Synthesis of the Control System with Variable Structure and Limitations of Coordinates

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Introduction

The research of the problems related with automatic and optimal control involves the cases of the need for limiting various state variables of an object. Such limitations are possible due to the characteristics of an object, the principle of the work carried or the limitations of coordinates are introduced in order to achieve certain targets. Practically the most widely accepted are considered the control objects possessing the limitations of various coordinates or so called automatic electric drives. When optimizing the control of such drives in accordance with the requirements of technological processes one has to limit the time for starting and braking, the velocity of an motor for gaining torque or stopping, sometimes even the torque of a motor or current.

In accordance with the safety rules of the installation and maintenance of lifts the acceleration of a passenger car at the time of starting and braking shall not have to exceed 1.5 m/s². Due to that the control system of the lift drive has to form the optimal pathways for the car to gain torque and stop, and it is characterized by the values of the determined velocity, acceleration and acceleration expressions independently on the number of passengers present in the car.

Within the high speed levitation means of transport with linear motors there have to be designed the following braking modes of a coach such as service, additional, emergency braking when the allowable slowing down of a coach have correspondingly values of 1, 3 and 6 m/s² (i.e. negative linear acceleration values) [1].

In mechatronic systems an electric motor operates both under the modes of a motorizing and braking (generating). That is why in the process of energy change especially in the dynamic modes is changed due to load, torque and current of the motor. The submitted samples indicate that it is necessary to develop and perfect the systems of automatic control, reacting onto the changes of the state of an object and to be able quickly to change its structure and parameters up to the allowable value by limiting the required coordinates of an object.

The objective of the article is to determine the regularities of the optimal structure of an automatic control system as well as the parameter changes of the controllers when the object of control is in the mechatronic system and is considered to be an electric drive with a two mass mechanical system.

Differential Equations and Block Diagram

In realistic mechatronic systems the mechanical part of an electric drive is mostly composed of two mass system, peculiar for elasticity and possessing kinematic clearance [2]. When such a drive is operating or when its mode is changed there could appear transient and forced oscillations as well as dynamic loads, damaging other elements of mechatronic system. That is why these factors have to be not only limited but damped as well.

The two mass mechanical link of an electric drive is characterized by the following parameters:

- reduced moment of inertia \( J_1 \) of the elements connected with a shaft of a motor;
- reduced moment of inertia \( J_2 \) of the elements connected with a running device of a mechatronic system;
- load torque \( T_{\text{ar1}} \) of a shaft of a motor;
- load torque \( T_{\text{ar2}} \) of a running device;
- resilient bond between both masses is characterized by the equivalent stiffness \( c_{12} \).

In order to derive the transfer function of a control object, namely the automatic electric drive with dual mass mechanical link and limitation of coordinates and for to compile its block diagram it is required to analyze the second type Lagrange (J. L. Lagrange, 1736 –1813) differential equation. It describes the movement of a drive and relates various sorts of acting energy present in the drive [3]:

\[
\frac{d}{dt} \left( \frac{\partial W_k}{\partial \phi_i} \right) - \frac{\partial W_k}{\partial \phi_i} + \frac{\partial W_p}{\partial \phi_i} + \frac{\partial W_L}{\partial \phi_i} = T_i; \tag{1}
\]
where $W_k$ is kinetic energy; $W_p$ is potential energy; $W_d$ is dissipation of energy; $\omega_i$ and $\varphi_i$ are angular velocity and shift of $i$ link; $T_i$ is a generalized torque; $i = 1, 2, ..., n$.

For the electric drive with two mass mechanical link it is possible to write following the equation (1):

$$
\begin{align*}
J_1 \frac{d\omega_1}{dt} + c_{12}(\varphi_1 - \varphi_2) &= T_v - T_{st1}; \\
J_2 \frac{d\omega_2}{dt} - c_{12}(\varphi_1 - \varphi_2) &= -T_{st2};
\end{align*}
$$

(2)

where $\omega_1$ is the angular torque velocity of a shaft of a motor and the first equivalent mass connected with it; $\omega_2$ is the angular velocity of the second equivalent mass; $T_v$ is the torque of a motor.

The torque of an elastic interaction of the two masses:

$$
T_{12} = c_{12}(\varphi_1 - \varphi_2).
$$

(3)

When solving the equations (2) of the system in an operative way it is possible to derive the following transfer function of the two mass mechanical system:

$$
W_2(p) = \frac{\omega_2(p)}{T_v(p)} = \frac{c_{12}}{p(J_1J_2p^2 + c_{12}(J_1 + J_2))}.
$$

(4)

Further follows the analysis of the case when the electric drives with the direct current motors in mechatronic system, are controlled by changing the voltage of the armature. Then according to the equations (2 – 4) together with the equations of dynamics of the direct current separately excited motor it is possible to compile the control systems with block diagram for the limitation of coordinates presented in Fig. 1.

In the diagram of Fig.1 there is presented the following: $I_s$ and $R_s$ are both the current and resistance of the armature; $C$ is the constructive constant of the motor; $\Phi$ is the excitation flux; $T_e$ is the electromagnetic time constant.

