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Radiation of a Vertical Dipole Antenna over Flat and Lossy Ground: Accurate Electromagnetic Field Calculation using the Spectral Domain Approach along with Redefined Integral Representations and corresponding Novel Analytical Solution

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***Abstract*—In this paper we examine the problem of radiation from a vertical short (Hertzian) dipole above flat lossy ground, known in the literature as the 'Sommerfeld radiation problem'. Our formulation is in the spectral domain and ends up into simple one dimensional integral expressions for the received electromagnetic (EM) field, representing the exact solution of the problem. The problem can be solved analytically in an approximate sense in the high frequency regime using the Stationary Phase Method (SPM). In this paper the above spectral integrals for the received EM field are also mathematically represented as integrals over the 'grazing angle', a formulation that allows for a more accurate calculation since it avoids the singularities of the integrand expression. Also, a new SPM analytical solution, based on the above novel integral representation is obtained. Numerical comparisons between our SPM solution and the integral representations for the received EM field show that neither the horizontal Transmitter–Receiver distance, nor the frequency of operation are alone sufficient indicators regarding the most appropriate method to use (SPM or Numerical Integration). Instead, such a decision is to be based on their combined effect, given by their product $k \cdot r$ (electric distance).**

***Index Terms*—Sommerfeld radiation problem; spectral domain; grazing angle; stationary phase method; electric distance.**

I. INTRODUCTION

The 'Sommerfeld radiation problem' is a well-known problem in the area of propagation of electromagnetic (EM) waves above flat and lossy ground with applications in the area of wireless and mobile telecommunications [1]–[10]. The original Sommerfeld solution to this problem is provided in the physical space by using the 'Hertz potentials' and it does not end up with closed form analytical solutions. Subsequently, K. A. Norton [11] focused in the engineering application of the above problem and provided approximate solutions represented by rather long algebraic expressions, suitable for engineering use. In the above expressions, the so-called 'attenuation coefficient' for the propagating surface wave plays an important role.

In this paper, the authors advance on previous research work of theirs, concerning the solution of Sommerfeld's problem in the spectral domain. Namely, in [12] the fundamental integral representations for the received EM field were given. Furthermore, in [13], [14] the Stationary Phase Method was proposed (SPM, [15], [16]) and as a result novel, closed-form analytical expressions were derived, for use in the high frequency regime.

Moreover, in this article the authors elaborate more on the integral expressions of [12] – [14]. Particularly, it is shown that an appropriate selection of the integration variable and subsequent use of the SPM method lead to useful insights regarding the propagation mechanism. The expressions obtained are also more suitable for calculation purposes through numerical integration (NI) techniques, since some inherent singularities in previously derived integral representations are now removed, as shown in Section IV.

As stated in [17], [18] determining the necessary conditions for the applicability of the SPM method, an inherently *high frequency technique*, is essential. There, the issue was investigated for the practical case of transmitter – receiver pairs that are separated by a rather long distance, in which case it was apparent that for most frequencies of interest in the telecommunication area, frequency alone is a good criterion for the selection of the most appropriate method for EM field calculation at the receiver's point.

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However, in Section V the authors examine the above matter more thoroughly, *i.e.* for a variety of carrier frequencies and distance ranges including close distances, in which the usual far field asymptotic expressions, even for the Line of Sight (LOS) EM field, do not apply. Particularly, extensive simulations are run that compare the estimated received EM field at an observation point above flat and lossy ground under the following two approaches: (a) SPM method, (b) Numerical integration of the corresponding integral representations. Simulation results seem to be in accordance with overall EM wave theory. Namely, in most cases, it is not the value of frequency alone, but *the electric distance (k·r)*, the combined effect of frequency and distance, that actually determines whether EM field calculations should be based on the SPM method or evaluated through numerical integration of the respective integral expressions.

Finally, in Section VI, all important findings of this paper are summarized and suggestions are provided on how they can be utilized for the design and implementation of an efficient simulation tool for radio signal estimation. Additional implications for future research, triggered by our results so far, are provided as well.

II. PROBLEM GEOMETRY

The problem geometry is provided in Fig. 1. A vertical small (Hertzian) dipole, characterized by dipole moment p, is directed to the positive x axis, at altitude x_0 above infinite, flat and lossy ground. The dipole radiates time-harmonic electromagnetic (EM) waves at angular frequency $\omega = 2\pi f$. The relative complex permittivity of the ground is $\varepsilon_r' = £'/£_0 = £_r + i\pm/\mu£_0$, where σ is the ground conductivity, f is the carrier frequency and $\varepsilon_0 = 8{,}854 \times 10^{-12}$ F/m is the absolute permittivity in vacuum or air. Finally, the wavenumbers of propagation in the air and lossy ground, respectively, are given as follows:

$$k_{01} = \omega/c_1 = \omega\sqrt{\varepsilon_1\mu_1} = \omega\sqrt{\varepsilon_0\mu_0}, \tag{1}$$

$$k_{02} = \omega/c_2 = \omega\sqrt{\varepsilon_2'\mu_2} = k_{01}\sqrt{\varepsilon_r + \mathrm{i}(\sigma/\omega\varepsilon_0)}. \tag{2}$$

