# Modelling of Propagation Constant of Twisted Pairs and Its Temperature Dependence at G.fast Frequencies

Marek Nevosad[1], Pavel Lafata[1]
[1]*Department of Telecommunication Engineering, Faculty of Electrical Engineering, Czech Technical University in Prague,*
*Technicka 2, Prague 6, 166 27, Czech Republic*
*marek.nevosad@fel.cvut.cz*

***Abstract*—Today, accurate and low complexity modelling of secondary parameters of twisted copper pairs at high frequencies is necessary, especially for modern transmission systems. The newly presented G.fast subscriber technology can occupy frequencies up to 212 MHz. For the purpose of its performance evaluations as well as for quick and accurate estimations of its transmission channel, the possibilities for accurate twisted pair's parameters approximation should be investigated. Basically, the motivation presented within this paper is to propose a completely new model for estimations of a propagation constant and its temperature dependence of twisted pairs and metallic cables suitable for G.fast frequencies. The main idea of a presented model is the innovative adoption of an inverse hyperbolic sine function in order to provide more accurate approximation of a propagation constant $\gamma(f)$ and its temperature character. The accuracy of a proposed model was verified for various metallic cables with different constructional arrangement and parameters. Based on the comparison of our proposed model with other typical existing models, the presented model outperforms the accuracy of all existing models using equal number of necessary parameters.**

***Index Terms*—Channel models; digital communication; modelling; transmission line measurements.**

## I. INTRODUCTION

The future modern access networks will be surely based on optical fibres and optical access technologies [1], such as passive optical networks (PONs), optical Ethernet technologies, *etc*. However, this will require massive capital investments to deploy optical fibres and to create a completely new optical infrastructure in access network segment [2]. Due to that, the deployment of optical access lines and fibres is still very slow today, especially in Europe. Another possible scenario is to combine optical access networks together with existing metallic lines and technologies in order to create various Fibre to the X (FTTX) solutions. Thanks to that the enormous costs could be potentially reduced, due to the exploitation of numerous existing metallic cables and lines. To support this idea, the ITU-T has recently developed and published an innovative xDSL technology called G.fast [3], presented within the new series of ITU-T G.9700 recommendation [4]. G.fast can occupy frequencies up to 106 MHz (G.fast 106a version) or up to 212 MHz (G.fast 212a version) and thanks to that it can easily reach transmission rates of Gbps for short metallic loops, not exceeding 300 meters [5]. Moreover, several innovations and new transmission concepts have been implemented into G.fast system, which can further increase its overall throughput [5], [6]. The main innovations include vectoring of transmitted DMT symbols through Vectored DMT modulation (VDMT) [3], the time-division duplex (TDD) technique, the reverse power-feeding mechanism [5], *etc*. However, the successful implementation of all these features require accurate and quick estimation of elementary transmission characteristics and parameters of the transmission channel [7], [8].

Today, there are several existing models for estimations of the primary or the secondary coefficients of twisted copper pairs and lines [9]. Moreover, both primary and secondary line coefficients strongly depend on the temperature conditions [9]. Basically, there are two main groups of twisted pair models, the first approach is to approximate the primary line coefficients, $R(f)$, $L(f)$, $C(f)$ and $G(f)$, while the second method estimates the secondary line coefficients, the propagation constant $\gamma(f)$ and the characteristic impedance $|Z_0(f)|$. Since the most important line characteristic is its attenuation constant $\alpha(f)$ [10], [11], which is a real part of a propagation constant $\gamma(f)$, the majority of existing twisted pair models directly approximate the attenuation constant of a transmission line $\alpha(f)$, or the combination of its longitudinal impedance $Z_S(f)$ and shunt admittance $Y_P(f)$. However, the calculation of the secondary line coefficients can be also performed by using its primary coefficients together with the telegraph equation [12]. The main problem of the majority of the existing line models is that they were originally derived for previous xDSL generation spectra, the ADSL2+ and VDSL2 up to 30 MHz, therefore their validity and accuracy for G.fast frequencies should be investigated [13]. The attenuation constant $\alpha(f)$ is also temperature dependent. This dependency plays the main role especially

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at high frequencies [13], therefore its approximation for G.fast transmissions and typical temperature range should be proposed.

