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An Improved Time-Frequency Representation Based on Nonlinear Mode Decomposition and Adaptive Optimal Kernel

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***Abstract*—Time-frequency representation (TFR) based on Adaptive Optimal Kernel (AOK) normally performs well only for monocomponent signals and has poor noise robustness. To overcome the shortcomings of AOK TFR mentioned above, a new TFR algorithm is proposed here by integrating nonlinear mode decomposition (NMD) with AOK TFR. NMD is used to decompose multicomponent signals into a bundle of meaningful oscillations and then AOK is applied to compute the TFR of individual oscillations, finally all these TFRs are summed together to generate one TFR. Through quantitative comparison with other TFR methods to both simulated and real signals, the superiority of proposed TFR based on NMD and AOK on removing noise and many other measurement index of TFR are shown.**

***Index Terms*—Time-frequency representation; nonlinear mode decomposition; adaptive optimal kernel; the cross terms.**

I. INTRODUCTION

Since most of the signals around us are non-stationary signals, such as communications, radar, sonar, acoustics, etc., their amplitudes and frequencies are changing with time. Instantaneous frequency estimation is a very important and popular tool to analyse many signals especially non-stationary signals. Many traditional instantaneous frequency estimation methods such as phase difference method [1], maximum likelihood estimator [2] and other similar methods are easily applied but lack of precise.

Empirical Mode Decomposition (EMD) is a kind of method introduced by Huang [3], with which we can obtain a meaningful instantaneous frequency. While doing a good job on conquering the influence of the cross term of multicomponent signals, the drawbacks of EMD are also obvious, it is not noise robust and has the phenomenon of mode mixing which is caused by intermittent signals or noise components [4]. To overcome the shortcomings of EMD, a noise assisted data analysis method called ensemble empirical mode decomposition (EEMD) is originally proposed in [5]. While doing a better job than EMD on resisting noise, it still doesn't meet expectations.

Time-frequency representations (TFR) is one of the most popular and efficient techniques in instantaneous frequency estimation, especially for multi-component signals [6]. Adaptive optimal-kernel (AOK) TFR is a signal depended instantaneous frequency estimation method [7]. To overcome the shortcomings of TFR methods with fixed windows or kernels that perform well only for limited class of signals, AOK is based on a signal-dependent radially Gaussian kernel that adapts over time. However, AOK normally performs well only for single component signals [8]. To reduce the interference of the cross terms in AOK when processing multi-component signals, a TFR method integrating EMD with AOK (EMD + AOK) can suppress the cross terms to a certain extent, but being quite sensitive to noise [9]. Since EEMD is more effective in resisting noise than EMD, EEMD + AOK can be used to improve the noise robustness to some degree but not enough [5].

The key ingredient of quantile-based empirical mode decomposition (QEMD) that analyses noisy signals efficiently is to apply a quantile smoothing method to a noisy signal itself instead of interpolating local extremes of the signal when constructing its mean envelope [10]. While holding the merits of the conventional EMD, it is also robust to outliers and noise. Using QEMD instead of EMD, a new TRF method is generated. However, it performs not good when the signals are complex and contain relatively high frequencies parts.

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Recently, a noise-robust, adaptive decomposition method called nonlinear mode decomposition (NMD) is proposed [11], [12]. NMD decomposes a given signal into a series of physically meaningful oscillations for any wave form while it is extremely noise robust. And it is based on the combination of time-frequency analysis, surrogate data tests, and the idea of harmonic identification [11]. However, the Nonlinear Modes (NMs) obtained by decomposing the given signal with NMD are not single component as IMF. The time-frequency distribution can't be got by directly doing the Hilbert transform as each of them may contains several frequency components.

To solve the problems of cross terms and the sensitivity to noise of EMD + AOK and EEMD + AOK methods, a new time-frequency representation algorithm combining NMD and AOK together is proposed in this paper. This approach fully utilizes the unique NMD decomposition ability to multicomponent signal and the good TFR of AOK. The results of examples verify that TFR based on NMD and AOK is practical.

