

## **Analytical Estimation of Self-Similar Wireless Traffic Relaxation Time and Hurst Coefficient Dependence**

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### **Introduction**

Wi-fi networks recently became very popular in home networking and SOHO segment for internet access. Many ISPs install IEEE 802.11 hardware as CPE because it's cheap, fast enough and allows easy last mile network deployment in most situations where cable installation cost is high or even impossible. Nevertheless that wi-fi networks were designed for static use there are several projects and researches trying to use them for mobile users. Newer 802.11n standard with it's MIMO technology, increased working range and data throughout make these tries more real. Of course there are special standards as 802.11r and 802.11p but those use licensed frequency ranges and thus, potentially, less interesting to mass ISPs than 802.11a/b/g/n working in ISM frequency band. But here we will not consider what technology or standard is better, instead we consider an abstract wireless network model which, possibly, could be applied not only to wireless networks [1–6].

A notable feature of transport wireless networks is that AP (Access Point or server in further considerations) communicates with several mobile clients (vehicles) simultaneously. A group of vehicles appearing in server working range causes an abrupt increase of server load [1]. Server itself has limited bandwidth (e.g. 54 Mbps in 802.11g network) and processing capacity (CPU is pretty slow in most servers and RAM can hold limited number of connections). It's obvious that server is not able to process all simultaneously arriving requests not to say that the air is half duplex media. Thus some mobile clients have to wait for their requests to be processed while others are communicating with server. This significantly slows down processing of client data. Considering the fact that a vehicle stays in server coverage for a pretty short time (about 20 seconds if vehicle speed is 100 km/h and server coverage diameter is 300 meters) such processing delays can affect the whole network. Further increase of incoming connection requests can overflow buffer memory and thus we'll have packet loss and unreliable service as a result.

Above described facts resulted in raising the problem of investigating the transient time or relaxation time – the time needed for a system to enter stationary mode – of a system with wireless access under an abrupt load jump. Analytical solve for this problem is possible only if incoming request flow will have Poisson characteristics and request processing time will obey an exponential distribution as in M/M/1 system. But according to our previous research on wireless network traffic it appeared that packet flow arriving at server is self-similar [2]. Thus the only way to analyze server behavior in transient mode under self-similar traffic is imitation modeling and further analytical analysis.

The aim of this article is to find which analytical equation can be used to find relaxation time if we know server load and self-similarity degree of input traffic.

### **Modeling and results**

In [1] we have modeled a pair of open loop systems with one processing device or server. First system was plain old M/M/1 but the other had self-similar input generated with the help of Pareto distribution and was called P/M/1. We used Hurst parameter as a well known degree of self-similarity [2]. M/M/1 system modeling was compared with the result given by formula from which gives mean number of requests in the system at any given moment of time. The comparison showed high accuracy in finding relaxation time of such system allowing us to rely on our modeling results of the second system with self-similar input. The intensity of arrivals was adopted for both systems to be equal for honest comparison.

Modeling results show the mean number of requests in the system as time goes (Fig. 1).

It's clearly seen that queue size is bigger and relaxation time is longer for system with self-similar input traffic even under such not stress condition.

Collecting modeling results of P/M/1 system with different load and Hurst parameters we get Fig. 2. where we already can see how relaxation time changes depending on server load and self-similarity degree.

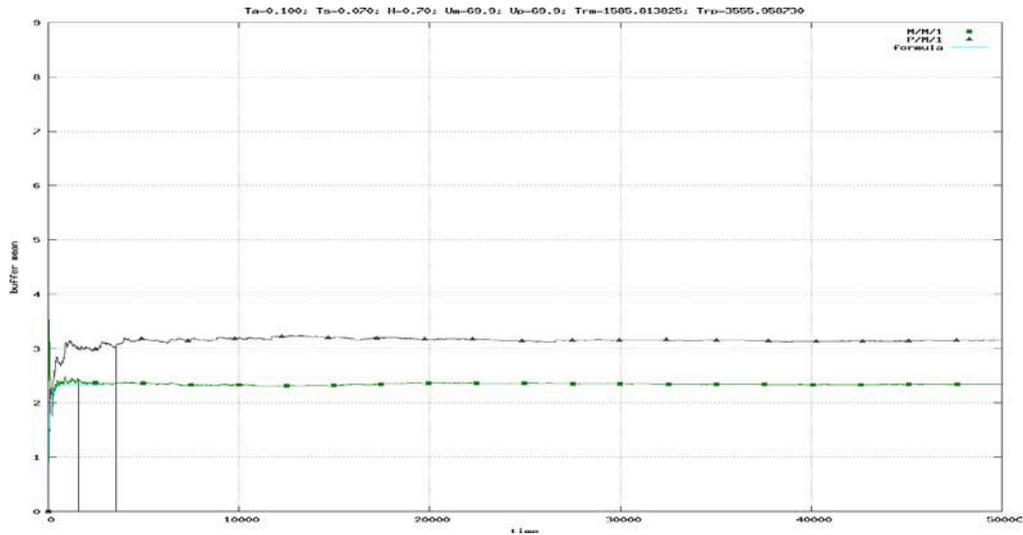


Fig. 1. M/M/1 and P/M/1 systems with load 0.7 and incoming traffic with Hurst 0.7