The main motivation of this paper is to present a completely new innovative model of the propagation constant $\gamma(f)$ and its temperature dependence suitable for G.fast frequencies. Moreover, the aim is to propose a new model for quick and accurate approximation of a propagation constant $\gamma(f)$ for various external temperature conditions with accuracy better than all existing transmission line models with equal number of necessary parameters. This innovative model is based on the application of an inverse hyperbolic sine function and thanks to that it can provide more accurate approximation of a propagation constant $\gamma(f)$ typical character. The derivation of a proposed model is based on the Taylor series expansion of a propagation constant $\gamma(f)$ and thanks to that, the inverse hyperbolic sine function was implemented. The verification of the presented model was then performed for various types of typical metallic cables at typical temperature of 20 °C. Moreover, the propagation constant $\gamma(f)$ of a standard unshielded twisted cable, UTP cat. 5e, was also measured and modelled for various temperatures from 20 °C to 120 °C to evaluate the temperature dependence of a propagation constant $\gamma(f)$. The comparison of a proposed model with several typical existing models confirms the accuracy of a presented model, since it outperforms all existing models with the same number of necessary parameters. The paper is organized as follows. The next Section II contains the description of existing typical models of transmission lines together with short introduction into modelling of twisted copper pairs. The new innovative model is presented in Section III. The comparison of the accuracy of a proposed model together with several existing models performed for various typical metallic cables is then presented in Section IV, while the discussion of the results is provided in the conclusion of this paper.

## II. MODELLING OF TWISTED PAIRS

This section contains a short introduction into a problematic of twisted pairs modelling and it also describes several typical existing models.

### A. Parameters of Twisted Pairs

Basically, the transmission characteristics of a twisted pair (transmission line generally) can be described by either primary line coefficients, the series resistance $R(f)$, the series inductance $L(f)$, the shunt capacitance $C(f)$ and the shunt conductance $G(f)$, or their combination, the longitudinal impedance $Z_S(f)$ and the shunt admittance $Y_P(f)$, or by the secondary line coefficients, the propagation constant $\gamma(f)$ and the characteristic line impedance $Z_0(f)$. The primary and the secondary line coefficients can be easily converted by using telegraph equations, however, today, mainly the attenuation constant $\alpha(f)$ is the most important line frequency characteristic. The propagation constant $\gamma(f)$ of any transmission line is a complex number, its real part is an attenuation constant $\alpha(f)$, while the imaginary part represents a phase constant $\beta(f)$ [9]

$$\gamma(f) = \alpha(f) + j \times \beta(f), \quad [\text{dB/km,rad/km}]. \tag{1}$$

By modifying the telegraph equations, the propagation constant $\gamma(f)$ can be calculated by using the primary line coefficients as

$$\gamma(f) = \sqrt{\left(R(f) + j\omega L(f)\right)\cdot\left(G(f) + j\omega C(f)\right)}, \tag{2}$$

where $R(f)$, $L(f)$, $C(f)$ and $G(f)$ are the primary line coefficients, and $\omega = 2\pi f$ is an angular frequency.

The attenuation constant $\alpha(f)$ shows strong temperature dependence, since mainly the series resistance $R(f)$ depends on the temperature $T$. Today, this phenomenon is usually approximated as [9]

$$R_T = R_{20°C} \times \left[1 + \sigma\left(T - 20\right)\right], \quad [\Omega;°\text{C}], \tag{3}$$

where $R_T$ is the series resistance for a temperature $T$ in °C, while $R_{20°C}$ is the resistance at 20 °C. The coefficient $\sigma$ is given individually for each material, in case of a copper pair (Cu) its value is 0.004 [9].

### B. Existing Cable Models

Generally, there are two main groups of twisted pair models, the first is focused on modelling of the primary line coefficients, while the second group directly approximates the secondary line coefficients.

Today, the most often used model for estimations of the primary line coefficients, is the British Telecom model (BT). This group of models usually uses 13, 11 or 7 parameters in total according to the required accuracy and required frequency band. The full version of BT model with 13 parameters is usually suitable for frequencies up to tens or hundreds of MHz and is given as [9]:

$$R(f) = \frac{1}{\dfrac{1}{\sqrt[4]{r_{0C}^4 + a_C \times f^2}} + \dfrac{1}{\sqrt[4]{r_{0S}^4 + a_S \times f^2}}}, \quad [\Omega/\text{km}], \tag{4}$$

$$L(f) = \frac{l_0 + l_\infty \times \left(\dfrac{f}{f_m}\right)^b}{1 + \left(\dfrac{f}{f_m}\right)^b}, \quad [\text{H/km}], \tag{5}$$

$$C(f) = c_\infty + c_0 \times f^{-c_e}, \quad [\text{F/km}], \tag{6}$$

$$G(f) = g_0 \times f^{g_e}, \quad [\text{S/km}]. \tag{7}$$

In (4)–(7) representing BT 13 model, all 13 parameters, $r_{0C}$, $a_C$, $r_{0S}$, $a_S$, $l_0$, $l_\infty$, $f_m$, $b$, $c_\infty$, $c_0$, $c_e$, $g_0$ and $g_e$, are specified individually for each type of metallic cable.

