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Stochastic Models Quality of Electronics Systems

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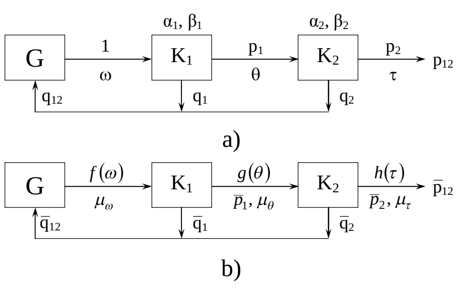
Introduction

In companies which utilize modern technologies databases of manufactured production and technological processes are constantly maintained and complemented. Data is often read and input automatically. By using computer networks and database control and analysis systems, information can be transmitted in real-time and can be used in decision making processes at each intermediate or final stage of manufacture. Thus there is a possibility to use information not only from the current but also from the previous stages of manufacture. That can increase the quality of electronics systems (ES) [1-6].

Complex ES are defined in technical documentation as an entire series of parameters, the values of which determine the level of quality. Parameters can be differentiated according to their importance regarding the implementation of purpose functions. International standard ISO-2859-0 [2] recommends to divide parameters into two – A, B – or three – A, B, C – classes (groups). Here A – most important or significant parameters, and B, C – secondary or less significant parameters. Such classification of parameters is convenient when analyzing problems of multiparametric product quality control [2, 5, 6]. When imitational modeling is applied, stochastic models of quality level are required for separate parameters, their groups and for entire product.

Stochastic models of control quality of ES

Inter-operational control fragment is presented in Fig. 1, which involves two stages of continuous control K_1 and K_2 (in both stages IS are classified according to analogical decision rules) [2, 5].



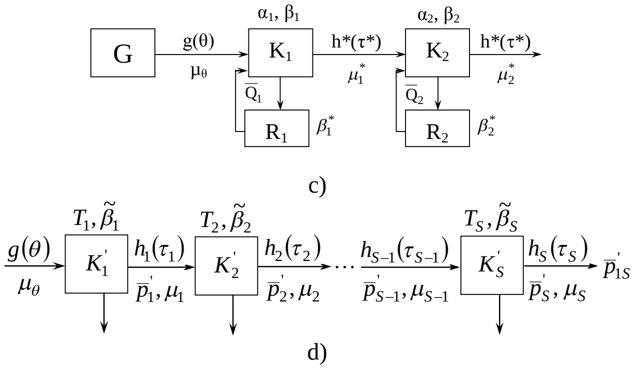


Fig. 1. Stochastic models of control quality: a – one; b – two; c – three; d – S stage

There is a probability ω that a defective IS after manufacture operations G will enter the control stage K_1 which is characterized by classification error probabilities α_1 and β_1; probability ω is transformed to parameter θ after control operations. IS acceptance probability is p_1 and rejection probability is q_1.

Analogously in the second stage K_2 with errors α_2 and β_2 parameter θ is transformed to τ, when probabilities p_2, q_2 are fixed. IS in such scheme is accepted with probability q_{12}. Rejected IS are returned for reparation to manufacture process G .

According to models created in the work [2, 5], we receive such direct and reverse dependencies, which are needed for the further analysis:

$$\begin{cases} p_1 = p_1(\omega) = 1-\alpha_1-(1-\alpha_1-\beta_1)\omega = (1-\alpha_1)\left[1-\left(1-\tilde{\beta}_1\right)\omega\right]; \\ p_2 = p_2(\theta) = (1-\alpha_2)\left[1-\left(1-\tilde{\beta}_2\right)\theta\right]; \\ p_{12} = p_1 p_2; \end{cases} \quad (1)$$

where $\tilde{\beta}_1 = \dfrac{\beta_i}{1-\alpha_i}, \quad i = 1,2;$ (2)

and $q_1 = 1-p_1$, $q_2 = 1-p_2$, $q_{12} = q_1 + p_1 q_2 = 1-p_{12}$.

It is obvious, that formulas may be applied for entire product, if r.q. $\theta \sim Be(a,b)$, or for separate groups A, B, C, if their defectivity levels θ_A , θ_B , θ_C are characterized by beta-distribution.

