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Nonlinear Control based on Feedback Linearization for Double-Electromagnet Suspension System

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Introduction

In recent years so many studies have been carried out on electromagnet suspension systems [1–5]. Performance requirements for electromagnet suspension systems include comfort and suspension deflection. However, these requirements are often conflicting, and a compromise of the requirements must be reached. To this end, a considerable amount of research has been carried out [5].

The goal of designing a feedback linearization for a double magnetic suspension system is to suspend the effects of an object in a certain distance from a magnetic rail by using two electromagnets. The advantage of the suggested method over the work done by [3] is that their presented decoupled controller shows, but here the problem solved without decoupling the performance of their proposed method have been decreased. However the current work will solve the mentioned problems and improves their method.

System Modelling

The basic idea for designing such trains is the magnetic suspension. As shown in Fig. 1, in a single suspension system, the mass is under the influence of two forces: the gravity and the magnetic force. In the case of passenger trains, a different type of suspension system is used (Fig. 2). according to respective controlled object. The coupling between the two groups of electromagnets is regarded as disturbance and suppressed by enhancing the robustness of individual controllers. However, this method cannot actively overcome the uncertainty issues, and the control performance is not desirable especially in the presence of external disturbances. DEM (Double Electro-Magnet) has five degrees of freedom in movement: heave, sway, pitch, roll and yaw. Among them, only heave and pitch are to be controlled. Hence the system in this case has two degrees of freedom.

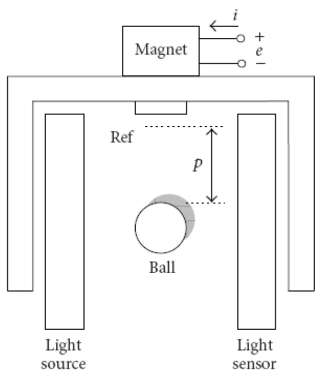

Fig. 1. Magnetic suspension system

Variable Parameters. In hand Variable parameters in the problem are: the mass of the object (m) and the inductance of the coil of the electromagnets (k).

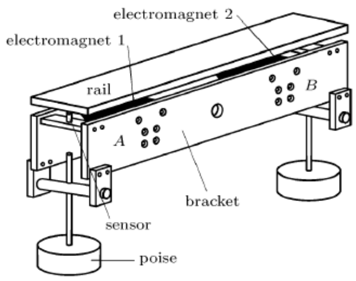

Fig. 2. The structure of the suspension system

Structure of the System. The structure of DEM shown in Fig. 2 includes two identical electromagnets connected by a rigid bracket. The magnets 1 and 2 provide the suspension forces needed for points 1 and 2, respectively. The suspended object can be considered as a solid object with two electromagnets. Furthermore, two sensors are used to measure the states of the suspension and two poises can be used to provide load forces.

DEM can be simplified as shown in Fig. 3. Parameters of the system are defined as follows: *m* is the mass of the suspended object, *I* is the spinning inertia in the center of the object O, and F_2 are magnetic forces, N_1 and N_2 are load forces on the two ends of the bracket, d is the distance of the center of the object from the rail and δ_1 and δ_2 are the distance from the points where F_1 and F_2 are applied, respectively. d_1 and d_2 are the distance between the solid object and the corresponding points on the rail. l is the distance between the center of the object O and the point where magnetic forces are applied and finally, L is the distance between the center of the object and the point where load forces are applied.