The recently presented ITU-T G.9701 recommendation [4] contains also a new G.fast reference model, which is based on older KPN model. This G.fast model uses 9 parameters and it approximates the longitudinal impedance $Z_S(f)$ and the shunt admittance $Y_P(f)$ as:

$$Z_S(j\omega) = j\omega \times L_{S\infty} + R_{S0} \times$$

$$\times\left(\begin{array}{l}1-q_S\times q_X+\\+\sqrt{q_S^2\times q_X^2+2\times\dfrac{j\omega}{\omega_S}\times\left(\dfrac{q_S^2+{j\omega}/{\omega_S\times q_Y}}{{q_S^2}/{q_X}+{j\omega}/{\omega_S\times q_Y}}\right)}\end{array}\right), \tag{8}$$

$$Y_P(j\omega)=j\omega\times C_{P0}\times(1-q_C)\times\left(1+\frac{j\omega}{\omega_D}\right)^{-2\phi/\pi}+ \\ +j\omega\times C_{P0}\times q_C. \tag{9}$$

The model, containing 9 following individual parameters specified for each metallic line, $L_{S\infty}$, $R_{S0}$, $C_{P0}$, $q_S$, $q_X$, $q_Y$, $q_C$, $\omega_S$ and $\omega_D$, provides accurate G.fast channel approximation at least up to 212 MHz. However, since it uses 9 parameters and due to its overall complexity, the adoption and practical application of the model is rather complicated in practice.

The second group of twisted pair models, which directly approximates the attenuation constant $\alpha(f)$ and/or the phase constant $\beta(f)$, is represented mainly by parametric models based on Chen's model [9]

$$\alpha(f)=k_1\times\sqrt{f}+k_2\times f,\quad [\text{dB/km; Hz}], \tag{10}$$

where $k_1$ and $k_2$ are the parameters specified individually for each type of metallic cable (line). The Chen's model is able to provide quick and accurate attenuation constant $\alpha(f)$ approximation, however, typically only for frequencies of tens of MHz (VDSL2 spectra). The validity of the model was also provided through mathematical derivations presented by Acatauassu *et al*. in [12], [14], where the causality of the model was also corrected and modelling of a phase constant $\beta(f)$ was added. The resulting KM1 model can be then expressed as [12], [14]:

$$\alpha(f)=k_1\times\sqrt{f}+k_2\times f,\quad [\text{dB/km; Hz}], \tag{11}$$

$$\beta(f)=k_1\times\sqrt{f}-\frac{2}{\pi}k_2\times f\times\ln(f)+ \\ +k_3\times f,\quad [\text{rad/km; Hz}]. \tag{12}$$

The KM1 model with 3 $k$-parameters provides sufficiently accurate results up to tens (hundred) of MHz, as presented through measurements and comparisons with various metallic cables presented in [12], [14]. Further improvements and derivations provided by Acatauassu *et al*. in lead into obtaining KM2 and KM3 models, which can achieve better accuracy compared to KM1 model (11), (12), however, KM2 model requires 4 $k$-parameters, while KM3 5 $k$-parameters in total.

## III. PROPOSED INVERSE SINH MODEL

The main motivation of a proposed inverse hyperbolic sine model is to outperform all existing models, namely the KM1 model with the equal number of necessary parameters. Moreover, the temperature dependence of a propagation constant $\gamma(f)$ given mainly by the (3) has been further studied and the proposed inverse sinh model directly provides the temperature character of $\gamma(f)$ without using (3) a priori.

In all following equations the mathematical notation will be simplified by eliminating *(f)* in order to reduce the length of formulas and equations. Previous (2) can be modified as

$$\gamma(f)=j\omega\times\sqrt{LC}\times\sqrt{1+\frac{R}{j\omega L}}\times\sqrt{1+\frac{G}{j\omega C}}. \tag{13}$$

Since the modelling is performed for G.fast frequencies at least up to 250 MHz, the following conditions can be applied:

$$\begin{cases}R<<wL,\\G<<wC.\end{cases} \tag{14}$$

Thanks to that, the Taylor series expansion of all square roots in (13) can be proposed. Therefore, (13) can be represented by Taylor series expansion and previous conditions as [12], [14]

$$\gamma(f)=j\omega\times\sqrt{LC}\times \\ \times\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\begin{bmatrix}\left[\dfrac{(-1)^n\times(2n)!}{2^{2n}\times(n!)^2(1-2n)}\times\left(\dfrac{R}{j\omega L}\right)^n\right]\times\\ \times\left[\dfrac{(-1)^m\times(2m)!}{2^{2m}\times(m!)^2(1-2m)}\times\left(\dfrac{G}{j\omega C}\right)^m\right].\end{bmatrix} \tag{15}$$

Both sums can be simply merged together, therefore the resulting Taylor approximation can be expressed as

$$\gamma(f)=j\omega\times\sqrt{LC}\times \\ \times\sum_{\substack{m=0\\n=0}}^{\infty}\left[\begin{array}{c}\dfrac{(-1)^{n+m}\times(2n)!\times(2m)!}{2^{2(n+m)}\times(n!)^2(1-2n)\times(m!)^2(1-2m)}\times\\ \times\left(\dfrac{R}{j\omega L}\right)^n\times\left(\dfrac{G}{j\omega C}\right)^m\end{array}\right]. \tag{16}$$

