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Switched Semilogarithmic Quantization of Gaussian Source with Low Delay

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Introduction

Quantization denotes the heart of analog to digital (*A*/*D*) conversion, the process of approximating a continuous range of values by a set of discrete symbols (bits). Scalar quantizers are primarily used for *A*/*D* conversion, while vector quantizers are used for sophisticated digital signal processing. Signal compression is a procedure of digitalization of continuous signals, by using as few bits as possible, while truing at the same time to maintain a reasonable level of quality. The level of quality is usually measured by mean-square-error (quantization noise) or signal-to-quantization noise ratio (*SQNR*) [1]. In a number of papers the quantization of Gaussian source was analyzed since the probability density function (*PDF*) of the instantaneous speech signal values for lower number of digitalization samples is better represented by Gaussian then the Laplacean function [1]. The volume of speech signal also has Gaussian distribution Finaly, for appropriate design of filters, every continuous signal that is filtered, will have an approximately Gaussian distribution. Memoryless Gaussian source is commonly used and is important in many areas of telecommunications and computer science. The analysis of overcoming deficiencies and limitations of the existing signal compandors, when the input speech signal is modelled by the Gaussian distribution, was presented previously in [2]. Optimization of robust and switched nonuniform scalar quantization model is analyzed in [3], for the case when the power of an input signal varies in a wide range. In this paper, we have analyzed *A*-law characteristic for swiched non-uniform scalar quantization of Gaussian source. An A-law algorithm is a standard companding algorithm, used in European digital communications systems to optimize, i.e., modify, the dinamic range of an analog signal for digitizing.

An approach based on input signal's statistics for obtaining and optimizing companding characteristics, is presented. Further optimization of characteristic's parameters and number of quantization levels is performed in order to achieve higher level of quality, while attaining as higher as possible compression. Then this model has been compared with standard semilogarithmic *A*-law companding, in order to show the benefits of its applying. Capitalizing on this switched semilogarithmic quantization of Gaussian source, based on proposed model has been analysed. For the case when switching quantization techniques are considered, maximal achieved *SQNR* value is of significance than *SQNR* values which are obtained for smaller values of input signal variance. Which quantizer is used for frame encoding is transmitted as side information. When higher frame lengths M=160-240 are considered, side information of 5-6 bits, does not significantly affects average bit per semple rate so semilogorithmic companding low with optimal *A* value which provides maximal *SQNR* value can be used for this case. A paper [7] considered application of linear prediction coefficients, and [8] presented quality estimation of speech signal. They both oserved speech signal and frames that are much more longer then our´s. In this paper we are considering small frame lenghts M=10-20 samples (small encoder delay < 2.5 ms). That is why the main goal of this paper is determining compromise *A* parameter value, which will provide high *SQNR* quality for side information of 1 or 2 bits/frame. It is shown that this model provides higher quality of transmission and possibility of it applying for digitalization of continuous signals in wide range.

This paper is organized as follows: A general analysis of non-uniform scalar quantization and *A*-law companding is given in Section 2. We have developed expressions for granular and overload distortion, using Bennett`s integral on Gaussian distribution. Comparatione of A-low companding characteristics is also given in this section. In Section 3, nonuniform scalar switching quantization of Gaussian source, using *A*-low companding implementation is analysed. First the discussion is provided, about how *A* should be chosen, in order to obtain total distortion to be as minimal as possible in wide volume range of input signal. Then, we have presented how the increase of number of quantizers in switching scheme affects the SQNR dependence of input power. The optimization is made for one or more quantizers in the considered volume range based on total distortion. Dependence between bit rate per sample and frame length is also observed Finaly, we analised influence of codebook size and number on quality of transmission. Section 4 concludes the paper by summarizing the key features of the quantizer design and its applications. We have discussed obtained results, and on these bases, conclusions about the possibilities of this switched quantization application in speech processing are derived.

Scalar quantizer and semilogarithmic compandor

The N-point scalar quantizer $Q^{(N)}$ could be characterized by choosing a set of N real-valued quantization points $\{y_1^{(N)}, y_2^{(N)},..., y_N^{(N)}\}$, and N+1 decision thresholds $\{t_0^{(N)}, t_1^{(N)},..., t_N^{(N)}\}$. By relating decision thresholds as $-\infty= t_0^{(N)}< t_1^{(N)}<...< t_N^{(N)}<+\infty$, we define quantization rule as many-to-one mapping, $Q^{(N)}(x)=y_i$ if $t_{i-1}^{(N)}<x\le t_i^{(N)}$ for i=1, 2,..., N. In other words, a quantized signal has the value $y_i^{(N)}$ when the original signal belongs to the quantization cell $S_i^{(N)}=(t_{i-1}^{(N)}, t_i^{(N)}]$ for i=1, 2,..., N. Nonuniform quantization can be achieved in following way: first by compressing the input signal x using nonuniform compressor characteristic $c(x)$ (also called companding law); then by quantizing the compressed signal $c(x)$ employing a uniform quantizer; and finally by expanding the quantized version of the compressed signal using a nonuniform transfer characteristic $c^{-1}(x)$, which is inverse to the characteristic of the compressor. The overall structure, which consists of the compressor, the uniform quantizer, and the expandor, in cascade, is called compandor (Fig.1). In situations such as speech coding, the exact value of the input variance changes with time and is not known in advance. In cases like that, by using logarithmic companding law, we can obtain *SQNR* which is constant over a broad range of input variance.