$$\begin{cases} \theta=\theta(\omega)=\dfrac{\beta_1\omega}{p_1(\omega)}=\dfrac{\tilde{\beta}_1\omega}{1-(1-\tilde{\beta}_1)\omega}, \\ \tau=\tau(\theta)=\dfrac{\beta_2\theta}{p_2(\theta)}=\dfrac{\tilde{\beta}_2\theta}{1-(1-\tilde{\beta}_2)\theta}, \\ \tau=\tau(\omega)=\dfrac{\tilde{\beta}_1\tilde{\beta}_2\omega}{1-(1-\tilde{\beta}_1\tilde{\beta}_2)\omega}=\dfrac{\tilde{\beta}_{12}\omega}{1-(1-\tilde{\beta}_{12})\omega}; \end{cases} \quad (3)$$

where $\tilde{\beta}_{12}=\tilde{\beta}_1\tilde{\beta}_2$ – generalized two-stage control transformation constant;

$$\begin{cases} \theta=\theta(\tau)=\dfrac{\tau}{\tilde{\beta}_2+(1-\tilde{\beta}_2)\tau}, \\ \omega=\omega(\theta)=\dfrac{\theta}{\tilde{\beta}_1+(1-\tilde{\beta}_1)\theta}, \\ \omega=\omega(\tau)=\dfrac{\tau}{\tilde{\beta}_{12}+(1-\tilde{\beta}_{12})\tau}. \end{cases} \quad (4)$$

It is easy to ascertain, that when the third control stage with errors α_3, β_3 is introduced, we have $\tilde{\beta}_{13}=\tilde{\beta}_1\tilde{\beta}_2\tilde{\beta}_3$, and in general case the generalized transformation constant $\tilde{\beta}_{1S}$ for control scheme consisting of S stages is

$$\tilde{\beta}_{1S}=\prod_{i=1}^{S}\tilde{\beta}_i, \quad i=1-s. \quad (5)$$

In separate occurrence, when $\tilde{\beta}_1=\tilde{\beta}_2=\ldots=\tilde{\beta}_S=\tilde{\beta}$, we have $\tilde{\beta}_{1S}=\tilde{\beta}^S$.

Beta-distributions

We will provide the main formulas, required for the further analysis, when r.q. θ_i and also η_i are distributed according to the beta-distribution with shape parameters a_i, b_i and marking that: $\theta_i \sim Be(a_i,b_i)$, $\eta_i \sim Be(b_i,a_i)$ [6]:

$$\begin{cases} g_i(\theta_i)=B_i^{-1}(a_i,b_i)\theta_i^{a_i-1}(1-\theta_i)^{b_i-1} \\ \varphi_i(\eta_i)=B_i^{-1}(a_i,b_i)\eta_i^{b_i-1}(1-\eta_i)^{a_i-1} \end{cases}, \quad (6)$$

here $B_i(a_i,b_i)\equiv B_i=\dfrac{\Gamma(a_i)\Gamma(b_i)}{\Gamma(a_i+b_i)}$ – beta function, $\Gamma(z_i)$ – gamma function, $\Gamma(z_i)=(z_i-1)\Gamma(z_i-1)$ or $\Gamma(n)=(n-1)!$, when *n* is a whole number (h.n.), $i=1\ldots l$;

$$\begin{cases} \mu_i=\dfrac{a_i}{a_i+b_i}, \overline{\mu_i}=1-\mu_i=\dfrac{b_i}{a_i+b_i} \\ \sigma_i^2=\dfrac{a_ib_i}{(a_i+b_i)^2(a_i+b_i+1)}=\dfrac{\mu_i\overline{\mu_i}}{a_i+b_i+1}; \end{cases} \quad (7)$$

$$v_i=\frac{\mu_i}{\sigma_i}=\sqrt{\frac{a_i}{b_i}(a_i+b_i+1)}. \quad (8)$$

If $a_i=1$ and μ_i is sufficiently small ($\mu_i<0.03$), then $b_i>>1$ and $v_i\approx 1$, i.e. $\sigma_i\approx\mu_i$.

Density $g_i(\theta_i)$ has maximum at the point θ_{iM} (mode)

$$\theta_{iM}=\frac{a_i-1}{a_i+b_i-2}. \quad (9)$$

Respectively the maximum of density $\varphi_i(\eta_i)$ is at the point $\eta_{iM}=1-\theta_{iM}$.

Multiparametric ES

Multiparametric case can be described using biparametric models ($l=2$), consistently joining product parameters into pairs, and then joining paired models in groups of two, etc.

For practical applications it is most convenient to model beta-densities $g_i(\theta_i)$ at first using. Then: $\theta\sim Be(a,b)$, $\theta_A\sim Be(a_A,b_A)$, $\theta_B\sim Be(a_B,b_B)$, $\theta_C\sim Be(a_C,b_C)$.