Since the attenuation constant $\alpha(f)$ is defined as a real part of a propagation constant $\gamma(f)$, and on the other hand, the phase constant $\beta(f)$ is an imaginary part of $\gamma(f)$, only real parts of (16) for $\alpha(f)$ approximation and only imaginary parts of (16) for $\beta(f)$ approximation should be selected. Therefore, only odd multiplies of $m+n$ indexes in (16) should be selected for $\alpha(f)$, while only even multiples of $m+n$ should be picked for $\beta(f)$ approximation:

$$m+n=2p+1\rightarrow\alpha(f), \tag{17}$$

$$m+n=2p\rightarrow\beta(f). \tag{18}$$

Since the Taylor series expansion of inverse hyperbolic sine function is generally given as [9]

$$\text{arsinh}(z)=\sum_{n=0}^{\infty}\frac{(-1)^n\times(2n)!}{2^{2n}\times(n!)^2(2n+1)}\times z^{2n+1}, \tag{19}$$

its implementation into previous (16) could be performed with only slight modification of its argument by adding terms with *k*-parameters and frequency roots fractions with proper approximation terms. Thanks to that, the resulting inverse sinh model for propagation constant $\gamma(f)$ approximation could be expressed as:

$$\alpha(f) = k_1 \operatorname{arsinh}\left( k_2 \left( f^{0.3} + \sqrt{f} \right) + \frac{k_2}{k_1^{2.5}} f^{1.1} \right), \quad (20)$$

$$\beta(f) = k_3 f - \operatorname{arsinh}(k_3 f). \quad (21)$$

*A. Modelling of Temperature Character of the Attenuation Constant*

The temperature character of an attenuation constant $\alpha(f)$ is mostly given by the temperature dependence of the line series resistance $R(f)$ (3) [9]. However, the temperature change of dielectric constant also influences the series inductance $L(f)$, the shunt capacitance $C(f)$ and the shunt conductance $G(f)$. Therefore, in order to approximate all these complex phenomena, the modification of the inverse sinh model (20), (21) was proposed. According to the formulas and based on numerous experimental results, the model (20), (21) could be extended by adding a temperature term to the $k_2$ parameter, so the resulting presented inverse sinh model for temperature dependence approximation could be expressed as:

$$\alpha(f) = k_1 \operatorname{arsinh}\left( \begin{array}{l} \left(1+\tau(T-20)^{1.25}\right)\times \\ \times\left[ k_2\left(f^{0.3}+\sqrt{f}\right)+\frac{k_2}{k_1^{2.5}}f^{1.1}\right] \end{array} \right), \quad (22)$$

$$\beta(f) = k_3 f - \operatorname{arsinh}(k_3 f). \quad (23)$$

The temperature influence is given by the parameter $\tau$ specified for each metallic cable, while $T$ is the temperature in °C. Next, the accuracy of the proposed model was verified and compared with other existing models through measurements performed for real metallic cables.

## IV. EXPERIMENTAL MEASUREMENTS AND RESULTS

*A. Summary Squared Error Minimization*

First, in order to compare the accuracy of each model for each metallic cable, the squared error $E_r$ for both, the attenuation constant $\alpha(f)$ as well as the phase constant $\beta(f)$, was calculated. Then, the summary squared error $E_S$ could be calculated as the sum [12]:

$$E_{S\alpha} = \sum_f \left(\alpha_M(f) - \alpha(f)\right)^2, \quad [\text{dB/100m}], \quad (24)$$

$$E_{S\beta} = \sum_f \left(\beta_M(f) - \beta(f)\right)^2, \quad [\text{rad/100m}]. \quad (25)$$

where $E_{S\alpha}$ and $E_{S\beta}$ is the summary error of attenuation constant and phase constant approximation respectively, while it can be calculated as the summary square difference between model values and real measured values of $\alpha(f)$ and $\beta(f)$ over the entire frequency band *f*. All results were also recalculated for the standard length of metallic lines – 100 meters. It is also necessary to calculate the *k*-parameters and other model parameters individually for each metallic cable and each model. This can be simply done by using summary squared error minimization process. For example in Matlab environment, this can be mathematically performed by using least-square fitting method described in [12].

*B. Results of Modelling at 20 °C*

First, the measurements and comparisons were performed for several typical metallic cables for the standard temperature of 20 °C. To illustrate the potential of a presented inverse sinh model, the following list of typical telecommunication cables was selected with the lengths and measured frequency band – UTP cat. 5e with the length of 50 meters in a frequency band 2 MHz–250 MHz, UTP cat. 6 cable with the length of 50 meters in a frequency band 2 MHz–250 MHz, STP cat. 7 with the length of 97 meters again in the frequency band between 2 MHz and 250 MHz, SXKFY 4 × 2 × 0.5 cable with the length of 47 meters in a frequency band 2 MHz–150 MHz, SYKFY 4 × 2 × 0.5 with the length of 25 meters in a frequency band between 2 MHz and 150 MHz and finally, TCEPKPFLE 75 × 4 × 0.4 cable with the length of 100 meters in a frequency band 2 MHz–150 MHz as well.