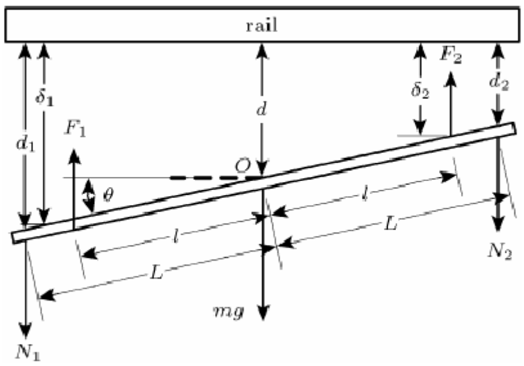

Fig. 3. Details of suspension system

Simplifying Assumptions:

- The stiffness of the rail is infinite and only the movement of the DEM relative to the rail is to be considered.
- The leakage flux, edge effect of the magnetic force, magnetic resistance of the core and the rail are negligible.
- A weight center is considered for the weight of the object. The weight of the two magnets are assumed to be identical; therefore, the total weight of the bracket and the magnets can be represented by a weight center, O, depicted in Fig. 3.
- The lengths of the magnets are very short and the points where the forces are applied are fixed.
- Load forces, created by the poises, have only one downward component.

DEM suspension system is a complex system including mechanical dynamics of DEM, relation between the current and the electromagnetic force and relation between the voltage and the current.

Mechanical- Dynamic Equations. As mentioned before, DEM has five degrees of freedom for movement: heave, sway, pitch, roll and yaw. For the controller, only heave and pitch movements are taken into account, namely, vertical movement of weight center O and the twisting movements around the main rotation pivot. The positive directions of movements and rotation are considered downward and counter-clockwise respectively. According to the principle of force transfer and the Newton's second law, the mechanical dynamics equations can be achived [2]. Let us select the state of the system as follows

$$x = \left[x_1, x_2, x_3, x_4, x_5, x_6\right]^T = \left[d_1, \dot{d}_1, d_2, \dot{d}_2, i_1, i_2\right]^T . \quad (1)$$

then state-space representation of the systemis as follow:

$$\begin{cases} \dot{x} = f(x) + G(x)u, \\ y = h(x), \end{cases} \quad (2)$$

$$f(x) = \begin{bmatrix} x_2 \\ \dfrac{-A_K K x_5^2}{\left(Px_1 + Qx_3\right)^2} + \dfrac{-B_K K x_6^2}{\left(Qx_1 + Px_3\right)^2} + C_K \times N_1 + D_K \times N_2 + g \\ x_4 \\ \dfrac{-B_K K x_5^2}{\left(Px_1 + Qx_3\right)^2} + \dfrac{-A_K K x_6^2}{\left(Qx_1 + Px_3\right)^2} + D_K \times N_1 + C_K \times N_2 + g \\ -\dfrac{R}{2K} x_5\left(Px_1 + Qx_3\right) + \dfrac{x_5\left(Px_2 + Qx_4\right)}{\left(Px_1 + Qx_3\right)} \\ -\dfrac{R}{2K} x_6\left(Qx_1 + Px_3\right) + \dfrac{x_6\left(Qx_2 + Px_4\right)}{\left(Qx_1 + Px_3\right)} \end{bmatrix}, \quad (3)$$

$$G(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{\left(Px_1 + Qx_2\right)}{2K} & 0 \\ 0 & \dfrac{\left(Qx_1 + Px_2\right)}{2K} \end{bmatrix}, \quad (4)$$

$$h(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}. \quad (5)$$

In the following investigation all parameters of the system are assumed to be unknown. The case in which the exact values of some parameters of the system are not known is to be investigated. In this paper a linearizing feedback is proposed. The introduced feedback is very useful in designing the control law.

Feedback Linearization

Dynamic equations for the suspended system are stated by equation (2). According to nonlinear systems theory, the number of derivatives needed to take from the output in order to get the input is called "Relative Degree" of the system. If the relative degree of the system is more than the system degree itself, the proposed method can not be used, and otherwise the system is "Completely Linearizable". To simplify calculations, the following operators are defined:

$$L_f^i h = \nabla_x \left(L_f^{i-1} h\right) \times f(x) , \quad (6)$$

$$L_g L_f^{i-1} h = \nabla_x \left(L_f^{i-1} h \right) \times G , \qquad (7)$$

$$L_f h = \nabla_x h \times f(x) . \qquad (8)$$

According to the above equations and explanation, the i'th derivative of the output is

$$y^{(j)} = L_f^j h + L_g L_f^{j-1} h \times u , \; L_g L_f^j h = 0 \; for \; j = i-2, \ldots, 1. \qquad (9)$$

With a glance at above equation, for attaining the input we must carry on derivations till the time we get $L_g L_f^{i-1} h \neq 0$.