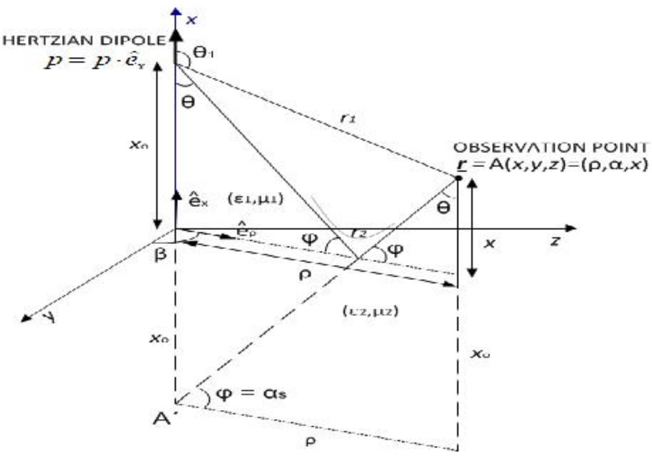

Fig. 1. Geometry of the problem.

Note that in Fig. 1 point A′ is the image of the source (Hertzian dipole) with respect to the ground (yz-plane), r_1 is the distance between the source and observation point, r_2 = (A′A) is the distance between the image and observation point, θ is the 'angle of incidence' at the so – called 'specular point' (which is the point of intersection of line (A′A) with the ground plane, and $\varphi = \pi/2 - \theta$ is the so – called 'grazing angle' [19].

III. INTEGRAL SPECTRAL DOMAIN REPRESENTATION FOR THE RECEIVED ELECTRIC FIELD AND CLOSED-FORM ANALYTIC EXPRESSIONS IN THE HIGH FREQUENCY REGIME

Following [13], [14] the following integral representation for the electric field at the receiver's position above the ground ($x > 0$), is derived

$$\underline{E}(\underline{r}) = \underline{E}^{LOS}(\underline{r}) - \frac{\mathrm{i}p}{8\pi\varepsilon_{r1}\varepsilon_0}\times$$
$$\times\left\{\hat{e}_\rho \int_{-\infty}^{\infty} k_\rho^2 \frac{\varepsilon_{r2}'\kappa_1 - \varepsilon_{r1}\kappa_2}{\varepsilon_{r2}'\kappa_1 + \varepsilon_{r1}\kappa_2} e^{i\kappa_1(x+x_0)} \times \mathrm{H}_0^{(1)}(k_\rho\rho)dk_\rho - \right.$$
$$\left. -\hat{e}_x \int_{-\infty}^{\infty} k_\rho^3 \frac{\varepsilon_{r2}'\kappa_1 - \varepsilon_{r1}\kappa_2}{\kappa_1(\varepsilon_{r2}'\kappa_1 + \varepsilon_{r1}\kappa_2)} e^{i\kappa_1(x+x_0)} \times \mathrm{H}_0^{(1)}(k_\rho\rho)dk_\rho\right\}, \tag{3}$$

where:

$$\begin{cases} \kappa_1 = \sqrt{k_{01}^2 - k_\rho^2}, \\ \kappa_2 = \sqrt{k_{02}^2 - k_\rho^2}, \end{cases} \tag{4}$$

and $\mathrm{H}_0^{(1)}$ is the Hankel function of first kind and zero order. Also note that in (3) ε_{r_2}' *is a complex quantity*, due to ground losses, as explained above. Similar expressions hold for the magnetic field [14], although only an azimuthal component is present for the magnetic field vector $\underline{H}$. Moreover, in (3), $\underline{E}^{LOS}$ is the Line of Sight (LOS) electric field vector, whose expression can be given either in an integral form [12] or via the following direct expressions in cylindrical coordinates [15], [19]

$$\underline{E}^{LOS}(\rho, \mathrm{x}) = -\frac{\mathrm{i}\mu \cdot p}{4\pi}\cdot e^{\mathrm{i}k_{01}\cdot r_1}\times$$
$$\times\left\{\left(\frac{-\mathrm{i}\omega\mu_1}{2r_1} + \frac{3\zeta}{2r_1^2} - \frac{3}{2\mathrm{i}\omega\varepsilon_1 r_1^3}\right)\sin 2\theta_1 \times \hat{e}_\rho + \right.$$
$$+\left[\frac{\mathrm{i}\omega\mu_1}{r_1}\sin^2\theta_1 + \left(\frac{\zeta}{r_1^2} - \frac{2}{\mathrm{i}\omega\varepsilon_1 r_1^3}\right)\times\right.$$
$$\left.\left.\times(\cos 2\theta_1 + \cos^2\theta_1)\right]\times \hat{e}_\mathrm{x}\right\}. \tag{5}$$

