ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2012. No. 6(122)
ELEKTRONIKA IR ELEKTROTECHNIKA

T 120
SYSTEM ENGINEERING, COMPUTER TECHNOLOGY
SISTEMŲ INŽINERIJA, KOMPIUTERINĖS TECHNOLOGIJOS

Optimization of Weighted Aggregated Sum Product Assessment

E. K. Zavadskas, Z. Turskis, J. Antucheviciene
Department of Construction Technology and Management of the Faculty of Civil Engineering, Vilnius Gediminas Technical University,
Sauletekio al. 11, LT-10223 Vilnius, Lithuania, e-mails: edmundas.zavadskas@vgtu.lt, zenonas.turskis@vgtu.lt, jurgita.antucheviciene@vgtu.lt
A. Zakarevicius
Department of Geodesy and Cadaster of the Faculty of Environmental Engineering, Vilnius Gediminas Technical University,
Sauletekio al. 11, LT-10223 Vilnius, Lithuania, e-mail: algimantas.zakarevicius@vgtu.lt

crossref http://dx.doi.org/10.5755/j01.eee.122.6.1810

Introduction

To design a high quality processes and to achieve effective decisions various computer-aided systems can be used. For complex decisions when several alternatives are taken into consideration and a lot of criteria are involved multi criteria decision support systems can be successfully applied.

Theoretical model of a real life situation, i.e. a comprehensive conceptual and quantitative description of alternative variants by a set of criteria constitutes a part of decision support system [4]. The next very important step is to select the most suitable MCDM (Multiple Criteria Decision Making) method to determine the optimal alternative. Selection of MCDM methods based on various parameters was analyzed in a number of papers [1, 5–7, 10, 11]. The further the more combination of several methods and development of decision making software are discussed [2, 4, 8, 15]. Robustness of methods or their combination is analyzed [3, 13].

The authors of the current research propose to select an appropriate multiple criteria method based on its accuracy of estimation. Moreover, combination of two methods is proposed to increase the ranking accuracy.

Accuracy of two rather well known methods, i.e. WSM (Weighted Sum Model) and WPS (Weighted Product Model) as well as accuracy of aggregated both methods is analyzed. It is proved that accuracy of aggregated methods is larger comparing to accuracy of single ones. Accordingly, optimization of aggregation is held and Weighted Aggregated Sum Product Assessment (WASPAS) method for ranking of alternatives is proposed. The method could be successfully applied in computer-aided systems to support multiple criteria decisions.

Weighted aggregated sum product assessment

The Weighted Sum Model (WSM) is one of the best known and often applied multiple criteria decision making method for evaluating a number of alternatives in terms of a number of decision criteria. In general, suppose that a given MCDM problem is defined on m alternatives and n decision criteria. Next suppose that w_j denotes the relative significance of the criterion and x_{ij} is the performance value of alternative i when it is evaluated in terms of criterion j. Then, the total relative importance of alternative i, denoted as $Q_i^{(1)}$, is defined as follows [12]

$$Q_i^{(1)} = \sum_{j=1}^{n} \bar{x}_{ij}\, w_j, \tag{1}$$

where linear normalization of initial criteria values is applied, i.e.

$$\bar{x}_{ij} = \frac{x_{ij}}{\max_i x_{ij}}, \tag{2}$$

if $\max_i x_{ij}$ value is preferable or

$$\bar{x}_{ij} = \frac{\min_i x_{ij}}{x_{ij}}, \tag{3}$$

if $\min_i x_{ij}$ value is preferable.