II. TFR ALGORITHM

A. Adaptive Optimal-Kernel Time-Frequency Representation (AOK TFR)

As a large number of bilinear TFR's have been proposed, each of them only differs in the choice of the kernel function [7], [8], different kernel functions only perform well in the TFR of different signals. There is a conclusion from large amount research about TFR that no single kernel can adequately perform well on a large number of signals. In this case, a signal-dependent kernel function and time-frequency representation method is expected by all. The kernel function of AOK TFR can adaptively changes with the signals whose characteristics change over time [7], [8]. In this case, the kernel function at each time can always be optimal and it performs better in tracking the changes of the given signals. The AOK algorithm can be briefly described as follows:

A1. Choose the Gaussian radial kernels as the kernel function

$$\Phi(r,\psi)=\exp(-\frac{r^2}{2\sigma^2(\psi)}), \tag{1}$$

where $\sigma(\psi)$ is the extension function controlling the spread of the Gaussian at radial angle ψ.

A2. In order to get the optimal kernel adapted to a signal, solving the following optimal problem is the best way

$$\max_{\Phi}\int_0^{2\pi}\int_0^{\infty}\left|A(r,\psi)\Phi(r,\psi)\right| r dr d\psi. \tag{2}$$

Subject to

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{\infty}\left|\Phi(r,\psi)\right|^2 r dr d\psi = = \frac{1}{4\pi^2}\int_0^{2\pi}\sigma^2(\psi)d\psi \le \alpha, \tag{3}$$

where $\alpha \ge 0$, $A(r,\psi)$ is the ambiguity function in polar coordinate. The volume of the kernel is limited to α. And the definition of $A(r,\psi)$ in the rectangular coordinates is described in (4)

$$A(t;\theta,\tau)=\int_{-\infty}^{+\infty}s^*(u-\frac{\tau}{2})w^*(u-t-\frac{\tau}{2})\times \times s(u-\frac{\tau}{2})w(u-t-\frac{\tau}{2})e^{j\theta u}du, \tag{4}$$

where $w(u)$ is a window function which is symmetrical and equal to zero for $|u|>T$.

A3. By solving the optimal problem in **Step A2**, an optimal kernel function $\Phi_{\text{opt}}(t;\theta,\tau)$ is obtained who varies with time as the same with short-time ambiguity function (STAF).

A4. Finally, a single current-time slice of the AOK TFR is computed as one slice (at time t only) of the 2-D Fourier transform of the STAF-kernel product

$$P_{AOK}(t,\omega)=\frac{1}{4\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}A(t;\theta,\tau)\Phi_{opt}\times \times(t;\theta,\tau)e^{-j\theta t-j\tau\omega}d\theta d\tau. \tag{5}$$

AOK time-frequency representation is an efficient method to track the instantaneous frequency and energy variation changing with time. While having a good performance on time-frequency focusing property, AOK is still a member of the bilinear transfer family. So the problem of cross terms for multi-component signals especially for nonlinear signals can't be avoided by AOK time-frequency representation. What's more, the noise robustness of AOK TFR still not met expectations.

B. Nonlinear Mode Decomposition (NMD)

The NMD method is a new signal decomposing algorithm mainly aimed to extract the physically meaningful oscillations from a complex signal system and simultaneously avoiding the effect of noise effectively [11], [12]. Unlike EMD and EEMD method that use a sifting process to extract intrinsic mode functions (IMFs), it is mainly based on time-frequency analysis, surrogate data tests, and the idea of harmonic identification [11]. Each nonlinear mode (NM) extracted from the original signal has physical significance and almost does not contain noise. For a given signal $s(t)$, the NMD process contains the following steps:

B1. Calculate the wavelet transform $W_s\left[\omega,t\right]$ of the target signal $s(t)$ and find all of its ridge curves $\omega_p^{(1)}(t)$ through the method discussed in [13], where $\omega_p^{(1)}(t)$ is the ridge curve of the first harmonic.

B2. Reconstruct the amplitude $A^{(1)}(t)$, the phase $\varphi^{(1)}(t)$, and the frequency $\nu^{(1)}(t)\equiv\varphi'^{(1)}(t)$ of the corresponding component $x^{(1)}(t)=A^{(1)}(t)\cos\varphi^{(1)}(t)$ of first harmonic by the ridge method. The formulas are as:

$$\nu^{(1)}(t) = \omega_p^{(1)}(t)e^{\delta \ln \nu_d(t)}, \tag{6}$$

$$A^{(1)}(t)e^{i\varphi^{(1)}(t)} = \frac{2W_s(\omega_p^{(1)}(t),t)}{\hat{\psi}^*\left[\omega_\psi \nu^{(1)}(t)/\omega_p^{(1)}(t)\right]}, \tag{7}$$

where $\delta \ln \nu_d^{(1)}(t)$ is the correction for discretization effects found by parabolic interpolation. In addition, $\psi(t)$ is the wavelet function and $\omega_\psi = \arg\max|\hat{\psi}(\xi)|$ denotes wavelet peak frequency.