In (5), $\zeta = \sqrt{\mu_1/\varepsilon_1}$ = 377Ω is the free space impedance and parameters r_1, θ_1 are shown in Fig. 1. Note that these parameters are related to the observation point's cylindrical coordinates (ρ,x) through the following expressions:

$$r_1 = \sqrt{\circ^2 + (x - x_0)^2}, \tag{6}$$

$$\begin{cases} \bar{}_1 = ® - \arctan(\frac{\circ}{x_0 - x}), & x < x_0, \\ \bar{}_1 = \arctan(\frac{\circ}{x - x_0}), & x > x_0. \end{cases} \tag{7}$$

Also, note that (5) holds for every distance r_1 (*i.e.* either large or small, with the only assumption that r_1 is much larger than the Hertzian dipole length) between the transmitter and the receiver. For the usual case of large distances, only the $1/r_1$ terms dominate, thus leading to the well-known far–field expressions for the electric field of a radiating Hertzian dipole in free space [19].

Application of the 'Stationary Phase Method' (SPM) to (3) and (4), leads to the following analytic expressions for the electric field vector scattered from the plane ground, in the far field region and in the high frequency regime (for $x > 0$) [17], [18]

$$\underline{E}_{x>0}^{SC} = \frac{p}{4\pi\varepsilon_0\varepsilon_{r1}\rho^{1/2}\left(x+x_0\right)^{1/2}} \times \frac{\kappa_{1s}^{1/2}k_{\rho s}^{3/2}}{k_{01}} \times$$
$$\times \frac{\varepsilon'_{r2}\kappa_{1s} - \varepsilon_{r1}\kappa_{2s}}{\varepsilon'_{r2}\kappa_{1s} + \varepsilon_{r1}\kappa_{2s}} \times e^{ik_{\rho s}\rho} \times e^{i\kappa_{1s}(x+x_0)} \times$$
$$\times(-\kappa_{1s}\hat{e}_\rho + k_{\rho s}\hat{e}_x) = \frac{pk_{01}\cos\varphi}{4\pi\varepsilon_0\varepsilon_{r1}(\mathrm{A'A})} \times \frac{\varepsilon'_{r2}\kappa_{1s} - \varepsilon_{r1}\kappa_{2s}}{\varepsilon'_{r2}\kappa_{1s} + \varepsilon_{r1}\kappa_{2s}} \times$$
$$\times e^{ik_{\rho s}\rho} \times e^{i\kappa_{1s}(x+x_0)} \times (-\kappa_{1s}\hat{e}_\rho + k_{\rho s}\hat{e}_x). \tag{8}$$

In (8), the following expressions hold [17], [18]:

$$k_{\rho s} = \frac{k_{01}\,\rho}{\sqrt{\left(x+x_0\right)^2+\rho^2}} = \frac{k_{01}}{\sqrt{1+\left(\dfrac{x+x_0}{\rho}\right)^2}} = k_{01}\cos\varphi, \tag{9}$$

$$\begin{cases} \kappa_{1s} = \sqrt{k_{01}^2 - k_{\rho s}^2} = k_{01}\sin\varphi, \\ \kappa_{2s} = \sqrt{k_{02}^2 - k_{\rho s}^2}, \end{cases} \tag{10}$$

with $k_{\rho s}$ being the stationary point obtained from the SPM method.

IV. Redefined Integral Representations and Corresponding SPM Analytic Solution for the Received EM Field. Advantages and Implications of the New Approach

We now introduce the following variable transformation in the integral expression of (3)

$$k_\circ = k_{01}\cos\sim. \tag{11}$$

Note that according to (4) and (11), the following expressions hold:

$$\begin{cases} ©_1 = k_{01}\sin\sim, \\ ©_2 = \sqrt{k_{02}^2 - k_{01}^2\cos^2\sim}. \end{cases} \tag{12}$$

Substituting in (3) the following formula is obtained for the total received electric field vector

$$\underline{E}\left(\underline{r}\right) = \underline{E}^{LOS}\left(\underline{r}\right) - k_{01}^3\frac{ip}{8\pi\varepsilon_{r1}\varepsilon_0} \times$$
$$\times\int_0^{®}\left\{\begin{array}{l} \cos^2\sim\dfrac{\varepsilon'_{r2}k_{01}\sin\sim - \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2\cos^2\sim}}{\varepsilon'_{r2}k_{01}\sin\sim + \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2\cos^2\sim}} \times \\ \times e^{ik_{01}(x+x_0)\sin\sim} \times \\ \times(\hat{e}_\rho\sin\sim - \hat{e}_x\cos\sim)\cdot \mathrm{H}_0^{(1)}(k_{01}\rho\cos\alpha)d\sim \end{array}\right\} +$$
$$+\Delta, \tag{13}$$

where Δ is the error arising from the fact that actually only the $(-k_{01},\ k_{01})$ range is considered in the above integration, since according to (11), using the selected transformation variable, it is assumed that $|k_\rho| \le k_{01}$. In other words, Δ can be expressed by the following formula

$$\Delta = \int_{-\infty}^{-k_{01}} \underline{g}(k_\circ)dk_\rho + \int_{k_{01}}^{\infty} \underline{g}(k_\circ)dk_\rho, \tag{14}$$

with $\underline{g}(k_\rho)$ being the integrands of (3). Its behaviour is further analysed below.