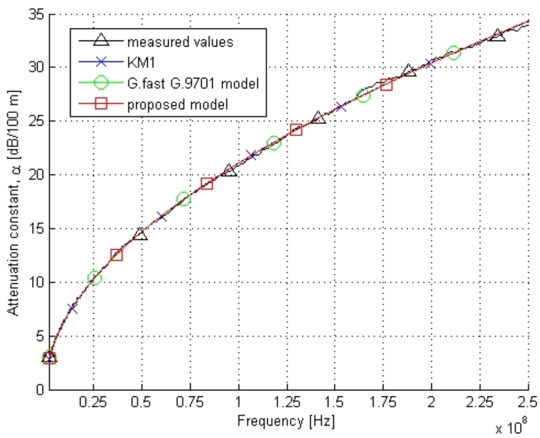


Fig. 1. Measured and modelled attenuation constants $\alpha(f)$ for UTP cat. 5e.

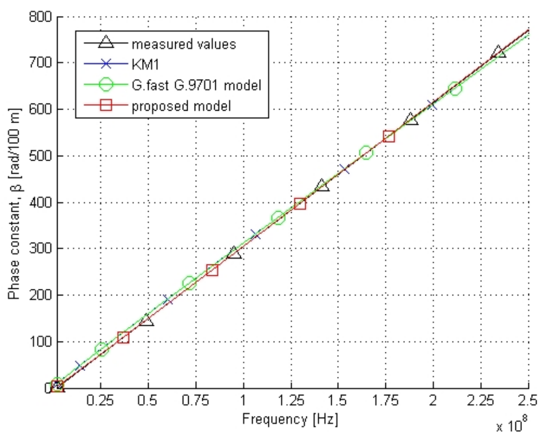


Fig. 2. Measured and modelled phase constants $\beta(f)$ for UTP cat. 5e.

All measurements were performed using calibrated Rohde&Schwarz vector network analyser together with proper NorthHill balun transformers to ensure correct impedance connection between the analyser and measured metallic cables. All these measurements were performed for a standard room temperature of 20 °C, therefore $T$ = 20 °C in the proposed inverse sinh model (22), (23). The first graph in Fig. 1 contains the results of attenuation constant $\alpha(f)$ for UTP cat. 5e cable, while the phase constant $\beta(f)$ is illustrated in Fig. 2 with summary squared errors presented in Table I. Additionally, the absolute errors between real measured constants $\alpha(f)$, $\beta(f)$ and all models are illustrated in Fig. 3 and Fig. 4 respectively.

TABLE I. SUMMARY SQUARED ERRORS FOR EACH MODEL.

| Summary squared errors for UTP cat. 5e cable | KM1 model | G.fast G.9701 model | Proposed Inverse sinh model |
|---|---|---|---|
| $Es_{\alpha}$ | 59.154616 | 56.210238 | 57.607778 |
| $Es_{\beta}$ | 84780.48576 | 83573.64868 | 1410.596965 |

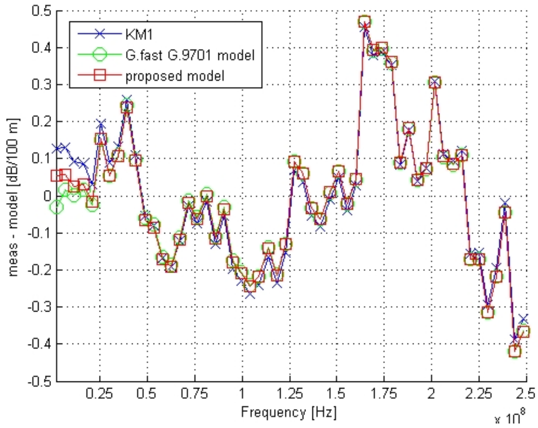


Fig. 3. Absolute error between measured and modelled attenuation constants $\alpha(f)$ for UTP cat. 5e cable.

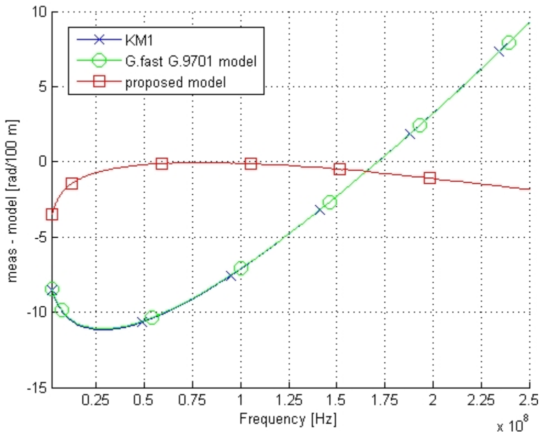


Fig. 4. Absolute error between measured and modelled phase constants $\beta(f)$ for UTP cat. 5e cable.