B3. Compute the corresponding degree of order D_0 of the first harmonic and create N_s Fourier transform surrogates of the signal $s(t)$ [11]. Calculate the TFRs of each of them and compute the respective $D_{s=1,2,...N_s}$. Count up the number of D_s meeting the condition of $D_s > D_0$ and the significance level is identified as the ratio of the number and N_s.

B4. Calculate the amplitude-phase consistency of the first harmonic $\rho^{(1)}$ and set the threshold $\rho_{\min} = 0.25$. And the harmonic is identified as true if both $\rho^{(1)} \geq \rho_{\min}$ and the significance level is greater than or equal to 0.95. If $x^{(1)}(t)$ was identified as true harmonic, set $y_1(t) = x^{(1)}(t)$.

B5. Subtract the true harmonic $y_1(t)$ obtained by **Step B4** from the original signal $s(t)$ and repeat the above steps on the residue to extract the h th true harmonic $y_k(t)$, $(k = 1,2,...,K)$, where K is the number of true harmonics. Stop when three consecutive harmonics were judged as false.

B6. Sum all the true harmonics to generate a $NM_1(t) = \sum_{k=1}^{K} y_k(t)$.

B7. Calculate the residue $r_1(t) = s(t) - NM_1(t)$. Iterate on $r_1(t)$ to obtain the next NM_i.

Finally, the given signal $s(t)$ can be expressed as

$$s(t) = \sum_i NM_i(t) + n(t), \tag{8}$$

where $n(t)$ denotes the noise.

III. IMPROVED TIME-FREQUENCY REPRESENTATION METHOD

A. EMD + AOK

Empirical Mode Decomposition (EMD) can be used to adaptively decompose signals into a set of mono-component signals called Intrinsic Mode Functions (IMF) [3].

To eliminate the effect of cross terms in AOK, a TFR approach integrating EMD with AOK is proposed [8]. Firstly, multi-component signals are decomposed into a series of IMFs by EMD. Secondly, AOK is applied to compute the TFR of individual single component, finally summing all these TFRs together to generate one TFR. While restraining the cross terms to some extent, the EMD + AOK method is still very sensitive to noise.

B. EEMD + AOK

Ensemble Empirical Mode Decomposition (EEMD) is an improvement method over EMD to eliminate the drawbacks of mode mixing and low robustness to some extent [5].

With the advantages compared to EMD, EEMD is chosen to replace EMD to be integrated with AOK to compute the time-frequency representation [9]. While EEMD + AOK method performs a little better than EMD+AOK on removing the noise and mode mixing, it still cannot be regarded as a real noise robust method compared with many approaches and its performance do not meet our expectations.

C. QEMD + AOK

Quantile-Based Empirical Mode Decomposition (QEMD) is an efficient way to decompose noisy signals based on a quantile smoothing method and conventional EMD [10]. The quantile sifting process can be seen as a process of removing noise and the residual signal obtained from it is expected to represent the pure signal without noise. And then the conventional EMD method can be used to decompose the "pure" signal into a set of IMFs. Finally, calculate the TFRs of all the IMFs using AOK and sum them together to generate the final TFR. This is a new TFR method based on QEMD and AOK.

D. NMD + AOK

Nonlinear Mode Decomposition (NMD) is an extremely noise robust decomposition method [11]. A new TFR method integrating NMD with AOK algorithm together is resulted in. NMD covers the shortage of AOK of cross terms and poor noise robustness. So this TFR method has the advantages of the two algorithms and performs better than the other TFR methods mentioned above. The detailed procedure of NMD + AOK can be described as follows:

C1. Give the original signal $s(t)$.

C2. Perform NMD (**Step B1–Step B7**) on the given signal to obtain a set of NMs and the signal $s(t)$ can be expressed as (8).