Examining (13) in more detail, an important note can be made at this point: in the present formulation, where variable α is introduced as the variable of integration, instead of k_ρ, no singularity exists for the integrand within the $(0,\ \pi)$ range of α. This is not the case in (3), where singularities exist at points $k_\rho = \pm k_{01}$, leading to a major problem for our numerical integration procedure, as mentioned in [18]. This appears to be an important advantage of the new proposed formulation, as also described in Section VI below (future research).

Now, according to [14], in order to apply the SPM method, we use the large argument asymptotic approximation of the Hankel function, which in our case is written as

$$\mathrm{H}_0^{(1)}(k_{01}\times\cos\sim\times\rho) = \sqrt{\frac{2}{\pi k_{01}\rho\cos\alpha}} \times$$
$$\times e^{-\mathrm{i}\frac{\pi}{4}} \times e^{\mathrm{i}k_{01}\times\rho\times\cos\alpha}. \tag{15}$$

Substituting (12) and (15) in (13), the following expression is obtained, after some rather lengthy, but otherwise straightforward calculations

$$\underline{E}\left(\underline{r}\right) = \underline{E}^{LOS}\left(\underline{r}\right) - k_{01}^3\frac{\mathrm{i}p}{8\pi\varepsilon_{r1}\varepsilon_0}\sqrt{\frac{2}{®\,k_{01}\,°}} \times e^{-\mathrm{i}\frac{\pi}{4}} \times I, \tag{16}$$

where

$$I = \int_0^{®}\left\{\begin{array}{l} (\cos\sim)^{\frac{3}{2}}\dfrac{\varepsilon'_{r2}k_{01}\sin\sim - \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2\cos^2\sim}}{\varepsilon'_{r2}k_{01}\sin\sim + \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2\cos^2\sim}} \times \\ \times e^{\mathrm{i}k_{01}r_2\cos(\sim-\,¨)} \times (\hat{e}_\rho\sin\sim - \hat{e}_x\cos\sim) \times d\sim \end{array}\right\}, \tag{17}$$

In (17) above, angle φ and distance r_2 are shown in Fig. 1

(and described in detail just below Fig. 1). It is also apparent that the following obvious geometrical relations hold:

$$\begin{cases} r_2 = \sqrt{z^2 + (x + x_0)^2}, \\ z = r_2 \cos\varphi, \\ (x + x_0) = r_2 \sin\varphi. \end{cases} \tag{18}$$

Note that (16)–(17) do not include the error factor Δ of (13)–(14). This is due to the fact that according to the SPM technique, which will be applied just below, only a small region around the so – called ‘stationary point’, is contributing to the overall integral calculation [15], [16]. According to (9), this stationary point (SP) lies within the ($-k_{01}$, k_{01}) range of k_ρ, which with the transformation of (11) it is mapped into the (0, π) range of variable α. In other words, in the high frequency regime, and for a grazing angle φ (Fig. 1) not very close to zero (so that the SP is not very close to the boundary value of integration α = 0, see application of the SPM method just below), the error Δ of (13) and (14) is almost zero, *i.e.* Δ ≈ 0.

Now, according to the SPM method [15], [16] we define the phase $k_{01} \cdot r_2$ in (17) as a ‘large parameter’. Then the phase and amplitude function for (17) are:

$$f(\alpha) = \cos(\alpha - \varphi), \tag{19}$$

$$F(\alpha) = \begin{pmatrix} (\cos\alpha)^{\frac{3}{2}} \dfrac{\varepsilon'_{r2} k_{01} \sin\alpha - \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2 \cos^2\alpha}}{\varepsilon'_{r2} k_{01} \sin\alpha + \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2 \cos^2\alpha}} \times \\ \times (\hat{e}_\rho \sin\alpha - \hat{e}_x \cos\alpha) \end{pmatrix}. \tag{20}$$

According to [15], [16], the stationary point is obtained by solving $f'(\alpha) = {df}/{d\alpha} = 0$. Hence, from (19) it is obtained

$$\alpha_s = \varphi, \tag{21}$$

while the second derivative at α_s is: $f''(\alpha_s) = -1 < 0$.

Then, according to the SPM method, integral (17) is calculated as [16]

$$I = F(\alpha_s) \times e^{i k_{01} r_2 f(\alpha_s)} \times \sqrt{\frac{2\pi}{k_{01} r_2 \left| f''(\alpha_s) \right|}} \times e^{i\frac{\pi}{4}\mathrm{sgn}[f''(\alpha_s)]}. \tag{22}$$

