive antenna array is given as

$$r(t) = \sqrt{P_t} \mathbf{H}_{CR-OC} x_{CR-OC} + \sum_{l=1}^{L_t} \sqrt{P} \mathbf{h}_{PU-CR}^l x_1, \quad (1)$$

where x_{CR-OC} and x_1 are the desired and the 1th interfering signal. Let P_t and P denotes the transmit power of CR-Tx and 1th PU interferer, respectively. The vectors \mathbf{h}_{PU-CR}^l ($l = 1 \dots L_t$) are i.i.d with $E[\mathbf{h}_{PU-CR}^l] = 0$ and covariance matrix $\varepsilon = E[\mathbf{h}_{PU-CR}^l (\mathbf{h}_{PU-CR}^l)^H]$. All the channel coefficients have zero mean and σ^2 variance. Assuming that the CR-Tx has perfect channel state information (CSI) of interference link i.e. from CR-Tx to PU-Rx, the maximum permissible transmit power of CR-Tx (P_t) at each instant is given by [22]

$$P_t = \frac{Q}{\mathbf{h}_{CR-PU}}, \quad (2)$$

where $\mathbf{h}_{CR-PU} = \sum_{k=1}^{N_{pr}} \mathbf{h}_{CR-PU(k)}$, is a Chi-Square distributed random variable with $2N_{pr}$ degrees of freedom and the

probability density function (PDF) of $\mathbf{h}_{\text{CR-PU}}$ is given by

$$\begin{aligned} f_{\mathbf{h}_{\text{CR-PU}}}(\mathbf{h}_{\text{CR-PU}}) &= \\ &= \frac{1}{r(N_{\text{pr}})} \mathbf{h}_{\text{CR-PU}}^{N_{\text{pr}}-1} e^{-\mathbf{h}_{\text{CR-PU}}}, \end{aligned} \quad (3)$$

where Γ is the standard Gamma function and is given as $\Gamma(N_{\text{pr}}) = (N_{\text{pr}} - 1)!$.

III. PERFORMANCE ANALYSIS

The OC weight vector that maximizes the SIR at the output of CR-Tx is written as

$$\mathbf{W}_{\text{OC}} = \mathbf{R}^{-1} \mathbf{H}_{\text{CR-OC}}, \quad (4)$$

where \mathbf{R} denotes the interference covariance matrix [15] conditioned on channel vector of L_t interferers and given by

$$\mathbf{R} = \sum_{l=1}^{L_t} \mathbf{h}_{\text{PU-CR}}^l (\mathbf{h}_{\text{PU-CR}}^l)^H, \quad (5)$$

where $(\cdot)^H$ represent the complex conjugate transpose. Next, we derive PDF for the SIR of CR-OC system in the presence of L_t equal power interferers.

A. PDF of Maximum SIR at the CR-Rx

From (2) the SIR ($\gamma_{\text{CR-OC}}$) at the output of CR-Rx is given by

$$\begin{aligned} \gamma_{\text{CR-OC}} &= P_t (\mathbf{H}_{\text{CR-OC}})^H \mathbf{R}^{-1} \mathbf{H}_{\text{CR-OC}} = \\ &= \frac{Q}{\mathbf{h}_{\text{CR-PU}}} (\mathbf{H}_{\text{CR-OC}})^H \mathbf{R}^{-1} \mathbf{H}_{\text{CR-OC}}. \end{aligned} \quad (6)$$

Let $\mathbf{R} = \mathbf{P}\mathbf{R}_1$ where $\mathbf{R}_1 = \sum_{l=1}^{L_t} \mathbf{h}_{\text{PU-CR}}^l (\mathbf{h}_{\text{PU-CR}}^l)^H$.

Therefore, $\gamma_{\text{CR-OC}}$ in (6) becomes

$$\begin{aligned} \gamma_{\text{CR-OC}} &= \frac{Q}{\mathbf{h}_{\text{CR-PU}} \mathbf{P}} (\mathbf{H}_{\text{CR-OC}})^H \mathbf{R}^{-1} \mathbf{H}_{\text{CR-OC}} = \\ &= \frac{Q}{\mathbf{P} \mathbf{h}_{\text{CR-PU}}} z, \end{aligned} \quad (7)$$

where $z = (\mathbf{H}_{\text{CR-OC}})^H \mathbf{R}^{-1} \mathbf{H}_{\text{CR-OC}}$.