Detailed information on the topic is described by [6].

$$f(x) = \begin{cases} x_2 \\ \dfrac{-A_K K x_5^2}{(Px_1 + Qx_3)^2} + \dfrac{-B_K K x_6^2}{(Qx_1 + Px_3)^2} + C_K \times N_1 + D_K \times N_2 + g, \\ x_4 \\ \dfrac{-B_K K x_5^2}{(Px_1 + Qx_3)^2} + \dfrac{-A_K K x_6^2}{(Qx_1 + Px_3)^2} D_K \times N_1 + C_K \times N_2 + g, \\ -\dfrac{R}{2K} x_5 (Px_1 + Qx_3) + \dfrac{x_5 (Px_2 + Qx_4)}{(Px_1 + Qx_3)}, \\ -\dfrac{R}{2K} x_6 (Qx_1 + Px_3) + \dfrac{x_6 (Qx_2 + Px_4)}{(Qx_1 + Px_3)}. \end{cases} \qquad (10)$$

$$G(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{(Px_1 + Qx_2)}{2K} & 0 \\ 0 & \dfrac{(Qx_1 + Px_2)}{2K} \end{bmatrix}, \; h(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}. \qquad (11)$$

Having taken the first derivative from the output, we gain

$$\dot{y} = \nabla_x h \times \dot{x} . \qquad (12)$$

By using the equations (6-11), then

$$\dot{y} = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} . \qquad (13)$$

It's evidently visible that the input doesn't show up in the first derivation from the output. Thus the derivation should be repeated accordingly. Therefore

$$\ddot{y} = \nabla_x \dot{x} . \qquad (14)$$

By using equations (6-11) we can write:

$$\ddot{y} = \begin{cases} \dfrac{-A_k K x_5^2}{(Px_1 + Qx_3)} + \dfrac{-B_k K x_6^2}{(Qx_1 + Px_3)} + C_k \times N_1 + D_k \times N_2 + g, \\ \dfrac{-B_k K x_5^2}{(Px_1 + Qx_3)} + \dfrac{-A_k K x_6^2}{(Qx_1 + Px_3)} + D_k \times N_1 + C_k \times N_2 + g. \end{cases} \qquad (15)$$

The input doses not show up either this time. Therefore, derivation should be repeated once more

$$\dddot{y} = L_f^3 h + L_g L_f^2 h \times u = \nabla_x \left(L_f^2 h \right) \times f(x) + L_g L_f^2 h \times u \, . \qquad (16)$$

In the above equation, $L_g L_f^2 h$ and $\nabla_x \left(L_f^2 h \right)$ are described as:

$$L_g L_f^2 h = \begin{bmatrix} -\dfrac{A_k x_5}{(Px_1 + Qx_3)} & -\dfrac{B_k x_6}{(Qx_1 + Px_3)} \\ -\dfrac{B_k x_5}{(Px_1 + Qx_3)} & -\dfrac{A_k x_6}{(Qx_1 + Px_3)} \end{bmatrix} \qquad (17)$$