TABLE II. SUMMARY SQUARED ERRORS FOR UTP 6 AND STP 7.

| *Es* for UTP cat. 6 cable | KM1 model | G.fast G.9701 model | Proposed Inverse sinh model |
|---|---|---|---|
| $Es_{\alpha}$ | 49.636942 | 47.802622 | 49.106957 |
| $Es_{\beta}$ | 80711.3449 | 79880.916468 | 2057.785855 |
| ***Es* for STP cat. 7 cable** | | | |
| *ESa* | 18.419249 | 17.997313 | 18.176721 |
| *ESb* | 69958.573 | 69536.179 | 701.906555 |

The same comparison was also performed for UTP cat. 6 cable and STP cat. 7 cable, the results in a form of squared errors are presented in Table II, while the absolute errors for STP cat. 7 cable and all models are illustrated in Fig. 5 for attenuation constant $\alpha(f)$ approximation and Fig. 6 for phase constant $\beta(f)$ approximation respectively.

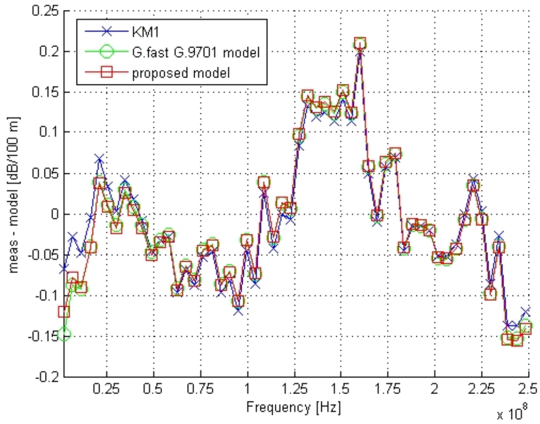


Fig. 5. Absolute error between measured and modelled attenuation constants $\alpha(f)$ for STP cat. 7 cable.

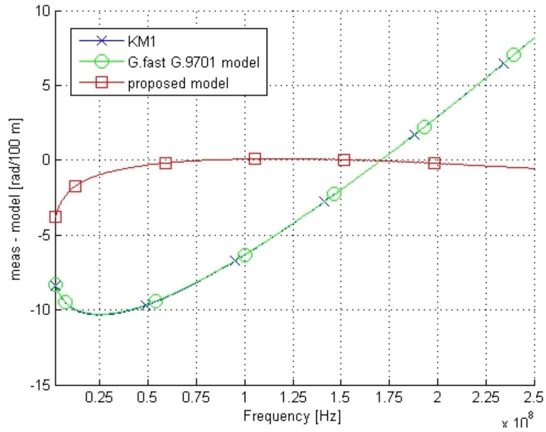


Fig. 6. Absolute error between measured and modelled phase constants $\beta(f)$ for STP cat. 7 cable.

Evidently, the proposed inverse sinh model outperforms the accuracy of existing KM1 model in both attenuation constant $\alpha(f)$ and phase constant $\beta(f)$ approximations for all three measured cables, UTP cat. 5e, cat. 6 and STP cat. 7. Moreover, it also outperforms the accuracy of G.fast G.9701 reference model of phase constant $\beta(f)$ estimation for all three cables, although the G.9701 G.fast model uses significantly more parameters. Next, TCEPKPFLE, SYKFY and SXKFY cables were measured as well at 20 °C. The example of absolute errors for TCEPKPFLE cable is presented in Fig. 7 for $\alpha(f)$ and in Fig. 8 for $\beta(f)$ approximation. The summary squared errors for all three cables are then included in Table III.

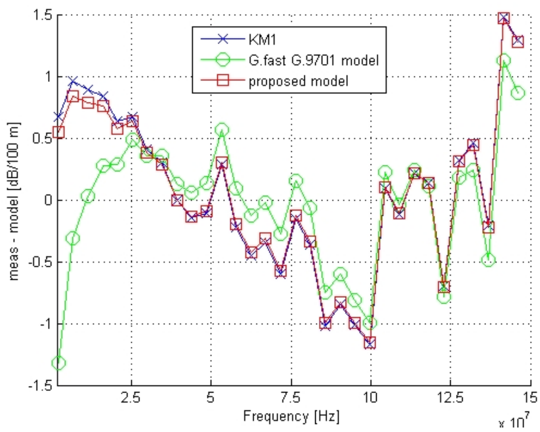


Fig. 7. The absolute error for TCEPKPFLE cable and attenuation constant $\alpha(f)$ approximation.