Consequently, by using (22) for the SPM calculation of (16), (17), we finally obtain, after some rather simple and straightforward calculations, the following expression for the total received electric field

$$\underline{E}(\underline{r}) = \underline{E}^{LOS}(\underline{r}) - \hat{e}_\theta \times k_{01}^2 \frac{p}{4\pi\varepsilon_{r1}\varepsilon_0} \times \frac{\varepsilon'_{r2} k_{01} \sin\varphi - \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2 \cos^2\varphi}}{\varepsilon'_{r2} k_{01} \sin\varphi + \varepsilon_{r1}\sqrt{k_{02}^2 - k_{01}^2 \cos^2\varphi}} \times \sin\theta \times \frac{e^{i k_{01} r_2}}{r_2}, \tag{23}$$

where (Fig. 1), $\theta = \frac{\pi}{2} - \phi$ and

$$\hat{e}_\theta = \hat{e}_\rho \cos\theta - \hat{e}_x \sin\theta. \tag{24}$$

The above SPM closed – form analytic result was obtained under the ‘high frequency approximation’ assumption, as well as the assumption that the ‘grazing angle’ φ is *not* very close to zero, as already explained. Under these assumptions, (23) just represents the summation of the LOS field and the field reflected from the ‘specular point’ [note that the fraction in (23) just represents the usual ‘Fresnel reflection coefficient’, [14], [18]).

V. NUMERICAL RESULTS: COMPARISON OF SPM WITH NUMERICAL INTEGRATION TECHNIQUES

In this section, we expand on the simulation results of [17], [18]. Indeed, in [18] the authors identified frequency areas where it was not apparent which method, SPM or Numerical Integration, is the most appropriate to use for EM field estimation. Moreover, only long distance scenarios were examined, which although of practical importance for telecommunication applications they may hinder important factors that affect the applicability of the SPM formulas. Hence, the results presented hereafter are expanded to cover short distance field calculations and at the same time an attempt to further decrease the convergence tolerances of the integral expressions was made, as given below. Finally, figures for all field components are shown, *i.e.* LOS field, Scattered field, Total field, whereas in [18] results pertained to the reflected field behaviour. A description of the simulation process is given in [17]. Table I summarizes the basic parameter set used in our simulations. Also, note that the numerical integration results, presented hereafter, refer to the original integral format expressed by (3). Producing similar results for (13) is left for future study, as mentioned in Section VI. In addition, although all expressions, given in Sections III & IV refer to the electric field, for completeness simulation results are shown for both the electric and the magnetic field.

TABLE I. SIMULATION PARAMETERS.

Symbol	Description	Value
f_{min}	Minimum Frequency	30 KHz
f_{max}	Maximum Frequency	3GHz
x_0	Height of transmitting dipole	60 m
X	Height of observation point (receiver)	15 m
I	Current of the radiating Hertzian dipole[1]	1 A
$2h$	Length of the Hertzian dipole[2]	0.1 m
ε_r	Relative dielectric constant of ground	20
Σ	ground conductivity	0.01 S/m
	Short distance range	10m – 500m
	Long distance range	1km – 5km
	Numerical integration technique	Adap. Simpson [14]
	Excluded interval around singularity k_{01}	0.00001 · k_{01}
	Tolerance for integral convergence	10^{-9} – 10^{-6}

Notes: [1] Relation between current I and dipole moment p: $I(2h) = -i\omega p$, where $\omega = 2\pi f$ and i is the unit imaginary number;
[2] Much smaller than the wavelength $\lambda = c/f$ in both cases

The objective of these comparisons is to identify the required conditions, which permit the use of the closed form SPM expressions for EM field calculations above flat lossy ground.

The first set of simulation results refers solely to the free space EM field. Figure 2 depicts the convergence of the exact formulas (5)–(7), which are applicable for every distance from the transmitting source, to their corresponding far field asymptotic approximations, commonly found in the literature [19]. As expected, the convergence is achieved faster at higher frequencies. For the problem considered here, which covers distances of up to 5 km, it is possible to have significant deviations for even large distances, as shown in Fig. 2(a) and Fig. 2(b). The implication for this is that under these circumstances, SPM analytic solution, that is (23) above, cannot be accurate. The reason is that (23) essentially indicates the propagation of spherical (and for large distances plane) waves, whilst according to (5)–(7) this is not the case (dependence on higher orders of *1/r*). However, close proximity between the exact expressions (5) to their respective far field counterparts, does not necessarily guarantee the use of the SPM method, as will be explained below.

The core of our simulations is shown in the graphs of Fig. 3, which are related solely to the scattered field. Particularly, the reflected EM field, as estimated by the SPM method, is compared to the respective field values obtained through the numerical evaluation of the corresponding integral expressions. In other words, (3) is juxtaposed with (8), having beforehand excluded the LOS contribution from (3).