$$\nabla_x \left(L_f^2 h \right) = \begin{bmatrix} \dfrac{2P \times A_k K \times x_5^2}{(Px_1 + Qx_3)^3} + \dfrac{2Q \times B_k K \times x_6^2}{(Qx_1 + Px_3)^3} & 0 & \dfrac{2Q \times A_k K \times x_5^2}{(Px_1 + Qx_3)^3} + \dfrac{2P \times B_k K \times x_6^2}{(Qx_1 + Px_3)^3} & 0 & -\dfrac{2A_k K \times x_5}{(Px_1 + Qx_3)^2} & -\dfrac{2B_k K \times x_6}{(Qx_1 + Px_3)^2} \\ \dfrac{2P \times B_k K \times x_5^2}{(Px_1 + Qx_3)^3} + \dfrac{2Q \times A_k K \times x_6^2}{(Qx_1 + Px_3)^3} & 0 & \dfrac{2Q \times B_k K \times x_5^2}{(Px_1 + Qx_3)^3} + \dfrac{2P \times A_k K \times x_6^2}{(Qx_1 + Px_3)^3} & 0 & -\dfrac{2B_k K \times x_5}{(Px_1 + Qx_3)^2} & -\dfrac{2A_k K \times x_6}{(Qx_1 + Px_3)^2} \end{bmatrix} . \qquad (18)$$

It is worth noting that the system relative degree is three, therefore, it can be completely linearized. Now, if we consider the control input as following, then we would have a linear system

$$u = \frac{1}{L_g L_f^2 h} \left(-L_f^3 h + \upsilon \right) . \qquad (19)$$

Considering such a control input leads to the following dynamic equation for the system

$$\dddot{y} = \upsilon . \qquad (20)$$

Now, suppose that the new control input is defined as:

$$\upsilon = \dddot{y}_d - k_1 \ddot{e} - k_2 \dot{e} - k_3 e , , \qquad (21)$$

where $e = y - y_d$.

Then, the dynamic equation would be

$$\dddot{e} + k_1 \ddot{e} + k_2 \dot{e} + k_3 e = 0, \qquad (22)$$

where $e = y - y_d$.

Here, if the coefficients k_1, k_2 and k_3 are selected so that the polynomial $s^3 + k_1 s^2 + k_2 s + k_3$ is Heurwitz, then error would approach zero as t goes to infinity.For instance, we consider the coefficients k_1 ، k_2 and k_3 as follow:

$$\begin{cases} k_1 = \alpha_1 + 2\alpha_2 , \\ k_2 = 2\alpha_1 \alpha_2 + \alpha_2^2 , \\ k_3 = \alpha_1 \alpha_2^2 . \end{cases} \qquad (23)$$

Therefore, dynamic equation will be

$$(s + \alpha_1)(s + \alpha_2)^2 e = 0 \, . \qquad (24)$$

Simulation and Results

Tracking a path. To show tracking capability of the proposed method, a path, as shown in Fig. 4, is considered. The coefficients k_1, k_2 and k_3 are selected equal to 7, 6 and 12, respectively. As shown in Fig (4), we use $L_f^2 h$ to gain the value of $\ddot{y}$.

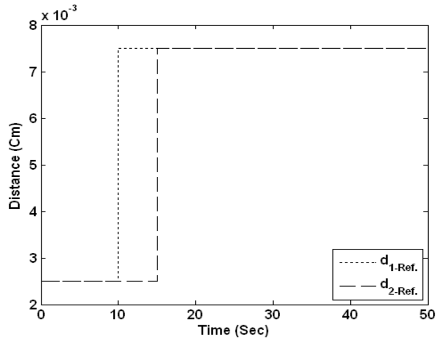

Fig. 4. Assumed reference value for tracking problem

Fig. 5 shows the performance of the system while applying the above input. The figure depicts that the proposed method is capable of tracking the input very well.

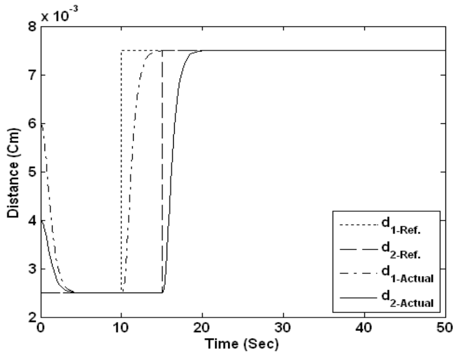

Fig. 5. The response of system in tracking purposed reference value

As mentioned in the modeling part, the parameters A_k, B_k, C_k and D_k are dependent on the mass value. Thus any changes in the mass of the plate lead to changes in these parameters.