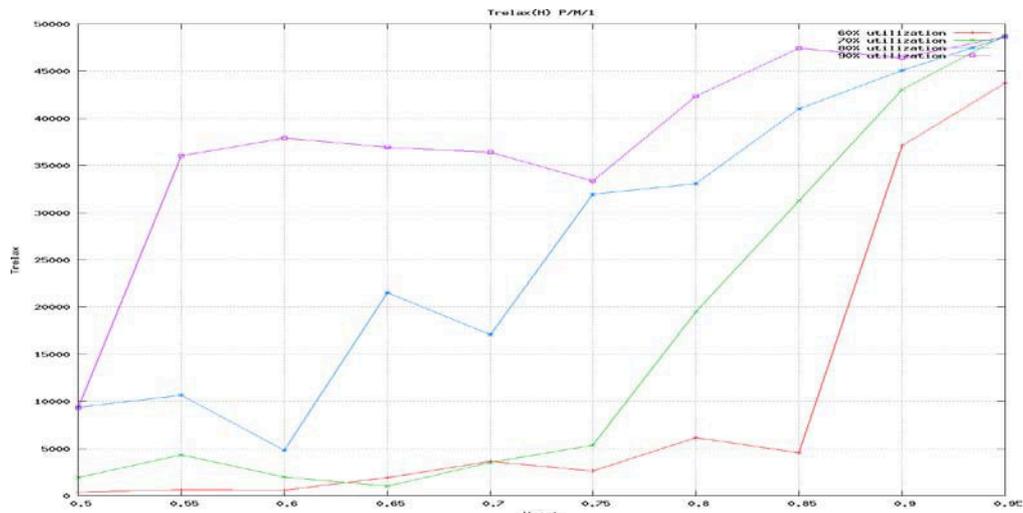


Fig. 2. Relaxation time obtained from modeling results with different parameters

Now let's analyze these results trying to get analytical dependence of relaxation time of Hurst coefficient.

### Analytical estimation

By analytical estimation we mean finding formula which will fit best our experimental modeling results. Such estimation could be called regression or non linear fit. We performed this analysis using Labplot software which gives the ability to define any formula for experimental data fitting. As there are countless number of different formulas to try it was decided to stop on several simple

well known formulas which would allow fast and easy computation of relaxation time. The chose fell on linear as the most simple, exponential, logarithmic and Verhulst equations. The later was chosen because it was determined at a glance that it could probably fit well our experimental data. Goodness of fit was determined by smallest chi square parameter which basically is squared sum of differences between the formula and the data allowing to see how close the formula approximates the data. Comparative results of chi square values we have put in Table.1.

Table 1. Chi<sup>2</sup> for different fit formulas

	U=0.6	U=0.7	U=0.8	U=0.9
Linear, $f(x) = a*x+b$	1.1965e+08	8.0142e+07	2.30163e+07	5.26435e+07
Exponential, $f(x) = a*\exp(-b*x)+c$	3.39007e+07	2.43968e+07	2.37445e+07	6.01641e+07
Logarithmic, $f(x) = a+b*\ln(x)$	2.93136e+08	3.8451e+08	2.80594e+08	1.42546e+08
Verhulst, $f(x) = (a*b*\exp(c*x)) / (a + b*(\exp(c*x) - 1))$	8.68669e+06	4.75526e+06	1.92941e+07	3.33734e+07

From Table 1 it's clearly seen that Verhulst formula gives the closest approximation. Here U is server utilization or load coefficient – higher value means higher

load, a, b and c are formula parameters. Verhulst curve can adopt very close to our data in most cases what is seen on Fig. 3–Fig. 6.