The PDF of random variable z is given by [15]

$$f_z(z) = \frac{\Gamma(L_t + 1) z^{N_r - 1}}{\Gamma(N_r) \Gamma(L_t + 1 - N_r) (1 + z)^{L_t + 1}}, \quad (8)$$

where $z \geq 0, 1 \leq N_r \leq L_t$.

The PDF in (10) is a modified form of central F -Distribution [15]. The density of the 'z' does not depend

upon the form of the covariance matrix \mathbf{R} . Thus the performance of the OC is the same regardless whether the fading at each receive antenna is independent or not. However, this is true only for the case $L_t \geq N_r$. Since F Distribution can be converted into Chi-Square distribution [22], therefore (8) can be rewritten as

$$f_z(z) = \frac{(L_t + 1 - N_r)^{K_r}}{\Gamma(N_r)} z^{N_r - 1} e^{-(L_t + 1 - N_r)z}. \quad (9)$$

From (6), the marginal PDF for the ratio of two random variables z and $\mathbf{h}_{\text{CR-PU}}$ is obtained by substituting:

$$\mu = \frac{z}{\mathbf{h}_{\text{CR-PU}}}, \quad (10)$$

$$\beta = \mathbf{h}_{\text{CR-PU}}. \quad (11)$$

By applying division of two random variables, the approximate PDF $f_\mu(\mu)$ of random variable μ can be written as

$$\begin{aligned} f_\mu(\mu) &= \frac{(L_t + 1 - N_r)^{N_r} \Gamma(N_r + N_{\text{pr}})}{\Gamma(N_{\text{pr}}) \Gamma(N_r)} \times \\ &\times \frac{\mu^{N_r - 1}}{[(L_t + 1 - N_r)\mu - 1]^{N_r + N_{\text{pr}}}}. \end{aligned} \quad (12)$$

The complete solution of above equation is solved in Appendix-A. Now we will derive the PDF of maximum SIR at the output of CR-Rx

$$\gamma_{\text{CR-OC}} = \frac{Q}{\mathbf{P} \mathbf{h}_{\text{CR-PU}}} z = \frac{Q}{\mathbf{P}} \mu. \quad (13)$$

By using the transformation as in (13), the density of the maximum SIR $\gamma_{\text{CR-OC}}(\gamma)$ is found as

$$\begin{aligned} f_{\gamma_{\text{CR-OC}}}(\gamma) &= \frac{\Gamma(N_r + N_{\text{pr}} - 1)}{\Gamma(N_r - 1) (N_{\text{pr}} - 1)} \left(\frac{Q}{\mathbf{P}(L_t + 1 - N_r)} \right)^{N_{\text{pr}}} \times \\ &\times \frac{\gamma^{N_r - 1}}{\left(\gamma + \frac{Q}{\mathbf{P}(L_t + 1 - N_r)} \right)^{N_r + N_{\text{pr}}}}. \end{aligned} \quad (14)$$

The PDF in (14) represents the final density function of SIR at the output of CR-Rx of CR-OC system.

B. Average Post Processed SIR

The average post processed SIR or the first moment of $\gamma_{\text{CR-OC}}$ at the output of CR-Rx under the influence of multiple PU interferers is given as

$$\begin{aligned} E[\gamma_{\text{CR-OC}}] &= \\ &= \frac{\Gamma(N_r + N_{\text{pr}} - 1)}{\Gamma(N_r - 1) (N_{\text{pr}} - 1)} \left(\frac{Q}{\mathbf{P}(L_t + 1 - N_r)} \right)^{N_{\text{pr}}} \times \end{aligned}$$

$$\times \int_0^{\infty} \frac{\gamma^{N_r}}{\left(\gamma + \frac{Q}{P(L_t + 1 - N_r)} \right)^{N_{pr} + N_r}} d\gamma. \quad (15)$$

Solving (15) the average post processed SIR of the CR-OC system is expressed as

$$E[\gamma_{CR-OC}] = \frac{\Gamma(N_r + N_{pr} - 1)}{\Gamma(N_r - 1)(N_{pr} - 1)} \times \frac{Q}{P(L_t + 1 - N_r)}. \quad (16)$$