The proposed method $L_f^2 h$ which is used to distinguish the value of $\ddot{y}$ compensates the effects of mass changes. Note that, parameters A_k, B_k, C_k and D_k are sufficient in finding the values of $L_f^3 h$ and $L_g L_f^2 h$. Even though both of these values are used to distinguish the control input, there's no need to correct these values according to the new mass value.

Compared with other introduced methods in the literature that needs correction of the parameters of the mass changes in the equations, in our proposed method there is no need for the correction. Correction of the parameters due to the mass changes requires a large amount of computational burden that prevents us from real time implementation of the control strategy. Note that in the proposed method, the correction of the value of mass in $L_f^2 h$ function is only needed. Now the system is simulated under the new condition, in which, the path is the same as before and some mass changes are applied:

a) The value of mass parameter in $L_f^2 h$ function, not being corrected;

b) The value of mass parameter in $L_f^2 h$ function, being corrected.

As it's been depicted in Fig. 7, when value of mass parameter in function $L_f^2 h$ is not corrected, the response of the system could not trace the reference value. Tracking wouldn't take place completely, even if big values are selected for parameters k_1, k_2 and k_3.

For instance, we set $k_1 = 70,\ k_2 = 1600,\ k_3 = 12000$ which is related to the case that the dynamic poles of the error system are fixed in 20, 20 and 30. As shown in Fig. 6. although the mass varies, the output of the system is able to track the reference value. Note that, correction is made only on $L_f^2 h$ function.

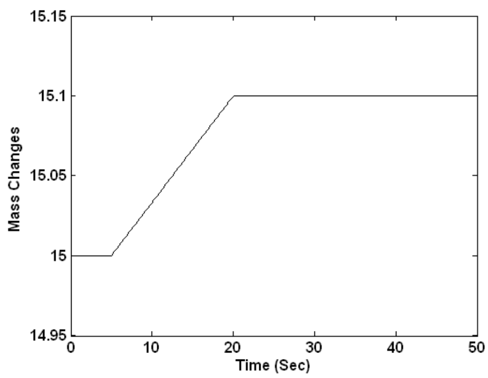

Fig. 6. Assumed mass changes

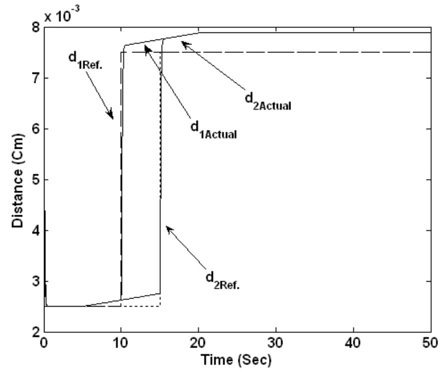

Fig. 7. The response of the system in assumed reference value tracking problem, considering mass changes

As mentioned earlier, in this case the tracking would be complete, even with considering small values for parameters k_1, k_2 and k_3. For instance we set $k_1 = 7,\ k_2 = 16,\ k_3 = 12$. The simulation result is shown in Fig. 8.

The effects of ball mass changes. None of the parameters of the system are changed as the mass of the ball changes but the value of $L_f^2 h$ function changes directly. Here, since the effects of two balls are the same, only the effects of the mass changes on one of the balls are considered.