C3. Calculate the time-frequency representation TFR_i of the corresponding NM_i by using AOK (**Step A1–Step A4**).

C4. Obtain the final time-frequency representation TFR by summing all the TFRs derived in **Step C3** together.

IV. NUMERICAL SIMULATION AND ANALYSIS

Having described the TFR method based on NMD and AOK and many other TFR methods, we now illustrate the new TFR method by consideration of some simulated signals as examples and contrasting with other TFR methods.

All methods use the parameter window width $2T = 128$ and the volume $\alpha = 5$.

To quantitatively compare the performances of TFRs mentioned above, three parameters are taken from [14]:

1. The *Instantaneous frequency correlation* IF.
2. The *two-dimensional correlation* ρ.
3. The time-frequency resolution [15, 16] measure res.

For the first two parameters 1.00 means best quality of estimated representation, while for the third one it is 0.00.

A. *Example 1*

Consider a signal with white noise described as follows

$$s_1(t) = \cos(20\pi t) + \sin(200\pi t) + \sin(400\pi t) + \\ + \sin(100\pi(t-0.5)^2) + n(t), \quad (9)$$

where $n(t)$ is a white noise. $s_1(t)$ is sampled at 1 kHz for 1 s. All the TFRs under signal to noise ratio (SNR) is 6 dB are shown in Fig. 1, and the performance results under different SNR are listed in Table I–Table III.

As shown in Fig. 1(b), there are serious cross terms and noise in AOK TFR. EMD + AOK TFR and EEMD + AOK TFR as in Fig. 1(c) and Fig. 1(d) suppress the cross terms to some extent but still have poor noise robustness. The noise robustness of QEMD + AOK in Fig. 1(e) is obviously better than the three approaches above, but still do not meet expectations. Meanwhile, it loses some of the useful high frequency components and performs not good when noise is skewed. As illustrated in Fig. 1(f), NMD + AOK TFR has a better performance on both removing noise and suppressing the cross terms.

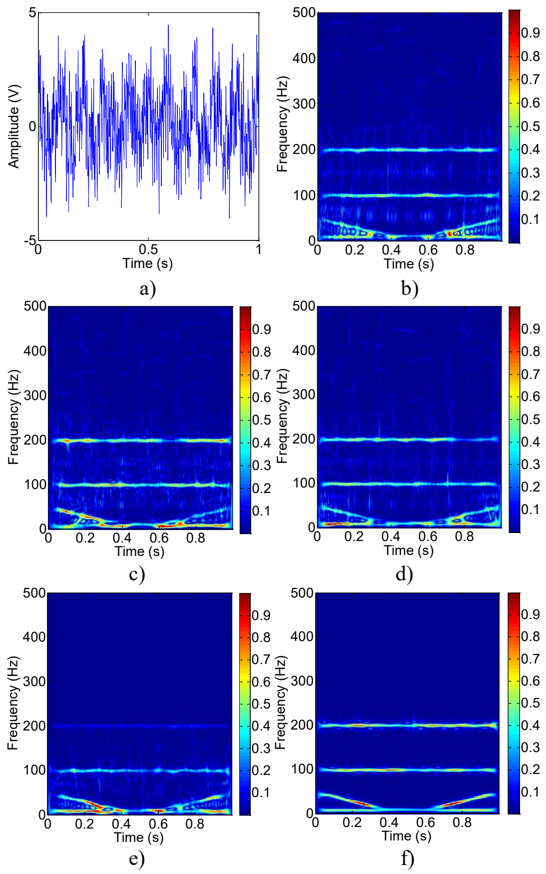


Fig. 1. TFRs of signal $s_1(t)$: a) waveform; b) AOK TFR; c) EMD+AOK TFR; d) EEMD+AOK TFR; e) QEMD+AOK TFR; f) NMD+AOK TFR.

According to the three tables, the parameter IF and ρ obtained by NMD + AOK are obviously closer to 1.00 and the value of res is closer to 0.00 than other methods under the condition of $SNR = 6$ dB. And it is the same in other cases with different SNR. So it is clear that the NMD + AOK TFR outperforms the other four TFRs according to the value of the three performance measures.

TABLE I. THE PARAMETER IF UNDER DIFFERENT SNR.