Two mass elastic system has one angular frequency value, and when it is reached there appears mechanical resonance:

$$
\omega_{x2} = \sqrt{\frac{c_{12}(J_1 + J_2)}{J_1J_2}}.
$$

(5)

To ensure the smooth operation of all electric drives the control system of the variable structure has to form the following transitional process of velocity $\omega_k = f(t)$ and $\omega_2 = f(t)$, in order during the starting torque and braking acceleration shall not exceed the prescribed mean value. The highest angular acceleration in the drive is achieved during the initial transitional process:

$$
\varepsilon_{pr} = \frac{T_{pr}}{J_s};
$$

(6)

where $J_s$ is the total moment of inertia.

Then the condition of acceleration limitation is written in the following:

$$
\varepsilon = \frac{d\omega}{dt} \leq \varepsilon_{pr}.
$$

(7)

Due to the elastic bond of the two mass system $c_{12}$ the second mass has a tendency to oscillate more than the first. That is why the velocity transitional process $\omega_2 = f(t)$ of the electric drives with two mass mechanical link is not optimal (Fig. 2).

Fig. 1. Block diagram of a variable structure automatic control system for direct current drives with two mass mechanical link and with limitation of acceleration.
Employment of a changeable structure controllers

In formulating a problem of the control system of a changeable structure with coordinate limitations, by means of the components $x_i$, the variation laws of the parameter $\beta_i$ on which the structure of the system (the regulator, the correction element and etc.) depends, must be formed [4, 5].

Within the control interval $t_0 \leq t \leq t_f$, where $t_0 = 0$, making use of $i = 1, 2, ..., m$ discrete values of the components $\beta_i$ of the vector $\beta$, $k = mr$ - dimensional vector is introduced:

$$x = \{x_1 = \beta_1[0], x_2 = \beta_1[1T], ..., x_{k-1} = \beta_m[(r-2)T], x_k = \beta_m[(r-1)T]\};$$

(9)

where $T$ is the sampling period.

Let us assume that, with the boundary conditions and coordinate limitations $g_j[\omega, \beta(x, t)] \geq 0$, $j = 1, 2, ..., q$.

The problem of the control system of a changeable structure with coordinate limitations (12) – (13) is solved according to the diagram presented in [6] by applying simlex search algorithms and using the software package Kvazio 1 [7]. In this case we shall limit the derivatives of the speed $\omega_1$ and $\omega_2$, that is $|\epsilon| \leq \epsilon_{pr} = 0.4$.

**Results of simulation**

When solving the problem (19) – (13) by using software package Kvazio 1, we find the variation law of the controller coefficient $\beta_2(t)$ that is close to the optimal one (Fig. 3) to which the system of a changeable structure with the coordinate limitation $|\epsilon| \leq 0,4$ corresponds. Transitional process $\epsilon = f(t)$ of the angular acceleration is presented in Fig. 4, and the velocity of the optimized system is presented in Fig. 5.

There have been registered optimal transitional processes of the velocity and angular acceleration of the control system with two mass mechanical link.
The transitional process of the optimized system with two mass mechanical link is not considered to be an optimal one. To ensure fluent operation of all the drives it is necessary to limit the acceleration during the moment of starting and braking. There have been submitted the differential equations for the control systems with limitation of coordinates with the help of which there has been worked out the block diagram. The software package *Kvazio 1*, applied for the algorithmic synthesis of the control system, is composed of the algorithms of simplex search, formation of the impacts of control, and programs of adapting.

The law of variation of the controller gain of the control system with the limitation of acceleration has been developed, the transitional processes of the system have been optimized.

### Conclusions

1. There has been analyzed the control system of the variable structure of the direct current electric drives with two mass mechanical link as well as with acceleration limitation.

2. For the synthesis of the automatic control system there has been applied a software package *Kvazio 1* by the help of which has been determined the law of optimal variation of the gain of the controller.

### References


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The article analyses the issues of designing the automatic control systems possessing direct current motors with two mass mechanical link. Due to the two mass elastic bond the second mass has a tendency to oscillate more than the first. Because of the velocity of the electric drive with the two mass mechanical link the transitional process is not considered to be an optimal one. To ensure fluent operation of all the drives it is necessary to limit the acceleration during the moment of starting and braking. There have been submitted the differential equations for the control systems with limitation of coordinates with the help of which there has been worked out the block diagram. The software package *Kvazio 1*, applied for the algorithmic synthesis of the control system, is composed of the algorithms of simplex search, formation of the impacts of control, and programs of adapting.

The law of variation of the controller gain of the control system with the limitation of acceleration has been developed, the transitional processes of the system have been optimized.


Рассматривались вопросы синтеза системы управления, имеющей двигатель постоянного тока с двухмассовой механической частью. В такой системе из-за упругой связи вторая масса способна развивать нежелательные колебания. Поэтому переходной процесс системы с двухмассовой механической частью не является оптимальным, а для обеспечения плавной работы всех электроприводов необходимо ограничивать ускорение как при пуске, так и при торможении. Получены дифференциальные уравнения системы управления, по которым составлена структурная схема. Для синтеза системы управления применен компьютерный пакет *Kvazio 1*, в состав которого входят алгоритмы симплексного поиска, программы формирования воздействий управления и программы адаптации.

Получен закон изменения коэффициента регулятора и переходные процессы оптимизированной системы управления. Изд. 5, библ. 7 (на английском языке; рефераты на английском, русском и литовском яз.).