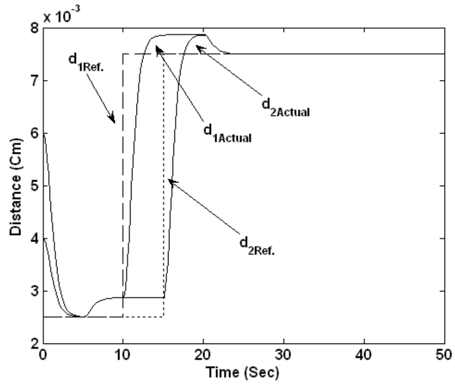

Fig. 8. The response of the system in assumed reference value tracking problem, considering mass changes

When the mass changes of the ball in finding the value of $L_f^2 h$ function is considered, there would be a complete tracking. But if such changes weren't taken into account, tracking wouldn't be complete even by estimating big values for parameters k_1, k_2 and k_3. To show this, two states are considered:

- When the value of the ball mass changes is not considered in finding the value of $L_f^2 h$ function;
- When the value of the ball mass changes is considered in finding the value of $L_f^2 h$ function.

Fig. 9 shows the mass changes in one of the balls. Figures 10 and 11 show the response of the system with and without considering the ball mass changes, respectively.

It should be noted that, practically, this assumed situation may not occur. This condition is considered just to show the capabilities of the control algorithm.

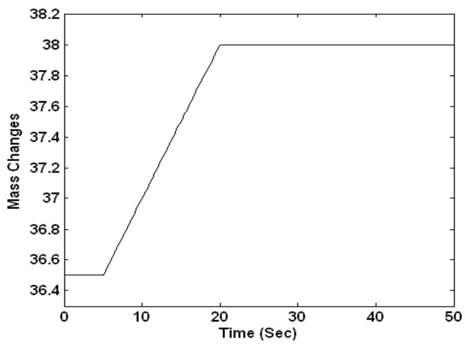

Fig. 9. Changes of mass

Including Disturbance in Input. Consider two random signals with homogenous distributions as input disturbance. Estimate the disturbance amplitude between -0.5 and 0.5. To decrease the effects of input disturbance, the values of parameters k_1, k_2 and k_3 must be large numbers. Here, we select these parameters as $k_1 = 35,\ k_2 = 400,\ k_3 = 1500$. The plate is supposed to be in its balance point: $[d_1, d_2]^T = [0.004, 0.006]^T$. In this control method, when noise relative to signal is increased, the system meets instability limits. Fig. 12 shows the output of the system in presence of input noise.