IF	***SNR***			
	2 dB	6 dB	10 dB	14 dB
AOK	0.2764	0.3245	0.3134	0.3307
EMD + AOK	0.3143	0.3391	0.3792	0.4302
EEMD + AOK	0.3095	0.3516	0.3601	0.3663
QEMD + AOK	0.3552	0.4005	0.4316	0.4112
NMD + AOK	0.3931	0.4414	0.4450	0.4719

TABLE II. THE PARAMETER ρ UNDER DIFFERENT SNR.

ρ	***SNR***			
	2 dB	6 dB	10 dB	14 dB
AOK	0.5906	0.6526	0.6808	0.6668
EMD + AOK	0.5921	0.6564	0.6885	0.6887
EEMD + AOK	0.6091	0.6572	0.6889	0.6999
QEMD + AOK	0.5599	0.6395	0.6600	0.7095
NMD + AOK	0.6399	0.6948	0.6914	0.6930

TABLE III. THE PARAMETER res UNDER DIFFERENT SNR.

res	***SNR***			
	2 dB	6 dB	10 dB	14 dB
AOK	0.0237	0.0212	0.0209	0.0203
EMD + AOK	0.0205	0.0173	0.0162	0.0156
EEMD + AOK	0.0218	0.0176	0.0168	0.0170
QEMD + AOK	0.0127	0.0138	0.0123	0.0025
NMD + AOK	0.0091	0.0114	0.0105	0.0102

B. *Example 2*

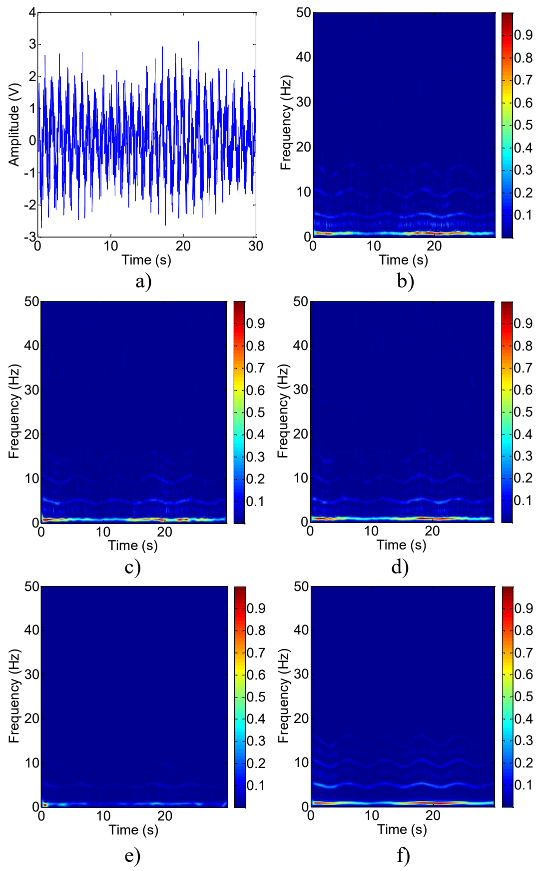


Fig. 2. TFRs of signal $s_2(t)$: a) waveform; b) AOK TFR; c) EMD+AOK TFR; d) EEMD+AOK TFR; e) QEMD+AOK TFR; f) NMD+AOK TFR.

Another simulated signal is as follows:

$$s_2(t) = A(t)[\cos\varphi(t) + 0.5\cos(5\varphi(t) - \pi/4) + \\ +0.33\cos(10\varphi(t) + \pi/2) + \\ +0.25\cos(15\varphi(t) + \pi/3)] + n(t), \quad (10)$$

$$A(t) = 1 + 0.25\cos(2\pi t/20), \quad (11)$$

$$\varphi(t) = 2\pi t + 0.5\sin(2\pi t / 6), \quad (12)$$

where $s_2(t)$ is sampled at 100 Hz for 30 seconds. The TFRs of each method under the condition of $SNR = 6\text{dB}$ are shown in Fig. 2, and the results of performance measure under different SNR are listed in Table IV to Table VI.

As shown in Fig. 2, each component of the signal can be seen from the TFR obtained by AOK, EMD + AOK and EEMD + AOK while some cross terms and noise exist. QEMD + AOK performs better on removing noise but losing parts of the high order harmonic components. NMD + AOK can suppress the noise and cross terms effectively and reflect all information of the signal. The same conclusion can be drawn from Table IV to Table VI by comparing the value of the three parameters.