C. Ergodic Capacity

The *Ergodic Capacity* (C_{CR-OC}) of CR-OC network is defined as the maximum long term achievable rate and determined by averaging over all the channel states of a fading channel. It is approximated using Taylor's series expansion of logarithm function [25] and is given by

$$C_{CR-OC} = \int_0^{\infty} \log_2 \left(1 + \mu \frac{Q}{P} \right) f_{\mu}(\mu) d\mu = \frac{(L_t + 1 - N_r)^{N_r}}{\Gamma(N_r) \Gamma(N_{pr})} (N_r + N_{pr} - 1)! \times \int_0^{\infty} \frac{\log_2 \left(1 + \mu \frac{Q}{P} \right) \mu^{N_r - 1}}{[(L_t + 1 - N_r)\mu + 1]^{N_r + N_{pr}}} d\mu. \quad (17)$$

By further solving (17), we get

$$C_{CR-OC} = \frac{\Gamma(N_r + N_{pr} - 1)}{\Gamma(N_r - 1)(N_{pr} - 1)} \sum_{p=0}^{N_r - 1} \binom{N_r - 1}{p} (-1)^{N_r - 1 - p} \times \frac{1}{\log(2)} \times \frac{1}{(N_r + N_{pr} - p - 1)^2} \times {}_2F_1 \left(1, N_r + N_{pr} - p - 1; N_r + N_{pr} - p; \frac{N_r - L_t + \frac{Q}{P} - 1}{\frac{Q}{P}} \right), \quad (18)$$

where ${}_2F_1$ is the hypergeometric function and it is defined as in [25]. Equation (19) represents the final expression for the ergodic capacity of the CR-OC system. The complete solution of (19) is given in Appendix B.

D. Outage Probability

The outage probability is an important statistical measure in the design of spectrum sharing system in fading environment in presence of interference. It is the probability of unsatisfactory reception over the intended coverage area. The outage probability is the probability that the received SIR is below a given threshold required to achieve radio reception in fading environment [14]. It is expressed as

$$P_{\text{Outage}}^{\text{CR-OC}} = \text{Probability}(\gamma_{CR-OC} \leq \gamma_t) = \int_0^{\gamma_t} f_{\gamma_{CR-OC}}(\gamma) d\gamma, \quad (19)$$

where γ_t is the SIR threshold. Its value depends on the modulation technique used and also on the desired performance criterion [12]. It is also known as cumulative distribution function. Solving (20) the outage probability of CR-OC system is found as

$$P_{\text{Outage}}^{\text{CR-OC}} = \frac{\Gamma(N_r + N_{pr} - 1)}{\Gamma(N_r - 1)(N_{pr} - 1)} \left(\frac{Q}{P(L_t + 1 - N_r)} \right)^{-N_r} \gamma_t^{N_r} \times {}_2F_1 \left(N_r, N_r + N_{pr}, N_r + 1, \frac{(N_r - L_t - 1)P\gamma_t}{Q} \right). \quad (20)$$

E. Average Bit Error Rate

An average Bit Error Rate is an important parameter for the analysis of performance of CR-OC system. In this section, the ABER of CR-OC system is derived under peak interference power constraint Q at PU-Rx. In case of BPSK modulation, the probability of error computed at a given value of $\gamma_{CR-OC}(\gamma)$ in terms of Gaussian - Q function is given by [27]

$$P_{e_{CR-OC}} = Q \left(\sqrt{2\gamma_{CR-OC}(\gamma)} \right). \quad (21)$$

Therefore, the ABER of CR-OC system is obtained by integrating (11) over $f_{\mu}(\mu)$ and is given by

$$P_{e_{CR-OC}} = \int_0^{\infty} Q \left(\sqrt{2\mu \frac{Q}{P}} \right) f_{\gamma_{CR-OC}}(\gamma) d\gamma. \quad (22)$$