The A-law companding is used for *PCM* systems in the Europe, with the standard value of A = 87.6, and A-law compression characteristic is given:

$$c_1(x) = \begin{cases} \frac{Ax}{1+lnA} sgn(x), & |x| < x_{min}, \\ \frac{x_{max}\left(1+ln\frac{Ax}{x_{max}}\right)}{1+lnA} sgn(x), & x_{min} < |x| < x_{max}. \end{cases} \quad (1)$$

For a source that is characterized as a continuous random variable with *PDF* $p(x)$ the N-point nonuniform scalar quantizer distortion could be defined as the expected mean square error between original and quantized signal. So the total distortion $D(Q)$ consists of two components, the granular $D_g(Q)$ and the overload $D_{ol}(Q)$ distortion

$$D(Q) = D_{g1}(Q) + D_{g2}(Q) + D_{ol}(Q), \quad (2)$$

with

$$D_{g1}(Q) = \frac{\Delta_1^2}{12} 2 \int_0^{x_{min}} \frac{p(x)}{|c\prime(x)|^2} dx, \quad (3)$$

$$D_{g2}(Q) = \frac{\Delta^2}{12} 2 \int_{x_{min}}^{x_{max}} \frac{p(x)}{|c\prime(x)|^2} dx, \quad (4)$$

$$D_{ol}(Q) = 2 \int_{x_{max}}^{\infty} \left(x - y_L(N)\right)^2 p(x)\, dx, \quad (5)$$

considering: $\Delta_1=2x_{min}/N_1$, $\Delta=2x_{max}/N$.

In this paper we consider the Gaussian input signals with unrestricted amplitude range. Determination of the support region enables quantizers to be adapted to the amplitudes of input signals. Gaussian probability density function of the original random variable x can be expressed

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right), \quad (6)$$

where x is zero-mean statistically independent Gaussian random variable of variance σ^2.

Substituting (1) and (6) into (3), (4) and (5), by approximating y_N with x_{max}, and taking into account that $x_{min}=x_{max}/A$, after some straightforward mathematical manipulations, we obtain:

$$D_{g1}(Q) = \frac{x_{max}^2}{3A^2N_1^2} erf\left(\frac{x_{max}}{\sqrt{2}\sigma A}\right), \quad (7)$$

$$D_{g2}(Q) = \frac{2\sigma^2(1+lnA)^2}{3\sqrt{\pi}N^2} \Gamma\left(\frac{x_{min}^2}{2\sigma^2}, \frac{3}{2}, \frac{x_{max}^2}{2\sigma^2}\right), \quad (8)$$

$$D_{ol}(Q) = \sigma^2 \left[\left(1 + \left(\frac{x_{max}}{\sigma}\right)^2\right)\left(1 - erf\left(\frac{x_{max}}{\sqrt{2}\sigma}\right)\right) - \right.$$
$$\left. - \frac{\sqrt{2}x_{max}}{\sqrt{\pi}\sigma} exp\left(-\frac{x_{max}^2}{2\sigma^2}\right)\right], \quad (9)$$

where $N_1=N/(1+\ln A)$ and good known functions:

$$\Gamma(z_1, a, z_2) = \int_{z_1}^{z_2} t^{a-1} exp(-t) dt, \quad (10)$$

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z exp(-t^2) dt. \quad (11)$$

Now by using well famous relationship between signal power and total distortion, *SQNR* can be efficiently *calculated from*

$$SQNR[dB] = 10log\frac{\sigma^2}{D(Q)}. \quad (12)$$

Let us compare the *SQNR* values obtained by using different value of parameter A for newly proposed model, with values obtained by using standard A-law companding. For example considering the case of N=256 and A=87.6, we obtain optimal value for A-law compounding *SQNR* of 38.17 dB, whereby N_1=46.77 and N_2=209.23. On the other hand observing a model whith A=11.86 and for the same case of N=256, we obtain *SQNR* value of 41.97 dB, whereby N_1=73.7 and N_2=182.3. The comparation is shown in Figure 2. Here it can be easily observed that novel proposed characteristic shows better performances in *SQNR* quality of 3.8 dB. We can see that newly proposed characteristic has better performances in range with high value of variances. If we are using one quantizer, A-law companding characteristic show better performances. But in our case, when we are using switched semilogarithmic quantization our proposed characteristic has an advance. Because of it's better performances, low delay and easier practical realization, we will now focus on novel proposed model. Transmition quality (*SQNR*), of our novel-proposed model for the case of N=256 in a wide dynamic input range of power for various values of A parameter, is presented in Fig. 3. We can observe how maximal reached SQNR value increase with A parameter decrease, while higher average