TABLE IV. THE PARAMETER IF UNDER DIFFERENT SNR.

IF	***SNR***			
	2 dB	6 dB	10 dB	14 dB
AOK	0.3982	0.4890	0.5184	0.5331
EMD + AOK	0.4766	0.5568	0.5958	0.6596
EEMD + AOK	0.4663	0.5558	0.6183	0.5848
QEMD + AOK	0.5361	0.6636	0.5933	0.6321
NMD + AOK	0.5541	0.6178	0.6644	0.6325

TABLE V. THE PARAMETER ρ UNDER DIFFERENT SNR.

ρ	***SNR***			
	2 dB	6 dB	10 dB	14 dB
AOK	0.6019	0.6481	0.6684	0.6722
EMD + AOK	0.6387	0.6240	0.6295	0.6153
EEMD + AOK	0.6507	0.6706	0.6579	0.6742
QEMD + AOK	0.6286	0.5233	0.6704	0.6534
NMD + AOK	0.6885	0.6859	0.6975	0.6953

TABLE VI. THE PARAMETER res UNDER DIFFERENT SNR.

res	***SNR***			
	2 dB	6 dB	10 dB	14 dB
AOK	0.0108	0.0070	0.0057	0.0053
EMD + AOK	0.0070	0.0051	0.0039	0.0025
EEMD + AOK	0.0073	0.0052	0.0045	0.0043
QEMD + AOK	0.0042	0.0028	0.0043	0.0010
NMD + AOK	0.0038	0.0036	0.0040	0.0051

V. REAL SIGNAL PROCESSING AND ANALYSIS

Verifying the proposed TFR method based on NMD and AOK by simulated signals successfully, we also need to check its feasibility with real signals.

A. Underwater Acoustic Signal

Here we choose the vertical underwater acoustic array data collected in shallow-water off the Italian west coast by the NATO SACLANT Centre. The real frequency of the signal is about 170 Hz. There are 48 sensors and each of them collected 64 K data points long with the sample frequency of 1 kHz. The first five seconds data of the whole signal data collected by the first sensor is as in Fig. 3(a).

In general, the sea noise is considered as complex coloured noise. From Fig. 3(b) to Fig. 3(e), it is obvious to find that the AOK TFR, EMD + AOK TFR, EEMD + AOK TFR, QEMD + AOK TFR are all suffering from noise seriously. And the NMD + AOK TFR as shown in Fig. 3(f) performs better on removing noise compared to other TFRs mentioned above. So we can find the NMD + AOK algorithm also performs well on removing the coloured noise.

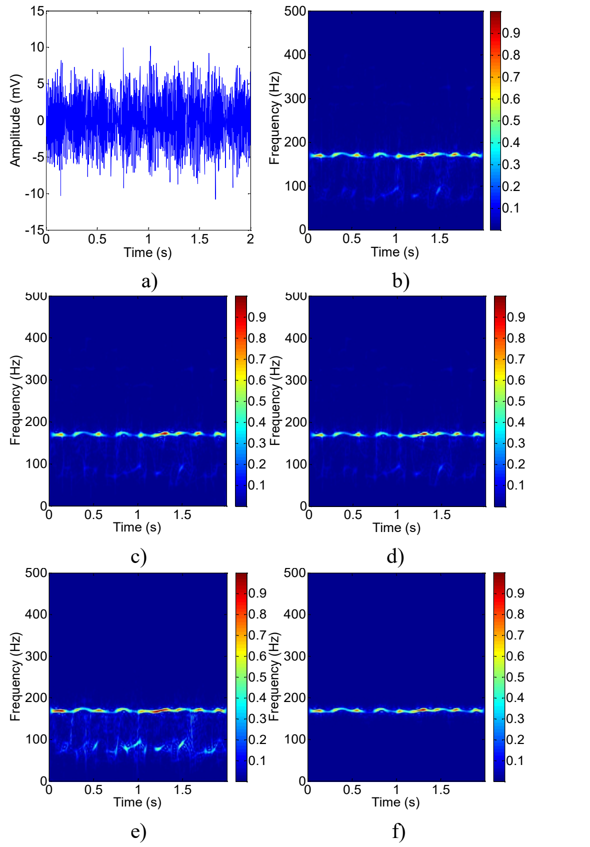
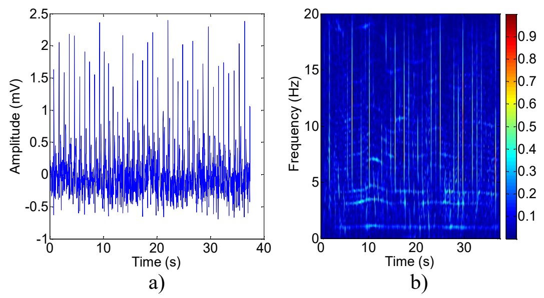