From Fig. 3(a)–Fig. 3(b) it is apparent that at 30 KHz and 100 KHz the SPM method significantly underestimates the signal level and this is true for both the short and long distance range of Table I (the gap between the two curves tends to increase for shorter distances). Similar behaviour is observed for all frequencies up to 1 MHz. At that frequency, *i.e*. at 1 MHz, Fig. 3(c) indicates that it takes about 4.8 km for the SPM values to match the numerical integration results, a distance which is equivalent to 16 λ. At 10 MHz (Fig. 3(d)), NI results outbalances SPM values for up to 350 m, or alternatively for about 13 λ–14 λ away from the source. At even higher frequency ranges this observation is essentially retained. Indeed, as seen in Fig. 3(e), SPM provides better estimates after about 150 m, *i.e*. 15 λ. This is also true at 100 MHz, shown in Fig. 3(f). Now the breakpoint distance (the reason for the term will be explained below), after which SPM behaves better than NI, seems to be around 55 m or about 18 wavelengths away from the source. The bottom line is that, contrary to [17], [18], the applicability of the SPM method does not seem to be solely determined by the frequency of operation. Instead, it is the combined effect of frequency and distance from the source that dictates its accuracy. This can be expressed either in terms of a distance, which is relative to the wavelength of radiation, or equivalently by the electric distance, $k\cdot r$. According to our simulation results a good reference point above which field values can be accurately determined by (22) is ~16 λ. In terms of the electric distance this is equivalent to 32π.

To all fairness, the previously argued conclusion of ours, does not completely cancel the findings of [17], [18]. Indeed, at [17] and [18] the authors examined solely far distance ranges, which is why the above illustrated combination effect between distance and frequency was not easily deceivable. Moreover, returning to Fig. 3(f), it is observed that before the breakpoint distance, SPM and NI results are eventually close to each other. This ultimately means that at 100 MHz and higher, the SPM method has global applicability and can be used for almost every distance of interest (or at least after 10 m which was our minimum distance in our simulations). This justifies in practice the characterization of the SPM method as a high frequency approximation technique, which was one of the major arguments of [17], [18].

Overall, our simulation results confirm at every means the approximations and overall mathematical analysis of our redefined SPM formulations, presented in Section IV. Recall that at that section, the used approximations for the derivation of the SPM formula were in the form of an electric distance formula, $k\cdot r$ ($k_{01}\cdot\rho$ for the large argument approximation of the Hankel function, $k_{01}\cdot r_2$ for the large parameter of the SPM phase factor). This is a major advantage over the previous SPM process given in [14], in which the large parameter of the SPM phase factor was solely ρ. As already mentioned, our simulation results do agree with this new formulation.

An important statement regarding the interpretation of the results presented in Fig. 3 must be made. Particularly, the used SPM method is an asymptotic approximation technique to the evaluation of the integral expression (3). Put it differently, when the conditions for SPM are valid, the results obtained through its application should be close enough to the respective ones given by (3), with the closeness becoming better as the distance ρ increases (since the electric distance also increases). While this is the case for small frequency sets (Fig. 3(a)– Fig. 3(c)) at higher frequencies the numerical integration results seem to be missing significant contributions. The reasons for this are explained in detail in [18] and seem an inevitable consequence of the presence of the singularity at $k_\rho = k_{01}$, which required for a sufficient range around the singular point to be excluded from the integration interval. This range appears more significant at higher frequencies, as explained in [18]. This now justifies the use of the term breakpoint distance introduced before. It is the distance above which, in practice, SPM actually provides better estimates than the exact integral expressions it tries to approximate and hence at those occasions signal level estimation should be based solely to SPM values.

Finally, in Fig. 4 we show the complex interference of the direct and scattered field using both approaches, SPM and NI. Since the LOS component is common to both methods, expression (5) above, the proper estimation for the total field follows the same rules used for the reflected field. Therefore, at a certain frequency, use the NI results for field calculations up to the breakpoint distance, whilst switch to the SPM when the observation point is further away. Especially at frequencies above 100 MHz and for the reasons given above, SPM may be the selection of choice for every distance. This is depicted in Fig. 5. At such high frequencies, the relative complex permittivity becomes almost real. As a result, the effect of the Brewster angle (θ_B) appears (particularly, this is the so – called in the literature pseudo Brewster angle [19]) in which the reflected field almost disappears. From Fig.3(f) it is evident that NI fails to describe this phenomenon, which is another indication on why NI should not be used for distances longer than the breakpoint distance and this is particularly true for large frequencies.

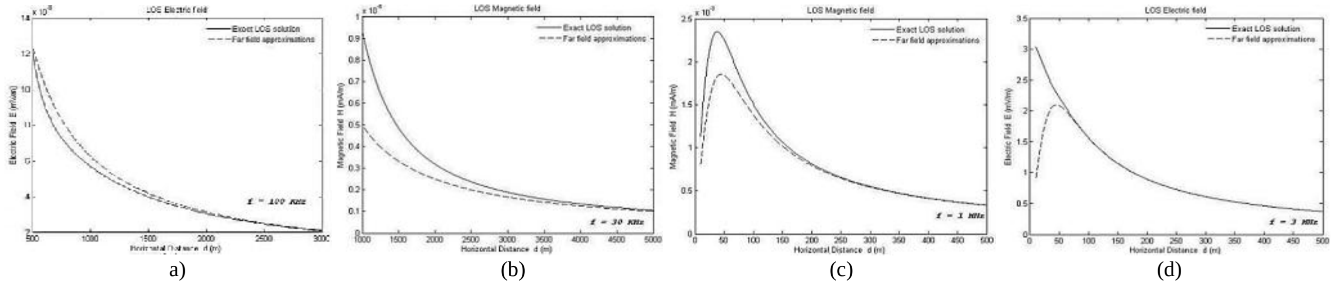

Fig. 2. Line of sight (LOS) EM field comparisons (exact formulae vs. far field approximations).