SQNR value in wider range of input variances is achieved with *A* parameter increase. Capitalizing on this *A* parameter can be chosen in order to achieve requested quality of *SQNR*, with respect to it's necessary robustness over a requested range of input variances.

Switched semilogarithmic quantization with low delay

We have solved the presented problem with switching quantization application. One simple technique is switched codebook adaptive scalar quantization. The quantization technique we propose in this letter utilizes the frame-by-frame manner to process the input signal. The basic scheme of robust and switched codebook adaptation is shown in Fig. 4. This technique uses a classifier that looks at the contents of the input frame buffer and decides that the next block of samples belongs to a particular statistical class of samples from a finite set of *K* possible classes. Namely, the index specifying the class is used to select a particular codebook from a predesigned set of *K* codebooks. In addition, this index is transmitted as side information to the receiver. Then, each sample in the block is encoded by the scalar quantizer, which performs a search through the selected codebook. One frame has length of M. The index to identify the class is sent on the end of block. If each of the *K* codebooks has size *N*, the bit rate per sample is

$$R = log_2 N + \frac{log_2 K}{M}. \qquad (13)$$

Codebook size *N* depends on number of bits that are used for the encoding *n*. The relation between *N* and *n* is $N = 2^n$, where *n* is the number of bits per sample. In Table 1, the bit rate per sample and *SQNR* in the function of frame length for different number of quantizers, when *N*=256 are shown.

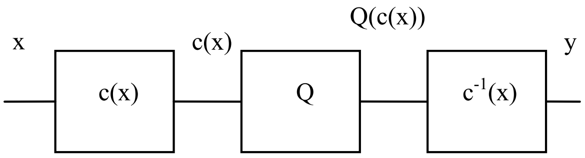

Fig. 1. Block diagram of non-uniform companding technique

For transmission systems with the main limitation in bit rate per sample and memory size, we should discuss the implementation of model with smaller number of quantizers and low delay. The main problem is acheving high quality of transmission by increasing *SNRQ*, in a wide range of signal volumes (variances) with respect to it's necessary robustness over a broad range of input variances. *SQNR* values for swiched semilogarithmic quantization technique based on proposed characetristic is presented on Fig. 5. Form that Fig.e we can see that our model outperforms standard *A*-law (*A*=87.6) for 1.23 dB and 3.22 dB, when we use 2 and 4 codebooks, with *A* = 15,58 and *A* = 42,47 respectivly. In Fig 5, we also can see that *SQNR* varies from it's peak value, for maximum 1.15 dB and 0.82 dB for each input power range $[\sigma^2_{1j}/\sigma^2_0, \sigma^2_{2j}/\sigma^2_0)$, $\cap^K_{j=1}$ $[\sigma^2_{1j}/\sigma^2_0$ [dB], $\sigma^2_{2j}/\sigma^2_{0j}$[dB])= [-20,20), for which the quantizer is designed, in case of codebook size of 256, when we use 2 and 4 codebooks, with *A* = 15,58 and *A* = 42,47 respectivly. However as it can be seen from Table 1 these SQNR values are acheived for the bit rates, which are negligibley higher than standard bit rate, considering chosen frame lenghts of *M* = 10 and *M* = 20.

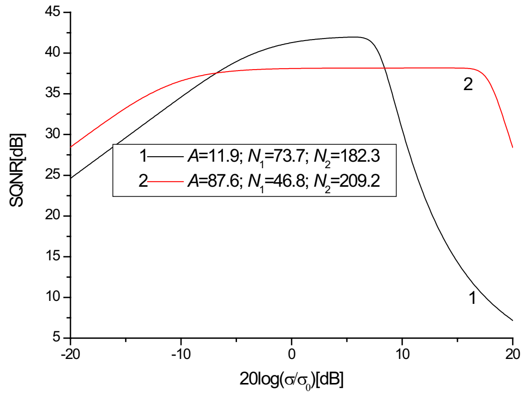

Fig. 2. Comparation of transmition quality (*SQNR*), between standard *A*-law companding characteristic and optimized *A*-law characteristic in a wide dynamic input range of power