Fig. 3. TFRs of underwater acoustic signal: a) underwater acoustic signal; b) AOK TFR; c) EMD+AOK TFR; d) EEMD+AOK TFR; e) QEMD+AOK TFR; f) NMD+AOK TFR.

B. Human ECG signal

Here is a human ECG signal sampled with the frequency 40 Hz for about 30 minutes which is the same as used in [11]. We pick the part of the previous 37 seconds and add some white noise of $SNR = 6\text{dB}$ as in Fig. 4(a). The TFRs of different algorithms are shown in Fig. 4(b)–Fig. 4(f).

ECG is a complex biological signal. The line near the bottom of each figure represents the heartbeat signal which frequency is about 1 Hz. And there are also many helpful high-order harmonic components related to health [11]. From Fig. 4(b) to Fig. 4(d), it is obvious that the high-order harmonic components in these TFRs are seriously damaged by the noise. The QEMD + AOK TFR method (see Fig. 4(e)) removes the noise to some extent but losing the high frequency parts of the ECG signal at the same time. So there is no doubt that NMD + AOK algorithm (see Fig. 4(f)) performs best among the five TFR methods.



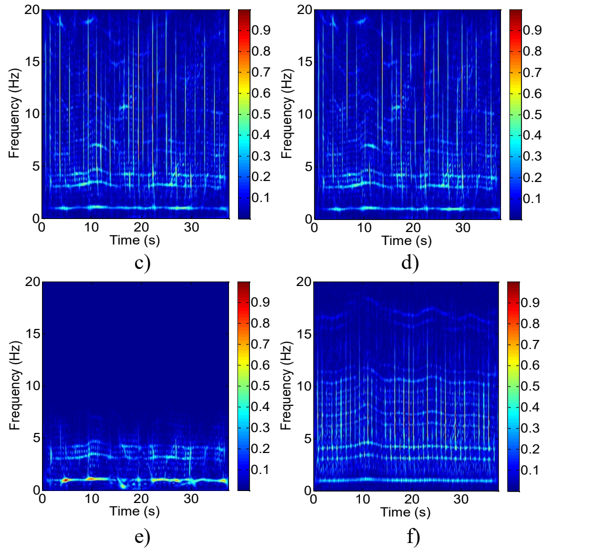


Fig. 4. TFRs of ECG signal: a) ECG signal; b) AOK TFR; c) EMD+AOK TFR; d) EEMD+AOK TFR; e) QEMD+AOK TFR; f) NMD+AOK TFR.

VI. CONCLUSIONS

AOK TFR has superior performance to a large class of signals with its signal-depended kernel changing over time adaptively. However, it only works well on mono-component signals and failed to suppress cross-terms of multi-component signals. In addition, the noise robustness of AOK TFR can't meet expectations. The EMD + AOK method and EEMD + AOK method suppress the cross terms to a certain extent but suffer from noise.

By using QEMD + AOK algorithm, the noise is removed effectively while the higher frequency components are weakened. Integrating NMD with AOK TFR algorithm, the new TFR develops both the advantages of them and covers their shortages to a great extent. So it is able to deal with signals which are difficult to be processed by AOK TFR individually. Moreover, it performs well on removing noise because of the strong noise robustness of NMD algorithm. By testing on two simulated signals and two real signals, we compare the performance of the novel TFR method to others by both analysing the TFR figures and the parameters in the tables. The proposed TFR method can be used to obtain the time-frequency representation of signals under low SNR and suppress cross-terms to a certain extent at the same time.

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