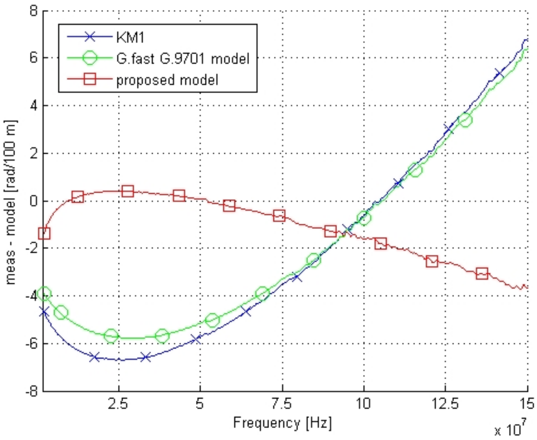


Fig. 8. The absolute error for TCEPKPFLE cable and attenuation constant *β(f)* approximation.

It is obvious that the proposed inverse sinh model again outperforms the accuracy of KM1 model for all three compared metallic cables, although they both use the equal number of necessary parameters. Only the accuracy of an attenuation constant *α(f)* modelling for SYKFY cable is lower compared to KM1, however, the difference is only minor. Moreover, the phase constant *β(f)* approximation is again more accurate even compared to the accuracy of G.fast G.9701 reference model.

TABLE III. SUMMARY SQUARED ERRORS FOR ALL CABLES.

| $E_S$ for TCEPKPFLE cable | KM1 model | G.fast G.9701 model | Inverse sinh model |
|---|---|---|---|
| $E_{S\alpha}$ | 407.902926 | 225.260434 | 374.547485 |
| $E_{S\beta}$ | 21028.46369 | 16612.898861 | 2671.686768 |
| **$E_S$ for SYKFY cable** | **KM1 model** | **G.fast G.9701 model** | **Inverse sinh model** |
| $E_{S\alpha}$ | 436.369036 | 321.319384 | 438.081241 |
| $E_{S\beta}$ | 71426.53645 | 64833.914645 | 13305.125584 |
| **$E_S$ for SXKFY cable** | **KM1 model** | **G.fast G.9701 model** | **Inverse sinh model** |
| $E_{S\alpha}$ | 451.677546 | 330.162296 | 435.505581 |
| $E_{S\beta}$ | 16237.63404 | 13087.09576 | 7436.371290 |

## C. Modelling of the Temperature Influence

The proposed inverse sinh model (22), (23) also enables temperature approximation of attenuation constant *α(f)*. In order to verify its accuracy, the experimental measurements of real attenuation constant *α(f)* of UTP cat. 5e cable were performed during a temperature experiment. The UTP cat. 5e cable was gradually heated in a special thermal chamber (oven) from 20 °C to 120 °C, during which its characteristics were continuously measured. The *k*-parameters of all models were calculated for a temperature of 20 °C and after that, the proper temperature models were applied. After that the parameter $\tau$ in (22) was again calculated using least-square fitting method to minimize the summary squared error. In order to compare the accuracy of a proposed inverse sinh model, the standard model for attenuation constant *α(f)* temperature character (3) together with existing KM1 model was applied as well. Therefore, the next Fig. 9 contains the comparison between measured *α(f)* and modelled *α(f)* characteristics using proposed inverse sinh model for selected temperatures (20 °C, 40 °C, 60 °C, 80 °C, 100 °C and 120 °C), while the same comparison for existing model (KM1, (11), (12), with (3)) is then presented in following Fig. 10. In order to illustrate the accuracy of modelling in better way, the absolute errors of both models and all selected temperatures were calculated and are presented in Fig. 11, while the summary squared errors are included in Table IV.

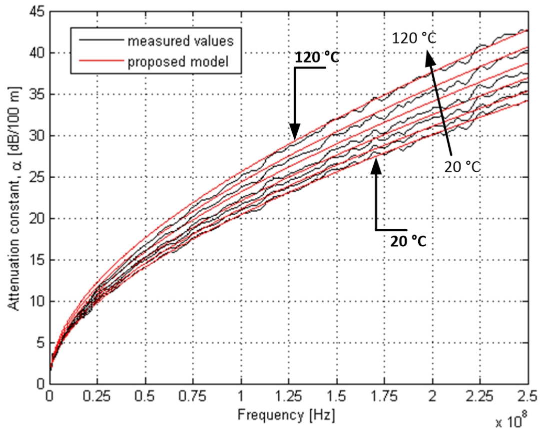


Fig. 9. Measured and modelled attenuation constants *α(f)* for UTP cat. 5e cable for selected temperatures using proposed inverse sinh model.

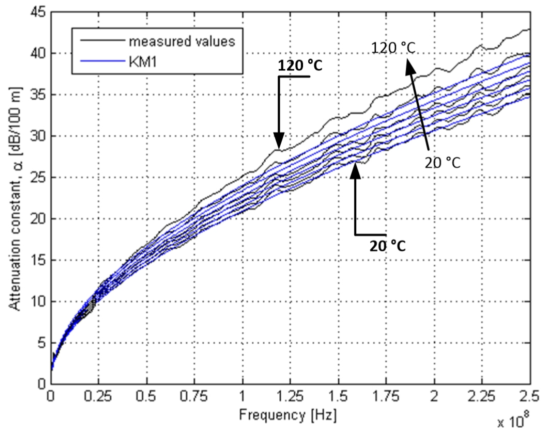


Fig. 10. Measured and modelled attenuation constants *α(f)* for UTP cat. 5e cable for selected temperatures using existing models (KM1+ (3)).