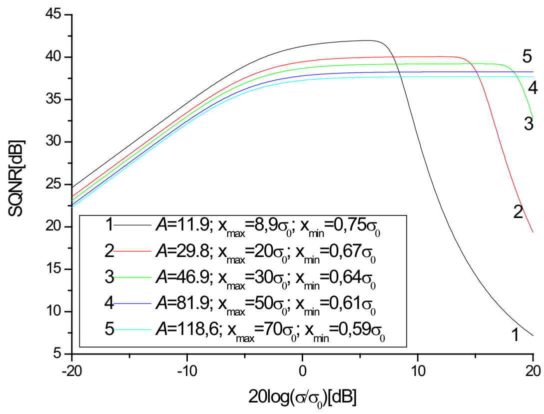

Fig. 3. Transmition quality (*SQNR*), of our novel-proposed *A*-law model for the case of *N*=256 in a wide dynamic input range of power

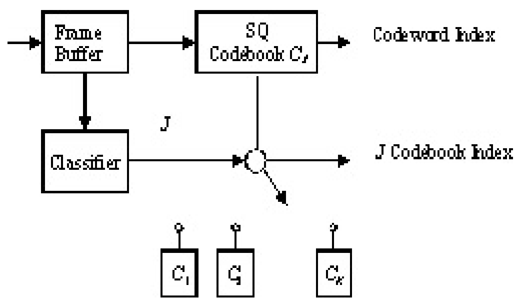

Fig. 4. Switched codebook adaptive scalar quantization

Table 1. The bit rate per sample and SQNR for *A*-law with one, two and four quantizers in a wide range of signal volumes for *N*=256.

	K	*M*	*R*	$SQNR_{max}$[dB]
A=87.6	1	1	8.0	38.17122
A=42.47	2	10	8.1	39.39963
A=15.58	4	20	8.1	41.39103

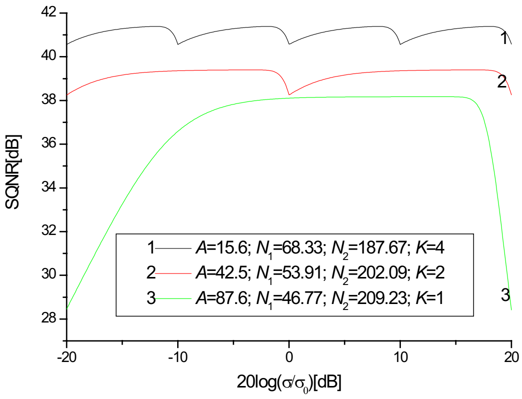

Fig. 5. Comparration of quality of transmission (*SQNR*), for *A*-law model implementations with one, two and four quantizers in a wide range of signal volumes

Conclusions

In this paper, we have presented and analyzed semilogarithmic *A*-law compounding characteristic for nonuniform scalar quantization of Gaussian source in a wide range of input signal's power. Discussion is provided, about how *A* should be chosen, in order to to satisfy transmission quality and necessary robustness requests. Capitalizing on this, switched semilogarithmic quantization of Gaussian source has been analyzed. The analysis of the codebook size and number of codebooks usage influence, on transmission quality and expected robustness is presented. It is shown that our model outperforms standard *A*-law model for 1.23 dB and 3.22 dB, when 2 and 4 codebooks are used, with $A = 42.5$ and $A = 15.6$, respectivly. Since compounding with proposed characteristic, produce accomplishing high quality of *SQNR*, for digitalized signal in a wide range of signal volumes (variances), with respect to it's necessary robustness over a broad range of input variances, so presented model can be applied for coding of speech signals.

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In this paper, a novel companding model based on nonuniform scalar quantization switching technique of Gaussian source is presented. *A* - law compandor implementation is observed. This scalar quantization model, with quantizers adapted to the maximal amplitudes of input signals, provides higher quality of signal-to-quantization noise ratio in a wide range of signal volumes, with respect to it's necessary robustness over a broad range of input variances. The main contribution of this model is achieving higher quality of transmission and possibility of its applying for digitalization of continuous signals in wide range. Ill. 5, bibl. 8, tabl. 1 (in English; abstracts in English and Lithuanian).

A. Mosic, Z. Peric, M. Savic, S. Panic. Mažai vėlinant perjungiamo pusiau logaritminio kvantavimo taikymas Gauso šaltiniui // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 2(108). – P. 71–74.

Apžvelgiamas mažai vėlinant perjungiamo pusiau logaritminio kvantavimo taikymas Gauso šaltiniui. Toks skaliarinis metodas su maksimaliai įėjimo signalų amplitudei pritaikytais kvantoriais gali būti taikomas aukštesnei perdavimo kokybei užtikrinti. Yra galimybė taikyti jį skaitmeniniu formatu plačiame diapazone. Il. 5, bibl. 8, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).