The Q - function and complementary error function are related as

$$Q(\sqrt{2\gamma}) = \frac{1}{2} \text{erfc}(\sqrt{\gamma}). \quad (23)$$

The above (23) can be evaluated and the ABER of CR-OC system is obtained as

$$P_{e_{CR-OC}} = \frac{1}{2} \frac{\Gamma(N_r + N_{pr} - 1)}{\Gamma(N_r - 1)\Gamma(N_{pr} - 1)} \left(\frac{Q}{P(L_t + 1 - N_r)} \right)^{N_{pr}} \times \int_0^{\infty} \text{erfc} \left(\sqrt{\mu \frac{Q}{P}} \right) \times \frac{\gamma^{N_r - 1}}{\left(\gamma + \frac{Q}{P(L_t + 1 - N_r)} \right)^{N_r + N_{pr}}} d\gamma, \quad (24)$$

$$P_{e_{CR-OC}} = \frac{1}{2} \frac{\Gamma(N_r + N_{pr} - 1)}{\Gamma(N_r - 1)\Gamma(N_{pr} - 1)} \times \left(\frac{Q}{P(L_t + 1 - N_r)} \right)^{N_{pr}} \times \frac{1}{N_{pr} \sqrt{\pi} (N_{pr} + N_r)}$$

$$\begin{aligned}
 & \times \left(\frac{P(L_t+1-N_r)}{Q} \right)^{0.5-N_r-N_{pr}} \times \\
 & \times \left(\frac{Q}{P(L_t+1-N_r)} \right)^{1-N_r-N_{pr}} \times \\
 & \times \left(-2N_{pr} \left(\frac{P(L_t+1-N_r)}{Q} \right) \right)^{N_{pr}} \times \\
 & \times \sqrt{\frac{Q}{P}} \Gamma(-0.5+N_{pr}) \Gamma(-0.5+N_r) \times \\
 & \times F_q \left[\frac{1}{2}, \frac{1}{2}+N_r; \frac{3}{2}, \frac{3}{2}-N_{pr}; \frac{Q^2}{P^2(L_t+1-N_r)} \right] + \\
 & + \sqrt{\frac{P(L_t+1-N_r)}{Q}} \left(N_{pr} \sqrt{\pi} \left(\frac{P(L_t+1-N_r)}{Q} \right) \right)^{N_{pr}} - \\
 & - \Gamma(N_r) \Gamma(N_{pr}) - \\
 & - \left(\frac{Q}{P} \right)^{N_{pr}} \times \Gamma(0.5-N_{pr}) \Gamma(N_r+N_{pr}). \quad (25)
 \end{aligned}$$

IV. NUMERICAL RESULTS

In this section, we present numerical results to verify simulation counterpart in terms of Average post processed SIR, Ergodic capacity, ABER and Probability of Outage for CR-OC system in flat Rayleigh faded environment. We assume that the number of PU interferers affecting CR network are i.e. $L_t = 3, 4, 5, 6$ and PU-Rx is equipped with $N_{pr} = 2$ receive antennas. The CR base station and CR-Rx is equipped with $N_t = 1$ and $N_r = 3$ antennas, respectively.

A. Performance Analysis with Varying Number of PU Interferers (L_t)

The Fig. 2 and Fig. 3 give the average post processed SIR and Ergodic Capacity of CR-OC system. It can be observed from Fig. 2 and Fig. 3 that at $Q = 5$ db when number of interferers are increased from $L_t = 3$ to $L_t = 6$, the average post processed SIR and Ergodic Capacity falls from 0.94 dB to 0.23 dB and 1.03 bits/sec/Hz to 0.38 bits/sec/Hz.

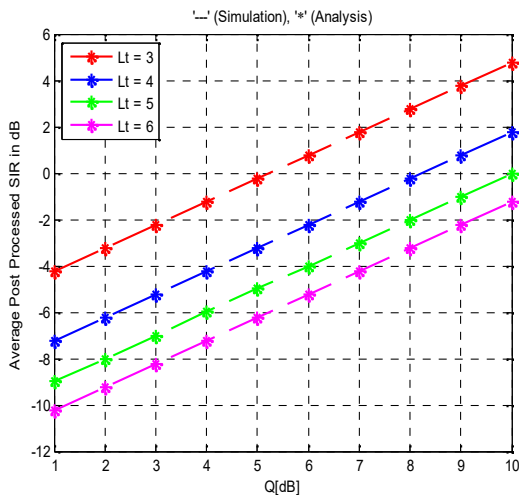


Fig. 2. Average post processed SIR of CR-OC system.