From the results presented above in Fig. 9–Fig. 11 and Table IV can be concluded that the accuracy of a proposed inverse sinh model is general equal or slightly worse to the accuracy of a standard temperature model (3) at lower temperatures up to 80 °C. However, at higher temperatures above 80 °C, the proposed inverse sinh model provides significantly better attenuation constant *α(f)* approximation compared to the existing model. This is also evident from the following Fig. 12, in which the previous Fig. 11 is illustrated in two separate graphs.

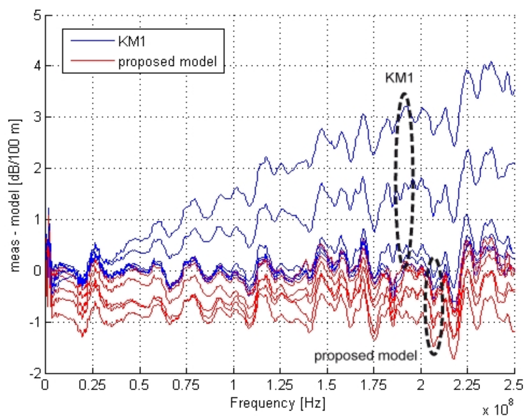


Fig. 11. Absolute errors between measured *α(f)* and both models.

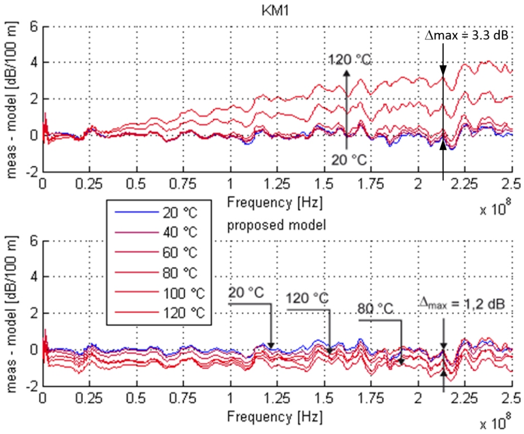


Fig. 12. Comparison of absolute errors between measured *α(f)* and both models.

TABLE IV. SUMMARY SQUARED ERRORS FOR TEMPERATURE MODELLING.

| $Es_{\alpha}$ | 20°C | 40°C | 60°C | 80°C | 100°C | 120°C |
|---|---|---|---|---|---|---|
| KM1 | 41.39 | 34.981 | 42.815 | 79.553 | 885.996 | 3089.951 |
| Inverse sinh | 40.31 | 39.396 | 105.447 | 152.826 | 262.533 | 238.268 |

The first graph contains the absolute error of *α(f)* approximation of existing KM1 model and evidently, this error significantly increases for higher temperatures. Its maximum $\Delta_{max}$ value for 120 °C is approx. 3.3 dB. While the approximation error of the proposed inverse sinh model does not change with the temperature much, as illustrated in the lower part of Fig. 12. Its *α(f)* approximation error for the same frequency, $\Delta_{max}$, is approx. 1.2 dB, which is significantly better compared to existing KM1 model. Moreover, the temperature dependence of *α(f)* is already included in a proposed model, (22), (23), which makes the presented model more versatile.

## V. CONCLUSIONS

This paper presented a new model of a propagation constant *γ(f)* of twisted copper pairs suitable for G.fast frequencies at least up to 250 MHz. The main innovative idea of a proposed model is the application of an inverse hyperbolic sine function, which can generally increase the accuracy of modelling compared to standard existing *γ(f)* models. Moreover, the temperature dependence of attenuation constant *α(f)* was successfully implemented into the proposed model, therefore the model can be complexly used for realistic and accurate estimations of G.fast transmission channels under various external conditions. The experimental results and comparisons presented within this paper clearly illustrate the accuracy of a proposed inverse sinh model, which generally outperforms the accuracy of existing KM1 model using equal number of *k*-parameters and in some cases it also outperforms the G.fast reference model presented in ITU-T G.9701 rec. The experimental measurements and modelling for UTP cat. 5e cable during the temperature test again proved the validity and accuracy of a proposed model for various external temperature. The presented inverse sinh model provides more accurate results for temperatures above 80 °C, however, its accuracy for lower temperatures is generally equal to the accuracy of a standard existing model. Since its performance for attenuation constant *α(f)* modelling is better compared to existing KM1 model and its *β(f)* approximation is also significantly more accurate than KM1 model, the proposed inverse sinh model generally provides better results with equal number of required parameters. Thanks to that the proposed inverse sinh model could be successfully adopted in practice.

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