According to the Weighted Product Model (WPM) the total relative importance of alternative i, denoted as $Q_i^{(2)}$, is defined as follows [12]

$$Q_i^{(2)} = \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j}. \tag{4}$$

There was an attempt to apply a joint criterion for determining a total importance of alternative, giving equal contribution of WSM and WPM for a total evaluation [9]

$$Q_i = 0.5Q_i^{(1)} + 0.5Q_i^{(2)}. \quad (5)$$

Based on previous research [9] and supposing the increase of ranking accuracy and, respectively, the effectiveness of decision making, the Weighted Aggregated Sum Product Assessment (WASPAS) method for ranking of alternatives is proposed in the current research

$$Q_i = \lambda \sum_{j=1}^{n} \bar{x}_{ij} w_j + (1-\lambda) \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j}, \ \lambda = 0, \dots, 1. \quad (6)$$

Accuracy of estimation based on initial criteria values

It is proposed to measure the accuracy of WASPAS based on initial criteria accuracy and when $\lambda = 0, \dots, 1$. When $\lambda = 0$, WASPAS is transformed to WPM; and when $\lambda = 1$, WASPAS is transformed to WSM.

Assuming that errors of determining the initial criteria values are stochastic, the variance σ^2 or standard deviation σ is a measure of dispersion in the distribution [14].

Suppose, there is the function

$$y = \varphi(x_1, x_2, \dots, x_n). \quad (7)$$

The standard deviations of the function's arguments (Eq. 7) are $\sigma(x_1), \sigma(x_2), \dots, \sigma(x_i), \dots, \sigma(x_n)$. The variance of function *y* is determined as follows

$$\sigma^2(y) = \sum_{i=1}^{n} \left(\frac{\partial \varphi}{\partial x_i}\right)^2 \sigma^2(x_i), \quad (8)$$

where $\frac{\partial \varphi}{\partial x_i}$ is a partial derivative of a function in respect of every argument.

Following the Eq. (1), (4) and (6), the expression can be written

$$Q_i = \lambda Q_i^{(1)} + (1-\lambda) Q_i^{(2)}, \ \lambda = 0, \dots, 1. \quad (9)$$

Accordingly, based on Eq. (8) and (9), estimate of variance of relative importance of alternative Q_i is determined as follows

$$\sigma^2(Q_i) = \left(\frac{\partial Q_i^{(1)}}{\partial \lambda}\right)^2 \sigma^2\left(Q_i^{(1)}\right) + \left(\frac{\partial Q_i^{(2)}}{\partial \lambda}\right)^2 \sigma^2\left(Q_i^{(2)}\right). \quad (10)$$

The following equation is obtained after calculating respective derivatives

$$\sigma^2(Q_i) = \lambda^2 \sigma^2\left(Q_i^{(1)}\right) + (1-\lambda)^2 \sigma^2\left(Q_i^{(2)}\right). \quad (11)$$

The variances $\sigma^2\left(Q_i^{(1)}\right)$ and $\sigma^2\left(Q_i^{(2)}\right)$ should be calculated. It can be written from Eq. (1) and Eq. (4):

$$\sigma^2\left(Q_i^{(1)}\right) = \sum_{j=1}^{n} \left(\frac{\partial Q_i^{(1)}}{\partial \bar{x}_{ij}}\right)^2 \sigma^2(\bar{x}_{ij}), \quad (12)$$

$$\sigma^2\left(Q_i^{(2)}\right) = \sum_{j=1}^{n} \left(\frac{\partial Q_i^{(2)}}{\partial \bar{x}_{ij}}\right)^2 \sigma^2(\bar{x}_{ij}). \quad (13)$$

Partial derivatives are calculated from Eq. (1) and Eq. (4) and inserted in Eq. (12) and Eq. (13):

$$\sigma^2\left(Q_i^{(1)}\right) = \sum_{j=1}^{n} w_j^2 \, \sigma^2(\bar{x}_{ij}), \quad (14)$$

$$\sigma^2\left(Q_i^{(2)}\right) = \sum_{j=1}^{n} \left(\frac{\prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} w_j}{(\bar{x}_{ij})^{w_j} (\bar{x}_{ij})^{(1-w_j)}}\right)^2 \sigma^2(\bar{x}_{ij}). \quad (15)$$