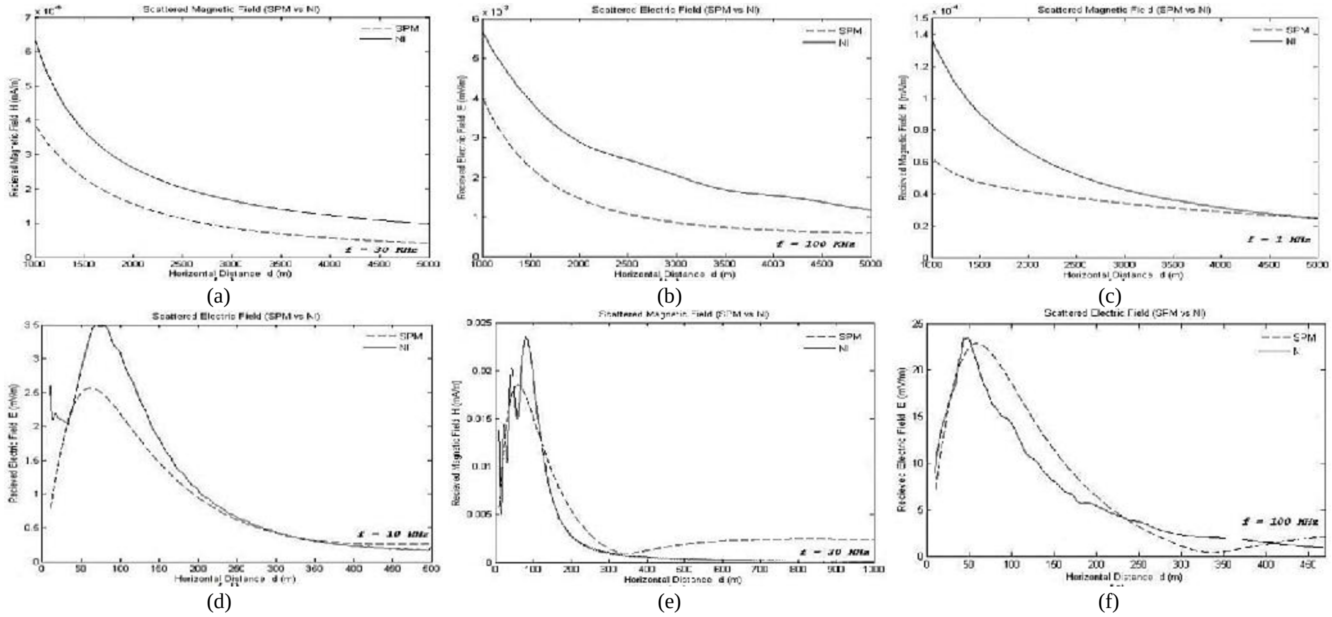

Fig. 3: Scattered EM field comparisons (SPM method vs. NI method).