Generally we analyze *l*-parametric product with average defectivity level μ: there are *r* different parameters in group A, *s* parameters in group B, and $l-r-s$ parameters remain in group C. Indexing: $A\in i=1\div r$, $B\in i\equiv j=(r+1)\div(r+s)$, $C\in i\equiv k=(r+s+1)\div l$. If there are no priorities, we assume the same values of parameter a_i for modeling inside one group, when $\mu_i\le 0{,}03$:

$$\begin{cases} \dfrac{\mu_j}{\mu_i}\approx\dfrac{a_j}{a_i}, \quad kai \quad a_j+b_j\approx a_i+b_i \\ \dfrac{\mu_k}{\mu_i}\approx\dfrac{a_k}{a_i}, \quad kai \quad a_k+b_k\approx a_i+b_i \end{cases}. \quad (10)$$

In this way, when $a_k>a_j>a_i$, we have, that $\mu_i<\mu_j<\mu_k$, i.e. defectivity level of one parameter in group A is on average less than in group B, and in group B – less than in group C.

Assume that the following condition is given: $\mu_k\approx 3\mu_i$ and $\mu_j\approx 2\mu_i$. We select $a_k=3a_i$, $a_j=2a_i$ and receive

$$a=[r+2s+3(l-r-s)]a_i=(3l-2r-s)a_i. \quad (11)$$

We select $a_i=1$. Then $a_j=2$, $a_k=3$, $a=3l-2r-s$ and $b=b_l=(3l-2r-s)\left(\dfrac{1}{\mu}-1\right)$.

By assessing the desired dispersion of r.q. θ_i we can select the needed values of variation coefficient in separate groups and further to select values of a_i, b_i. Situations can be easily modeled when one of parameter groups or one or several parameters are eliminated. Analogously situations can be modeled when additional parameters are introduced.

General cases

The simplest situation is when the series of l parameters $b_1 > b_2 \ldots > b_l$ is supplemented with parameters $i = 0$ with $b_0 = b_1 + a_1$ and $i = l + 1$ with $b_{l+1} = b_l - a_k$, i.e. a series expanded up to $l + 2$ is obtained with $b_0 > b_1 > b_2 \ldots > b_l > b_{l+1}$. Then there are $r + 1$ parameters in group A and $\theta_A \sim Be(r+1, b_r)$, when $a_i = 1$, and there are $(l+2) - s - (r+1) = l - r - s + 1$ parameters in group C and $\theta_C \sim Be[3(l - r - s + 1), b_{l+1}]$, when $a_k = 3$. There are *s* parameters in the remaining B group and $\theta_B \sim Be(2s, b_{r+s})$, $a_j = 2$.

For entire product $\theta \sim Be(a', b_{l+1})$, where

$$\begin{aligned} a' &= (r+1) + 2s + 3(l - s - r + 1) = 3l - 2r - s + 4 \equiv \\ &\equiv a + 4,\ kai\ a_i = 1,\ a_j = 2,\ a_k = 3\ ir \\ \mu' &= a'/(a' + b_{l+1}) > \mu = a/(a + b_l). \end{aligned} \tag{12}$$

Assume, that with such values of $a_{i,j,k}$ number of parameters $l = const$, and two parameters are introduced into B group from the neighboring groups: the last parameter $i = r$ from group A and the first parameter $i \equiv k = r + s + 1$ from group C. Then there are $s + 2$ parameters in group B, $r - 1$ in group A and $l - (r-1) - (s+2) = l - r - s - 1$ in group C. Beta-densities are defined by the following distributions:

$\theta_A \sim Be(r - 1,\ b_{r-1}), \theta_B \sim Be(2s + 4,\ b_{r+s+1}),$

$\theta_C \sim Be[3(l - r - s - 1),\ b_e]\ ir\ \theta \sim Be(3l - 2r - s),\ b_e]$.

If the number of parameters in group B is decreased down to $s - 2$ when $l = const$ (boundary parameters of group B are transferred to groups A and C respectively), then

$$\begin{aligned} &\theta_A \sim Be(r + 1,\ b_{r+1}), \theta_B \sim Be(2s + 4,\ b_{r+s-1}), \\ &\theta_C \sim Be[3(l - r - s + 1),\ b_e]\ ir\ \theta \sim Be(3l - 2r - s),\ b_e]. \end{aligned} \tag{13}$$

In other situations, when limitations (6-8) are not met, it is purposeful to use beta-densities $g_\sigma(\theta)$

In practical applications, when there are no additional information about separate parameters, it is most likely that $\mu_1 = \mu_2 = \ldots = \mu_l \equiv \mu_i$, i.e. all parameters have equal average defectivity level. In such case: $a_1 = \ldots = a_l \equiv a_i$, $b_1 = \ldots = b_l \equiv b_i$, $i = 1 \div l$.