Estimates of variances of normalized initial criteria values are calculated as follows

$$\sigma^2(\bar{x}_{ij}) = \left(\frac{\bar{x}_{ij} k}{t}\right)^2, \quad (16)$$

where *k* – coefficient that summarizes uncertainty of measured criterion, *t* is a multiplier depending on the distribution law of errors and on the credibility level *q*. *k*=0.10 when the uncertainty of estimation of the initial data to be approximately equal to 10 present of an average criteria value. *t*=2.0 in the case of normal distribution with the credibility *q*=0.05. Accordingly

$$\sigma^2(\bar{x}_{ij}) = (0.05\bar{x}_{ij})^2. \quad (17)$$

Optimization of weighted aggregated assessment

Variances of estimates of alternatives (Eq. 11) in WASPAS depend on variances of WSM and WPA (Eq. 12 and Eq. 13) as well as coefficient λ. Accordingly, the aim of the current part of the research is to calculate optimal values of λ, i.e. to find minimum dispersion $\sigma^2(Q_i)$ and to assure maximal accuracy of estimation. Optimal values of λ can be find when searching extreme of function. Extreme of function can be found when derivative of Eq. (9) in regard to λ is equated to zero:

$$2\lambda\sigma^2\left(Q_i^{(1)}\right) - 2\sigma^2\left(Q_i^{(2)}\right) + 2\lambda\sigma^2\left(Q_i^{(2)}\right) = 0, \quad (18)$$

$$\lambda = \frac{\sigma^2\left(Q_i^{(2)}\right)}{\sigma^2\left(Q_i^{(1)}\right) + \sigma^2\left(Q_i^{(2)}\right)}. \quad (19)$$

Optimal λ (Eq. 19) should be calculated for every alternative before applying WASPAS (Eq. 6). Optimal λ may vary depending on ratio of $\sigma^2\left(Q_i^{(1)}\right)/\sigma^2\left(Q_i^{(2)}\right)$ in every particular case.

Ranking of alternatives

A multiple criteria decision making problem aimed at determining the most accurate relative importance of alternatives as well as ranking alternative decisions is analyzed in the chapter. The given MCDM problem is defined on 4 alternatives and 12 decision criteria. Relative significances of criteria were determined by means of entropy [9].

Initial normalized decision making matrix as well as relative significances of criteria (criteria weights) [9] are presented in Table 1.

Calculation results applying WASPAS (Eq. 6) when $\lambda = 0, 0.1, 0.2, \dots, 1$ are presented in Table 2.

Ranking order of alternatives as well as their relative importance is shown in Fig. 1. As can be observed from the graph, even ranking order of alternatives can vary depending on λ values.

Accuracy of calculations is measured according to proposed algorithm when $\lambda = 0, 0.1, 0.2, \dots, 1$ (Eq. 10–17). For the results see Table 3.

Table 1. Initial normalized decision making matrix

Alternatives a_i	Normalized criteria values $\bar{x}_{ij}$											
	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	$\bar{x}_4$	$\bar{x}_5$	$\bar{x}_6$	$\bar{x}_7$	$\bar{x}_8$	$\bar{x}_9$	$\bar{x}_{10}$	$\bar{x}_{11}$	$\bar{x}_{12}$
a_1	0.8486	0.6364	0.7982	0.6707	1.0000	0.8534	0.6622	0.8618	0.1432	0.1585	1.0000	0.4531
a_2	1.0000	1.0000	1.0000	0.7976	0.7000	0.9005	0.9324	0.6788	1.0000	0.6500	0.7270	0.7346
a_3	0.6542	0.7000	0.9169	0.8951	0.6000	0.9791	0.6216	0.9479	0.1340	0.1585	0.9795	0.6728
a_4	0.3694	0.4375	0.9407	1.0000	1.0000	1.0000	1.0000	1.0000	0.5478	1.0000	0.3754	1.0000
Weights w_j	0.0627	0.0508	0.1114	0.0874	0.0625	0.1183	0.0784	0.0984	0.0530	0.1417	0.0798	0.0557