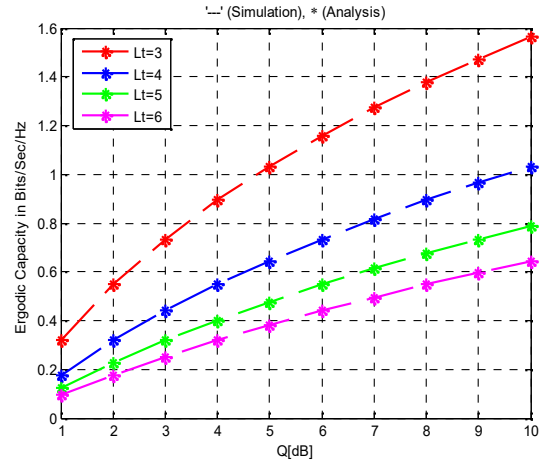


Fig. 3. Ergodic capacity of CR-OC system.

Furthermore, the average post processed SIR and Ergodic capacity of the CR-OC system improve when 'Q' is increased i.e. the received interference power constraint at PU-Rx is increased which further allows CR-Tx to transmit with increased power. The Fig. 4 and Fig. 5 show the ABER and outage probability of CR-OC system.

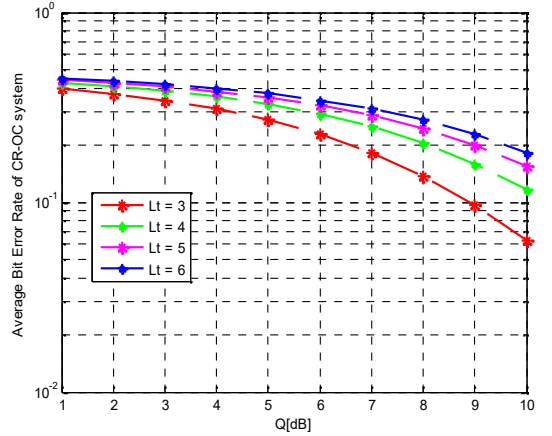


Fig. 4. Average bit error rate of CR-OC system.

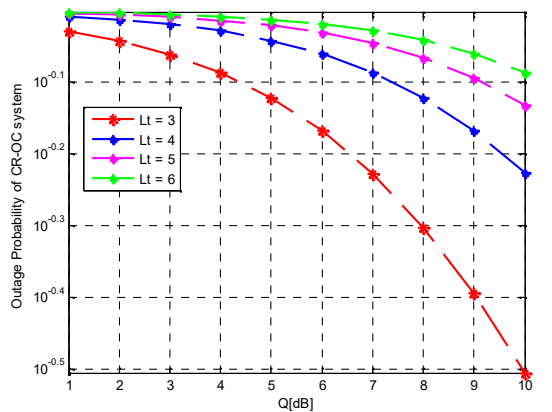


Fig. 5. Probability of outage for CR-OC system.

We can observe from Fig. 4 and Fig. 5 that the average bit error rate degrades from 0.270 to 0.374 and outage probability increases from 0.75 to 0.97 when the number of PU interferers are increased from $L_t = 3$ to $L_t = 6$ at $Q = 5$ dB. Thus, the average bit error rate and outage probability of the CR-OC system becomes better when 'Q' is increased and the performance of the proposed system degrades as number of interfering sources are increased.

B. Performance Analysis in terms of Diversity Gain with Varying Number of Receiver Antennas (N_r)

In this section, we demonstrate the diversity gain in terms of average post processed SIR at CR-OC output and probability of outage of the proposed system. In Fig. 6, the performance analysis of Average post processed SIR at the Optimum combiner output is examined with varying number of N_r receive antennas for the proposed system. It can be seen from the figure that the diversity gain is substantially increased, when number of CR-Rx receiver antennas i.e. N_r increases from 3 to 6 as effect of channel fading weakens when number of receive antennas increases. The average post processed SIR of CR-OC system rise from 0.237 dB to 1.897 dB.

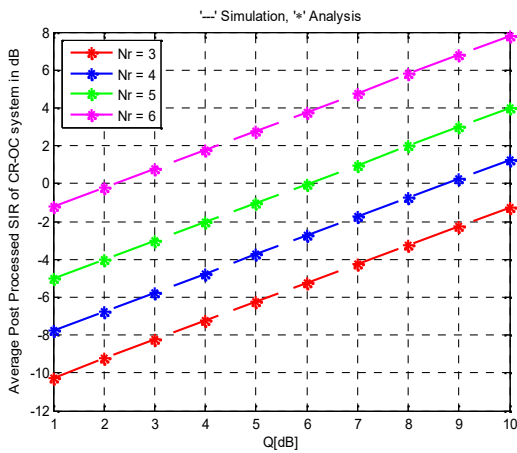


Fig. 6. Average Post processed SIR with varying number of N_r antennas for CR-OC system.