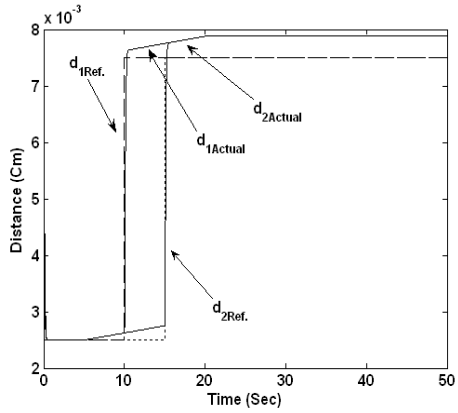

Fig. 10. The response of the system in presence of mass changes

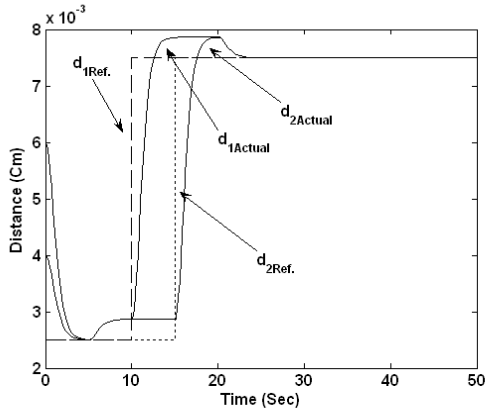

Fig. 11. The response of the system in presence of mass changes

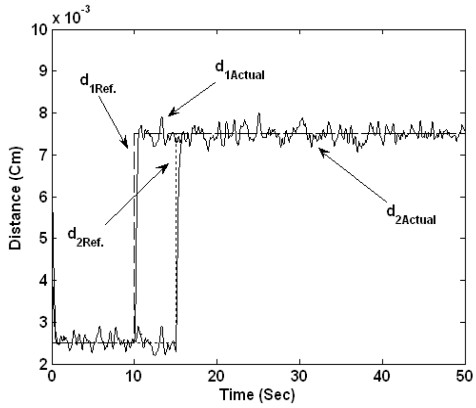

Fig. 12. The response of the system in presence of input noise

Including Noise in Output. The assumed system is severely sensitive toward measurement noise. As mentioned before, for high amplitude noises, system oscillates. So that the output might be lost in a noise signal. In order to decrease the noise effects, values of parameters k_1, k_2 and k_3 must be selected in large numbers. In this part, noise signal amplitude is estimated as 0.0001. Also values of parameters are taken as $k_1 = 35,\ k_2 = 400,\ k_3 = 1500$. To compare the results with the last chapter, suppose that the plate is placed on its balance point; $[d_1, d_2]^T = [0.004, 0.006]^T$. The reference signal is similar to that of Fig. 4. Fig. 5 shows the output of the system in presence of measurement noise. Note that, measurement noise cause incomplete tracking. Even though large numbers are selected for parameters k_1, k_2 and k_3 values. Of course it should be mentioned that assumed measurement noise relative to the output signals is considerable.

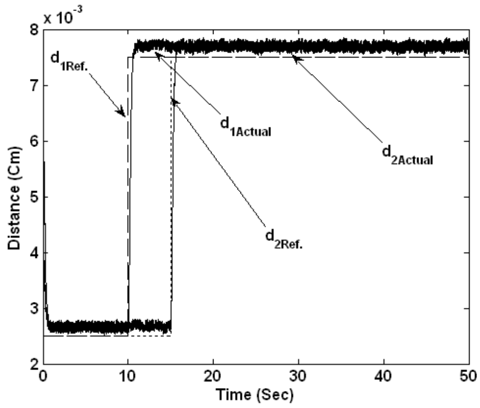

Fig. 13. the response of the system in presence of measurement noise

Conclusions

As mentioned earlier, this paper is presented in order to improve the work of De-Sheng and Jun Z., [3]. The system is severely sensitive toward the measurement noises. Such noises make it impossible to have a complete tracking. On the other hand, choosing large values for parameters k_1, k_2 and k_3 leads the system to considerably make up for the measurement noise and input noise effects. It also shows robustness against the presence of measurement noises which is the case in the considered plant where all sensors collect noise from the environment.

Accurate information about the plate mass changes and injecting such information into $\ddot{y}$ function can overcome the effects of mass changes so that there would be no need to enter such effects in $L_f^3 h$ and $L_g L_f^2 h$ functions.

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In this paper a feedback linearization control for double electromagnet suspension system is presented that addresses the coupling effects between two groups of electromagnetic trains. The controller has been developed based on feedback linearization and some reasonable assumptions of nonlinear mathematical rules. The proposed method in tracking has a satisfying performance in presence of unknown changes in the mass. It also shows robustness against the presence of measurement noises because sensors in plant collect noise from the environment. The simulation results show the capability of the proposed algorithm in the presence of input and output perturbation. Ill. 13, bibl. 7 (in English; abstracts in English and Lithuanian).

A. Maghsoudlou, R. Barzamini, K. D. Farahani. Dvigubos elektromagnetinės stabdymo sistemos grįžtamo ryšio linearizavimo netiesinėje kontrolėje taikymas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 2(108). – P. 85–90.

Pateiktas dvigubos elektromagnetinės stabdymo sistemos grįžtamo ryšio linearizavimo netiesinėje kontrolėje taikymas. Remiantis grįžtamo ryšio linearizavimu ir matematinėmis išraiškomis sudarytas valdiklis. Il. 13, bibl. 7 (anglų kalba; santraukos anglų ir lietuvių k.).