Table 2. Ranking of alternatives applying WASPAS

Alternatives a_i	Relative significances of alternatives Q_i										
	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$	$\lambda = 10$
a_1	0.4912	0.5033	0.5154	0.5274	0.5395	0.5516	0.5637	0.5758	0.5878	0.5999	0.6120
a_2	0.8173	0.8185	0.8197	0.8209	0.8221	0.8233	0.8244	0.8256	0.8268	0.8280	0.8292
a_3	0.5873	0.5983	0.6093	0.6203	0.6313	0.6423	0.6532	0.6642	0.6752	0.6862	0.6972
a_4	0.8015	0.8066	0.8116	0.8167	0.8217	0.8268	0.8318	0.8369	0.8419	0.8470	0.8520

Table 3. Accuracy of estimation

Alternatives a_i	Standard deviations $\sigma(Q_i)$										
	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$	$\lambda = 10$
a_1	0.0075	0.0068	0.0064	0.0062	0.0062	0.0066	0.0071	0.0079	0.0088	0.0098	0.0108
a_2	0.0124	0.0113	0.0103	0.0095	0.0090	0.0088	0.0090	0.0096	0.0104	0.0114	0.0126
a_3	0.0089	0.0081	0.0075	0.0072	0.0071	0.0073	0.0078	0.0085	0.0094	0.0104	0.0116
a_4	0.0122	0.0111	0.0102	0.0095	0.0092	0.0093	0.0097	0.0105	0.0115	0.0127	0.0140

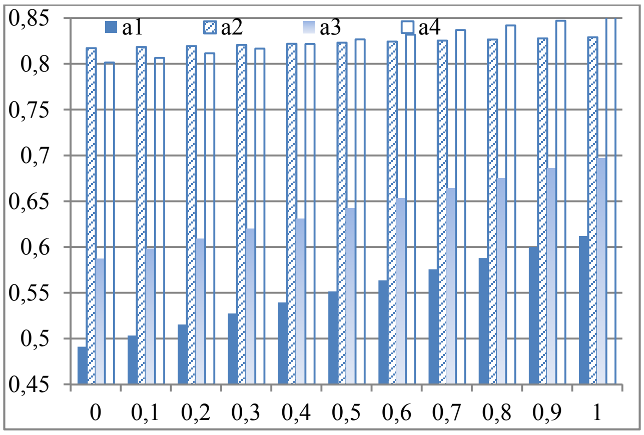

Fig. 1. Ranking order of alternatives when $\lambda = 0, 0.1, 0.2, \ldots, 1$

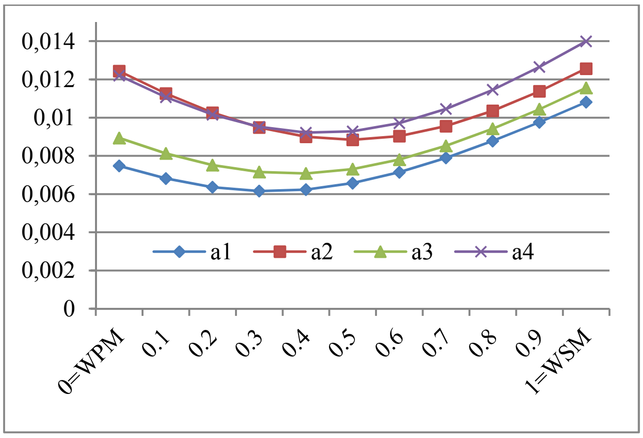

Fig. 2. Ranking accuracy

Dependence of errors on values of λ is shown in Fig. 2. As on can see from a graph, the higher ranking accuracy can be reached when aggregating the both particular methods in comparison with accuracy of WSM or WPM separately.

The optimal λ are calculated (Eq. 19) and ranking of alternatives is performed applying the estimated optimal values (Table 4).

Confidence intervals are calculated with the credibility q=0.05. Relative importance of alternatives and their confidence intervals are shown (Fig. 3).