In Fig. 7, the diversity gain of the proposed system is shown in terms of probability of outage for the proposed system with varying number of N_r receiver antennas. It can be seen that from the achieved result that outage probability of the CR-OC system drops from 0.970 to 0.469 when the number of N_r antennas increase from 3 to 6, respectively.

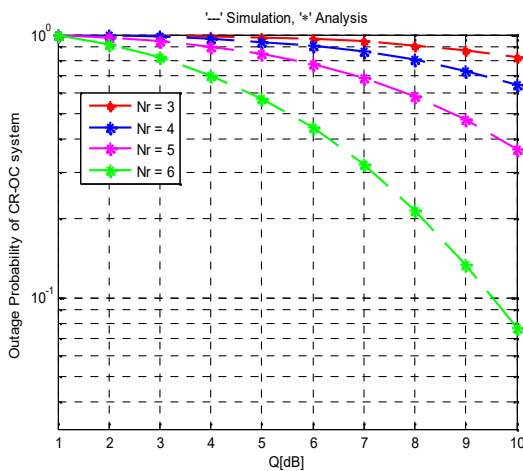


Fig. 7. Probability of outage with varying number of N_r antennas for CR-OC system.

V. DISCUSSION

In this section, the performance of CR-OC system is

demonstrated when number of PU interferers and number of CR-Rx receive antennas varying from $L_t = 3, 4, 5, 6$ and $N_r = 3, 4, 5, 6$, respectively.

TABLE I. PERFORMANCE EVALUATION OF THE CR-OC SYSTEM WITH VARYING NUMBER OF PU INTERFERERS.

Simulation Details				
$N_t = 1, N_{pr} = 2, L_t = 3, 4, 5, 6, N_r = 3, Q = 5$ dB				
P = 10dB				
Metrics	Average Post Processed SIR [dB]	Ergodic Capacity [Bits/Sec/z]	Average Bit Error Rate $P_{e_{CR-OC}}$	Outage Probability P_{Outage}^{CR-OC}
L_t				
3	0.948	1.033	0.270	0.754
4	0.474	0.644	0.328	0.907
5	0.316	0.477	0.356	0.952
6	0.237	0.381	0.374	0.970

Table I shows the performance of CR-OC system in terms of Average post processed SIR, Ergodic capacity, ABER and probability of outage. It is shown in the Table I that Average post processed SIR falls when number of PU interferer increases from $L_t = 3$ to 6. Thus it results in the SNR loss of 0.474 dB when L_t goes from 3 to 4, 0.158 dB when L_t goes from 4 to 5 and 0.079 dB when L_t goes from 5 to 6. It is also shown that ergodic capacity gains of the proposed system is achieved as 60 % when L_t goes from 3 to 4, 35 % when L_t increases from 4 to 5 and ~25 % as L_t increases from 5 to 6, respectively.

The ABER of OC-CR system increases from 0.270 to 0.374 as number of PU interferers increases from 3 to 6. As seen from the Fig. 4, we can observe that when $L_t = 3$ at $Q = 5$ dB the ABER is 0.270, the same error rate is achieved at $Q = 6, 6.5$ & 7 dB at $L_t = 4, 5$ & 6 , respectively. We can conclude that there is a power loss of 1 dB, 1.5 dB and 2 dB when number of PU interferer increases from 3 to 6.

TABLE II. PERFORMANCE EVALUATION OF DIVERSITY GAIN OF CR-OC SYSTEM WITH FIXED NUMBER OF PU INTERFERER I.E. L_t AND WITH THE VARYING NUMBER OF CR RECEIVER ANTENNAS I.E. N_r .