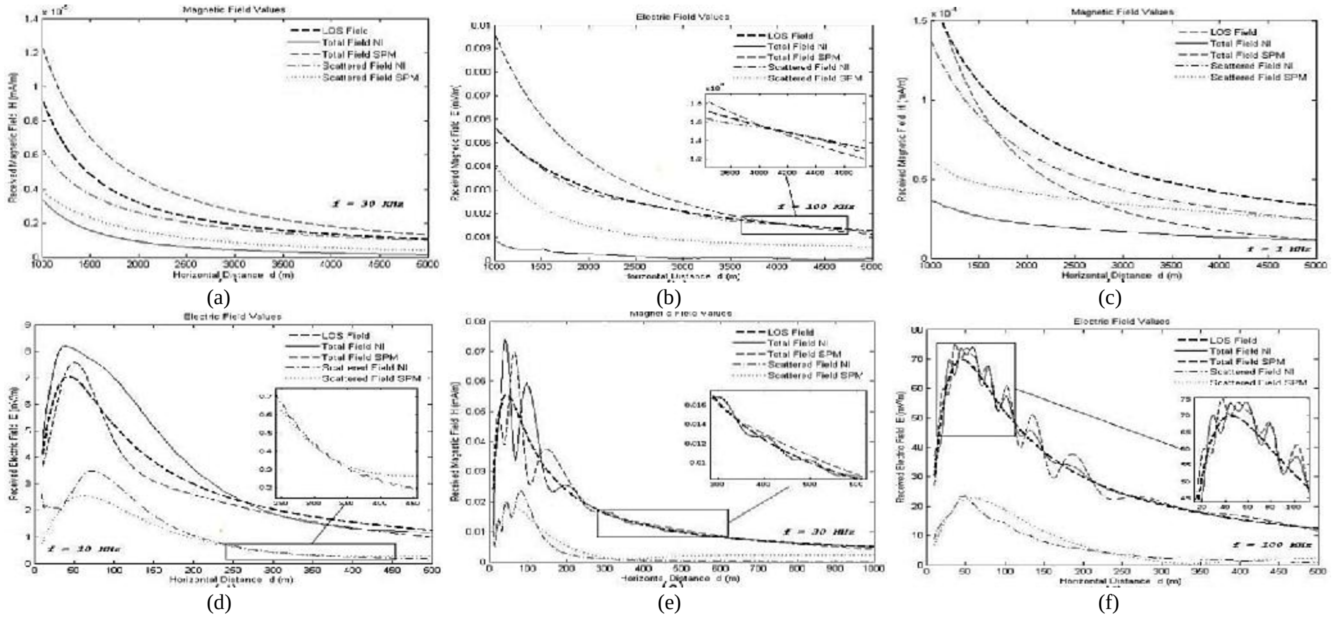

Fig. 4. EM field components.

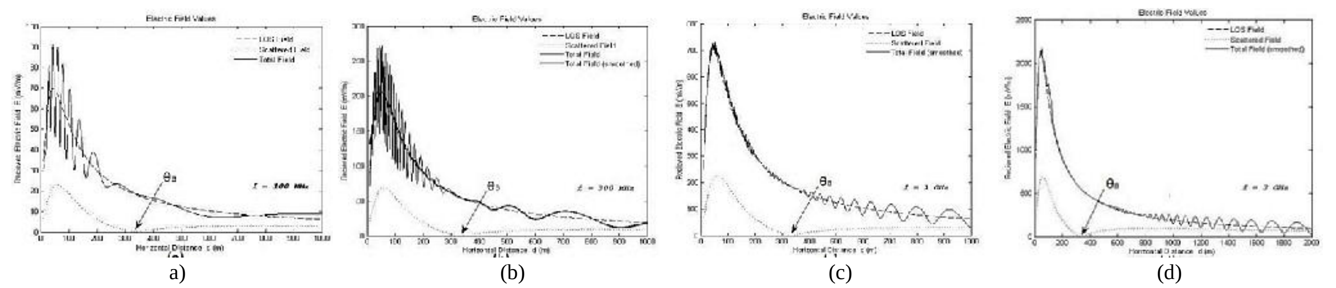

Fig. 5. LOS field, scattered field and total field behavior (high frequency regime).

VI. CONCLUSIONS

In the present article we re-examined the solution of Sommerfeld's radiation problem in the spectral domain, initially studied by our research team in [12]–[14]. Particularly, in Section IV, after introducing a new variable of integration to our spectral integrals, we ended up with integral formulas, which exhibit important advantages over our corresponding previous expressions of [17]–[18] (part of which is (3), given above). In Section V, we presented extensive simulation results for the received EM field and for a variety of distances between transmitter and receiver, as well as for a wide range of frequencies of interest. We concluded that after a sufficiently large, relative to the wavelength, distance from the transmitting antenna the problem can be ultimately solved in an analytical manner through the use of the SPM method. When these conditions are met, the propagation mechanism is essentially according to the ray optics theory, since, based on (23), the total received field is just the summation of the LOS field and the field reflected from the 'specular point'. The simulation results show that the requirements for this relative distance can be as small as 16 wavelengths or in terms of the electric distance, 32π making thus the SPM method rather suitable for most contemporary telecommunication applications. Even more, at rather high frequencies (at leastin the VHF area and above), this behaviour has essentially global spatial significance.

Today, there exist several software simulation tools for radio signal-level estimation, both open source and license based [21]–[23]. These tools are broadly classified to general scope simulators and specific simulators and essentially utilize a combination of theoretical approximation models (*e.g.* 2 ray model) and empirical ones (*e.g.* the well-known Hata-Okumura) for predicting signal loss. With the advance of IT technology it is now possible to perform calculations on the basis of numerical integration techniques, however great care is needed regarding the error they may incur [23]. At any means, the advantages of using accurate analytical models are more or less evident and described in more detail in [17], [18]. As such, our SPM closed-form solution appears a meaningful choice for a software (SW) simulation package used for radio coverage prediction since, as shown throughout this paper, it is applicable for most practicable use cases.

Corresponding future research our research group will focus on the major findings of our new formulation procedure presented in Section IV. Particularly, numerical results for the newly derived integral expression (13) will be produced and compared with the corresponding SPM estimates. It is expected that the new NI results will not expose the errors described in Section V, leading thus to a fully accurate method for signal-level estimation at almost every frequency – distance combination. Moreover, from a theoretical point of view, we intend to further examine the behaviour of (13) and (14) (particularly as far as the error factor Δ is concerned), which our research group believes that may reveal important findings regarding the nature of the propagation mechanism.

Finally, further future research work of ours will focus on the solution of Sommerfeld's radiation problem for the case of a *horizontal* dipole. Moreover, another possible area of investigation is the calculation of the received EM field above or below the ground, for *any frequency of the radiating dipole*, in an exact and analytical manner [24].

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