Table 4. Ranking of alternatives applying optimal λ

Alternatives a_i	Optimal λ	Relative significances of alternatives Q_i
a_1	0.32	0.5303
a_2	0.49	0.8232
a_3	0.37	0.6284
a_4	0.43	0.8233
Ranking order of alternatives	$a_4 = a_2 \succ a_3 \succ a_1$	

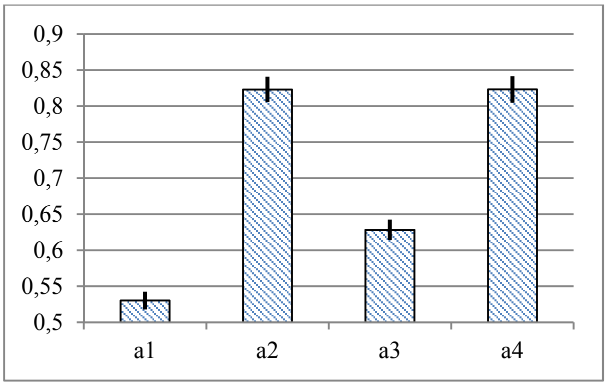

Fig. 3. Relative importance of alternatives and their confidence intervals with the credibility q=0.05

Alternatives a_2 and a_4 are of equal preference in the analyzed case when the optimal λ values are applied within the particular probability. One can state that optimized Weighted Aggregates Sum Product Assessment enables to reach the highest accuracy of estimation.

Conclusions

Effectiveness of computer-aided multiple criteria decision support system as well as accuracy of decisions is based on an application of a proper MCDM method.

It was observed that WSM and WPM methods can produce different ranking results. Accordingly, methodology for evaluation of accuracy of methods, based on initial criteria values, was developed.

It was proposed to apply a joint method of the latters, i.e. WASPAS (Weighted Aggregates Sum Product Assessment), to increase the ranking accuracy.

Accuracy of estimation applying WSM, WPM and WASPAS was evaluated. It was estimated that accuracy applying WASPAS increases up to 1.3 times as compared to WPM and up to 1.6 times as compared to WSM. Consequently, it was ascertained that the proposed joint method enables to increase the ranking accuracy.

Methodology for optimization of weighted aggregated function was proposed, that enables to reach the highest accuracy of estimation.

An example of application of methodology was presented. Relative importance of alternatives within the particular probability was calculated.

References

1. **Antucheviciene J., Zakarevicius A., Zavadskas E. K.** Measuring Congruence of Ranking Results Applying Particular MCDM Methods // Informatica, 2011. – Vol. 22. – No. 3. – P. 319–338.
2. **Balezentis A., Balezentis T.** An Innovative Multi–criteria Supplier Selection Based on Two–tuple MULTIMOORA and Hybrid Data // Economic Computation and Economic Cybernetics Studies and Research, 2011. – Vol. 45. – No. 2. – P. 37–56.
3. **Brauers W. K. M., Zavadskas E. K.** Robustness of MULTIMOORA: A Method for Multi–Objective Optimization // Informatica, 2012. – Vol. 23. – No. 1. – P. 1–25.
4. **Gajzler M.** Text and Data Mining Techniques in Aspect of Knowledge Acquisition for Decision Support System in Construction Industry // Technological and Economic Development of Economy, 2010. – Vol. 16. – No. 2. – P. 219–232.
5. **Ginevicius R.** A New Determining Method for the Criteria Weights in Multicriteria Evaluation // International Journal of Information technology & Decision Making, 2011. – Vol. 10. – No. 6. – P. 1067–1095.
6. **Lashgari A., Fouladgar M. M., Yazdani–Chamzini A., Skibniewski M. J.** Using an Integrated Model for Shaft Sinking Method Selection // Journal of Civil Engineering and Management, 2011. – Vol. 17. – No. 4. – P. 569–580.
7. **Peng Y., Kou G., Wang G., Wu W., Shi Y.** Ensemble of Software Defect Predictors: an AHP–based Evaluation Method // International Journal of Information Technology & Decision Making, 2011. – Vol. 10. – No. 1. – P. 187–206.
8. **Podvezko V.** The Comparative Analysis of MCDA Methods SAW and COPRAS // Engineering Economics, 2011. – Vol. 22. – No. 2. – P. 134–146.
9. **Saparauskas J., Zavadskas E. K., Turskis Z.** Selection of Facade's Alternatives of Commercial and Public Buildings Based on Multiple Criteria // International Journal of Strategic Property Management, 2011. – Vol. 15. – No. 2. – P. 189–203.
10. **Shi Y.** The Research Trend of Information Technology and Decision Making in 2009 // International Journal of Information Technology & Decision Making, 2010. – Vol. 9. – No. 1. – P. 1–8.
11. **Simanaviciene R., Ustinovicius L.** A New Approach to Assessing the Biases of Decisions Based on Multiple Attribute Decision Making Methods // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 1(117). – P. 29–32. DOI: 10.5755/j01.eee.117.1.1048.
12. **Triantaphyllou E., Mann S. H.** An Examination of the Effectiveness of Multi–Dimensional Decision–Making Methods: A Decision–Making Paradox // Decision Support Systems, 1989. – Vol. 5. – No. 3. – P. 303–312.
13. **Wang K., Wei F.** Robust Data Envelopment Analysis Based MCDM with the Consideration of Uncertain Data // Journal of Systems Engineering and Electronics, 2010. – Vol. 21. – No. 6. – P. 981–989.
14. **Zavadskas E. K., Zakarevicius A., Antucheviciene J.** Evaluation of Ranking Accuracy in Multi–Criteria Decisions // Informatica, 2006. – Vol. 17. – No. 4. – P. 601–618.
15. **Zavadskas E. K., Ustinovichius L., Peldschus F.** Development of Software for Multiple Criteria Evaluation // Informatica, 2003. – Vol. 14. – No. 2. – P. 259–272.