Assumptions				
$N_t = 1, N_{pr} = 2, L_t = 6, N_r = 3, 4, 5, 6, Q = 5$ dB				
P = 10dB				
Performance Metrics	(L_t, N_r) (3, 3)	(L_t, N_r) (3, 4)	(L_t, N_r) (3, 5)	(L_t, N_r) (3, 6)
Average Post Processed SIR of CR-OC system	0.237	0.421	0.790	1.897
Probability of Outage of CR-OC system	0.970	0.925	0.807	0.469

In Table II, the performance evaluation of CR-OC system

is analysed with fixed number of PU interferer i.e. L_t and with varying number of CR-Rx antennas i.e. N_r . We can see from the table that as the number of N_r increase from 3 to 4, 4 to 5 and 5 to 6, the average post processed SIR heightens from 0.237 dB, 0.421 dB, 0.790 dB and 1.897 dB. Thus the SIR gain of the proposed system is achieved at N_r goes from 3 to 4, 4 to 5 and 5 to 6 is 0.184 dB, 0.369 dB and 1.107 dB respectively. Also, the diversity of the proposed system is studied in terms of outage probability. It is shown that when number of N_r increases from 3 to 4, 4 to 5 and 5 to 6, the probability of outage for the CR-OC system falls from 0.970, 0.925, 0.807 and 0.469 at $Q = 5$ dB, respectively. Hence, it is thus concluded that the diversity gain of the system improves considerably when number of PU interferer becomes greater or equal to the number of CR receive antennas i.e. $L_t \geq N_r$.

VI. CONCLUSIONS

In this paper, we have examined the performance of OC-CR system under the impact of PU interferers when PU and CR user transmit concurrently on the same spectrum. The transmit power strategy for CR-Tx is taken into consideration to avoid harmful interference to PU-Rx. We have derived analytical expressions for Average post processed SIR, ergodic capacity, ABER and outage probability for the proposed system. Here, we demonstrate the diversity gain of the OC-CR system in terms of number of PU interferers. We observe that when the number of PU interferers (L_t) are close to the receive antenna diversity (N_r), the OC-CR system performs considerably better. However, when the number of PU interferers becomes very large i.e. $L_t \gg N_r$, the diversity gain begins to vanish.

APPENDIX A. DERIVATION OF PDF OF MAXIMUM SIR

Let us define N_r i.e. the number of receive antennas at the CR-Rx and N_{pr} as the number of receive antennas at the PU-Rx are integers valued.

The joint PDF [26] of random variables μ and β is given as

$$f_{\mu,\beta}(\mu,\beta) = |\beta| \times \int_0^\infty \beta(z) f_{h_{CR-PU}}(h_{CR-PU}) \Big|_{z=\mu\beta, h_{CR-PU}=\beta} \cdot \quad (A1.1)$$

Using (3) and (10), the marginal PDF for the ratio of two random variables z and h_{CR-PU} is obtained as

$$f_\mu(\mu) = \frac{(L_t+1-N_r)^{N_r}}{\Gamma(N_{pr})\Gamma(N_r)} \times \int_0^\infty \beta(\mu\beta)^{N_r-1} e^{-(L_t+1-N_r)\mu\beta} \beta^{N_r-1} e^{-\beta} d\beta. \quad (A1.2)$$

By re-arranging (A1.2), we obtain

$$f_\mu(\mu) = \frac{(L_t+1-N_r)^{K_r}}{\Gamma(N_{pr})\Gamma(N_r)} \mu^{N_{pr}-1} \times \int_0^\infty \beta^{N_r+N_{pr}-1} e^{-\beta[(L_t+1-N_r)\mu+1]} d\beta. \quad (A1.3)$$

Using identity [28], we get

$$I_1 = \int P_n(v) e^{bv} dv = \frac{e^{bv}}{b} \sum_{j=0}^n \frac{(-1)^j}{b^j} P_n^{(j)}(v), \quad (A1.4)$$

where $P_n^{(j)}(v)$ denotes the j^{th} derivative of $P_n(v)$.