When $a_i = 1$, we received general expression of $g(\theta)$ for any value of *l*. However, when $a_i > 1$ and $l > 4$, integration is sufficiently complex. Additionally, $g(\theta)$ expression becomes inconvenient for practical applications. For example, when $l = 4$, $a_i = 2$, $b_1 = b_2 = b_3 = b_4$, we have

$$\begin{aligned} &\varphi(\eta) = -b_1^4 (b_1 + 1)^4 \eta^{b_1 - 1} [20(1 - \eta) + 10(1 + \eta)\ln\eta + \\ &+ 2((1 - \eta)\ln^2\eta + \frac{1}{6}(1 + \eta)\ln^3\eta]. \end{aligned} \tag{14}$$

Assume, that $l = 10$, $\mu = 0.5$, $a_i = 1$ and $b_1 = \ldots = b_l \equiv b_i$, i.e. all parameters have equal μ_i. Parameters are divided into A, B, C groups with $r = 2$, $s = 3$, i.e. $A \in i = 1{,}2$, $B \in i = 3{,}4{,}5$, $C \in i = 6 \div 10$. We receive:

$\overline{\mu_i} = 1 - \mu_i = \frac{b_i}{b_i + 1},\ \mu = 1 - \overline{\mu_i}^{10} = \frac{1}{2},\ b_i = [\sqrt[10]{2} - 1]^{-1} = 13{,}93,$

$\mu_i = 0{,}067,\ g_i(\theta_i) = 19{,}93(1 - \theta_i)^{12{,}93},\ i = 1 - 10.$

In order to obtain beta-densities in groups and for entire ES, the condition $b_1 > b_2 \ldots > b_l$ according to (6-8) must be satisfied. Since $a = la_i = 10$, when $\mu = \overline{\mu} = 0.5$ we have $b = b_l = 10$ and $b_i = b_{10} + (10 - i)$, $i = 1 \div 9$. Then $b_9 = 11$, $b_8 = 12$, $b_7 = 13$, $b_6 = 14$, $b_5 = 15$, $b_4 = 16$, $b_3 = 17$, $b_2 = 18$, $b_1 = 19$.

Densities and main numerical characteristics according to groups for both cases are slightly different, but for entire product densities $g(\theta)$ of defectivity level θ and numerical characteristics μ, σ^2 practically coincides for both cases. It is obvious, that coincidence of all characteristics is the better the smaller μ value is.

Conclusions

1. For practical applications of stochastic defectivity modeling of ES products it is advisable to use beta-densities with offered limitations, since in this case we avoid integration procedures and therefore do not obtain complex models of densities.
2. In multiparametric case it is possible to perform the consistent joining of densities of separate parameters in pairs, thus narrowing the analysis down to biparametric models.

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Method is offered for synthesis of stochastic distributions of defectivity levels of multiparametric ES with interindependent parameters. This synthesis can be performed in groups of parameters or for entire product according to known distributions of defectivity levels of separate parameters. For practical applications it is advisable to differentiate average defectivity levels of separate parameters according to selected defectivity level of entire product, when ratio between defectivity levels in separate groups is selected or according to needed dispersion of parameters (selected variation coefficient). Ill. 1, bibl. 6 (In English; summaries in English, Russian and Lithuanian).

Д. Эйдукас. Стохастические модели качества электронных систем // Электроника и электротехника. – Каунас: Технология, 2008. – № 5(85). – С. 41–44.

Предложена методика синтеза вероятностных распределений уровня дефектности в отдельных группах параметров и для многопараметрических ЭС в целом по известным распределениям вероятностей уровней дефектности отдельных независимых параметров. Представлены модели синтеза плотностей вероятностей. Определены ограничения, при выполнении которых уровни дефектности в отдельных ЭС в целом описываются бета-распределением. Для практических приложений рекомендуется средние значения уровней дефектности отдельных параметров дифференцировать согласно заданному уровню дефектности в целом. Ил. 1, библ. 6 (на английском языке; рефераты на английском, русском и литовском яз.).

D. Eidukas. Stochastiniai elektroninių sistemų kokybės modeliai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 5(85). – P. 41–44.

Pasiūlyti elektroninių sistemų (ES) su nepriklausomais kokybės parametrais defektingumo stochastiniai modeliai. Siūloma skirstinių sintezei parametrų grupėse ir visai ES pagal žinomų atskirų parametrų defektingumo kokybės lygius vertinti β skirstiniais. Pateikti funkcijų sintezės modeliai, kai atskirų parametrų defektingumo lygiai pasiskirstę pagal beta dėsnį, esant minimalioms sveikaskaitinėms vieno iš formos parametrų vertėmis. Parodyta, kad tokiu atveju labai supaprastėja modeliavimas, nes nebereikalinga daugkartinio integravimo procedūra, taip pat nebereikia sudėtingų modelių. Praktiniams taikymams rekomenduojama atskirų parametrų vidutinius defektingumo lygius diferencijuoti pagal reikiamą parametrų sklaidą – pasirinktą variacijos koeficientą. Il. 1, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).