Received 2012 01 23
Accepted after revision 2012 03 30

E. K. Zavadskas, Z. Turskis, J. Antucheviciene, A. Zakarevicius. Optimization of Weighted Aggregated Sum Product Assessment // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 6(122). – P. 3–6.

One of important parts of every computer-aided multiple criteria decision support system is selection of a proper MCDM (Multiple Criteria Decision Making) method. WSM (Weighted Sum Model) and WPM (Weighted Product Model) are analyzed in the current research. The aim of the research is to measure the accuracy of the latter methods and to propose a method to increase the ranking accuracy of alternatives. It is proposed to apply joint WASPAS (Weighted Aggregates Sum Product Assessment) method. Methodology for evaluation of accuracy, based on initial criteria values, is developed. Optimization of weighted aggregated function is suggested, that enables to reach the highest accuracy of measurement. An example of application of the proposed methodology is presented. Ill. 3, bibl. 15, tabl. 4 (in English; abstracts in English and Lithuanian).

E. K. Zavadskas, Z. Turskis, J. Antuchevičienė, A. Zakarevičius. Svorinio agreguoto alternatyvių sprendimų vertinimo optimizavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 6(122). – P. 3–6.

Kompiuterinių sprendimų paramos sistemų, skirtų daugiatiksliams uždaviniams spręsti, viena iš svarbių dalių yra tinkamo daugiakriterių sprendimų priėmimo metodo parinkimas. Šiame darbe analizuojami svorinės sumos (angl. *Weighted Sum Model – WSM*) ir svorinės sandaugos (angl. *Weighted Product Model – WPM*) metodai. Keliamas tikslas įvertinti šių metodų tikslumą ir pasiūlyti alternatyvų būdą rangavimo tikslumui padidinti. Pasiūlyta taikyti agreguotą WASPAS (angl. *Weighted Aggregates Sum Product Assessment*) metodą bei parengta metodika tikslumui nustatyti, atsižvelgiant į pradinių duomenų neapibrėžtį. Taip pat parengta metodika funkcijai optimizuoti, kai pasiekiamas didžiausias matavimų tikslumas. Pateiktas metodo taikymo pavyzdys. Il. 3, bibl. 15, lent. 4 (anglų kalba; santraukos anglų ir lietuvių k.).