By comparing the identity defined in (A1.4) with (A1.3), we get $P_n(\beta) = \beta^{N_r+N_{pr}-1}$, $n = N_r + N_{pr} - 1$ and $b = -[(L_t+1-N_r)\mu+1]$, j^{th} derivative of $P_n(\beta)$ is obtained as

$$P_n^{(j)}(\beta) = \frac{(N_r + N_{pr} - 1)!}{(N_r + N_{pr} - 1 - j)!} \beta^{(N_r+N_{pr}-1-j)}. \quad (A1.5)$$

From (A1.3), the integration involved can be written in the form of (A1.4) as

$$\int_0^\infty \beta^{N_r+N_{pr}-1} e^{-[(L_t+1-N_r)\mu+1]\beta} d\beta. \quad (A1.6)$$

Also by putting above values in (A1.6), the above equation can be modified as

$$\int_0^\infty \frac{e^{-[(L_t+1-N_r)\mu+1]\beta}}{-(L_t+1-N_r)\mu+1} \times \sum_{j=0}^{N_r+N_{pr}-1} \frac{1}{[(L_t+1-N_r)\mu+1]^j} \times \frac{(N_r + N_{pr} - 1)!}{(N_r + N_{pr} - 1 - j)!} \beta^{N_r+N_{pr}-1-j}. \quad (A1.7)$$

By solving (A1.7), we get:

$$\frac{(L_t+1-N_r)^{K_r}}{\Gamma(N_{pr})\Gamma(N_r)} \mu^{N_r-1} \times \frac{e^{-[(L_t+1-N_r)\mu+1]\beta}}{-(L_t+1-N_r)\mu+1} \times \sum_{j=0}^{N_r+N_{pr}-1} \frac{1}{[(L_t+1-N_r)\mu+1]^j} \times \frac{(N_r + N_{pr} - 1)!}{(N_r + N_{pr} - 1 - j)!} \beta^{N_r+N_{pr}-1-j} \Bigg|_{\beta=0}^{+\infty}, \quad (A1.8)$$

$$f_{\mu}(\mu) = \frac{(\mathbf{L}_t+1-\mathbf{N}_r)^{K_r}}{\Gamma(\mathbf{N}_{pr})\Gamma(\mathbf{N}_r)} \mu^{\mathbf{N}_r-1} \times \frac{(\mathbf{N}_r+\mathbf{N}_{pr}-1)!}{\left[(\mathbf{L}_t+1-\mathbf{N}_r)\mu+1\right]^{\mathbf{N}_r+\mathbf{N}_{pr}}}. \quad (\text{A1.9})$$

Hence, the PDF of maximum SIR for the CR-OC system is given in (12).

APPENDIX B. DERIVATION OF ERGODIC CAPACITY

By substituting $y = (\mathbf{L}_t+1-\mathbf{N}_r)\mu+1$ in (18) and applying binomial expansion, we get

$$C_{\text{CR-OC}} = \frac{(\mathbf{N}_r+\mathbf{N}_{pr}-1)! \mathbf{N}_r^{-1}}{\Gamma(\mathbf{N}_r)\Gamma(\mathbf{N}_{pr})} \sum_{p=0}^{\mathbf{N}_r-1} \binom{\mathbf{N}_r-1}{p} (-1)^{\mathbf{N}_r-1-p} \times \int_1^{\infty} \log_2 \left(\frac{1 + \frac{yQ}{P(\mathbf{L}_t+1-\mathbf{N}_r)}}{-\frac{Q}{P(\mathbf{L}_t+1-\mathbf{N}_r)}} \right) y^{p-\mathbf{N}_r-\mathbf{N}_{pr}} dy. \quad (\text{A2.1})$$

The integral in above equation, denoted as I_2 can be solved by partial integration method and is given by

$$I_2 = \frac{Q}{\log_2(2)P(\mathbf{N}_r+\mathbf{N}_{pr}-p-1)} \times \int_1^{\infty} \frac{1}{y^{\mathbf{N}_r+\mathbf{N}_{pr}-p-1}} \frac{1}{(\mathbf{L}_t+1-\mathbf{N}_r)+y\frac{Q}{P}-\frac{Q}{P}} dy. \quad (\text{A2.2})$$

By solving (A2.2), the above equation is denoted by I_3 and is written as

$$I_3 = \frac{1}{\frac{Q}{P}(\mathbf{N}_r+\mathbf{N}_{pr}-p-1)} {}_2F_1 \times \left(1, \mathbf{N}_r+\mathbf{N}_{pr}-p-1; \mathbf{N}_r+\mathbf{N}_{pr}-p; \frac{(\mathbf{N}_r-\mathbf{L}_t+\frac{Q}{P}-1)}{\frac{Q}{P}} \right). \quad (\text{A2.3})$$

By putting (A2.3) in (A2.1), we obtain the final expression for the ergodic capacity of CR-OC system, which is given in (18).

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