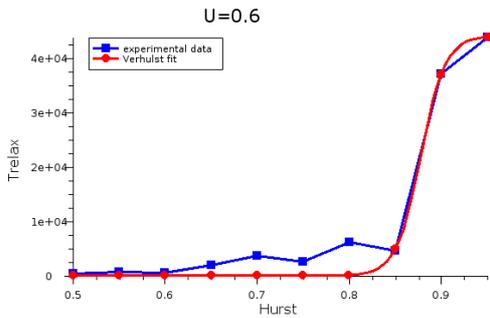


Fig. 3. Verhulst fit for load 0.6

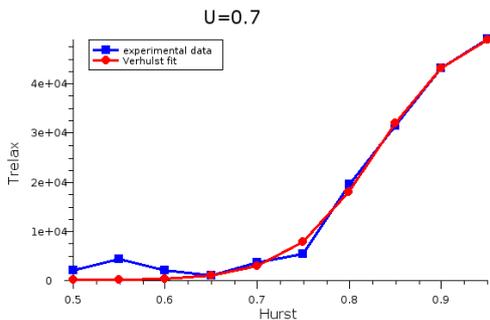


Fig. 4. Verhulst fit for load 0.7

Fig. 3– Fig. 5 and show fitted curves for relaxation time dependence on self-similarity degree (Hurst coefficient).

Estimated parameters for Verhulst formula are given in Table 2.

Table 2. Estimated Verhulst parameters

	U=0.6	U=0.7	U=0.8	U=0.9
a	44027.4 +/- 1.06334	52172.4 +/- 1.66882	59314.5 +/- 3.05851	41267.7 +/- 0.358781
b	1.74984e-24 +/- 9.82138e-27	0.000600981 +/- 7.43842e-07	99.7285 +/- 0.0360702	6.70094e-10 +/- 1.74836e-12
c	74.4936 +/- 0.00640213	22.0328 +/- 0.00154468	8.37611 +/- 0.000560786	61.0783 +/- 0.00506562

## Conclusions

In this article we have analyzed what formula can be used for finding system relaxation time depending on what server load is and how high is input traffic self-similarity degree. We have found that the best result is achieved with Verhulst equation. Current solution doesn't allow to find relaxation time with arbitrary input parameters but only those which are analyzed. To be able to use arbitrary input parameters common solution should be find which is the next problem.

## References

1. **Ipatovs A., Petersons E.** Performance Evaluation of WLAN depending on Number of Workstations and Protocols // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 8(72). – P. 15–18.

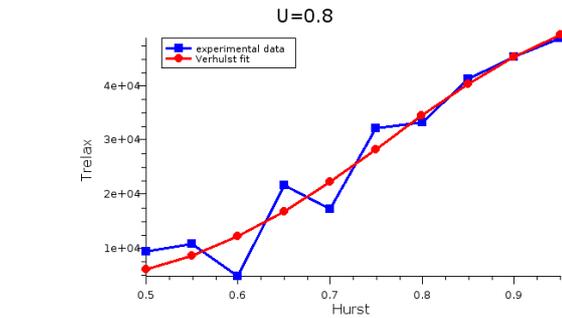


Fig. 5. Verhulst fit for load 0.8

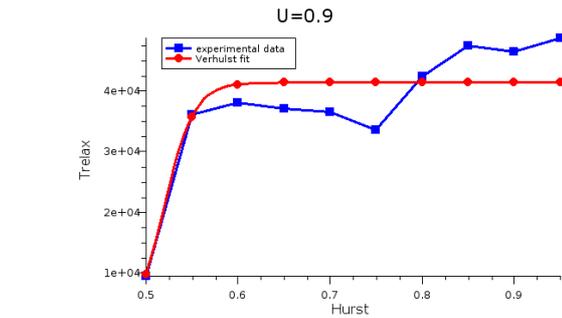


Fig. 6. Verhulst fit for load 0.9

Although Table 2 gives us estimated parameters for the fitting formula these are not a common solution to the problem. To find a common solution modeling results should be improved to give more than 10 points to fit for better statistical analysis. Finding common solution is the next problem to be solved.

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**R. Yeryomin, E. Petersons. Analytical Estimation of Self-Similar Wireless Traffic Relaxation Time and Hurst Coefficient Dependence // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 2(108). – P. 31–34.**

Analyzed formula can be used for finding system relaxation time depending on what server load is and how high is input traffic self-similarity degree. It was found that the best result is achieved with Verhulst equation. Ill. 6, bibl. 6, tabl. 2 (in English; abstracts in English and Lithuanian).

**R. Yeryomin, E. Petersons. Panašių bevielio ryšio srautų atsipalaidavimo laiko ir Hursto koeficiento priklausomybės analitinis įvertinimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 2(108). – P. 31–34.**

Analizuojamos matematinės išraiškos gali būti naudojamos atsipalaidavimo trukmės arba serverio apkrovos bei įėjimo srauto panašumo laipsniui nustatyti. Nustatyta, kad geriausių rezultatų pasiekama taikant Verhulsto matematinės išraiškas. Il. 6, bibl. 6, lent. 2 (anglų kalba; santraukos anglų ir lietuvių k.).