

Further Applications of the Fractal Spectra of the EEG Signals

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Introduction

The analysis of electroencephalographic (EEG) signals is usually done using tools generally adequate for linear signals. As it is well known that the EEG signals are extremely nonlinear signals, the need for suitable algorithms comes up.

One possible answer to this challenge may be the use of a new technique employed in the nonlinear spectral analysis which is based on the generalized entropy of a probability distribution, the so called Rényi entropy. It defines a set of fractal dimensions that characterize the time series from both the amplitude and the frequency point of view.

In what follows, we will review the theoretical background for the algorithm that allows the computing of the fractal spectrum in the next section. The third one is devoted to the characterization of the EEG signals, the fourth deals with the potential implications of the results in detecting the epileptiform activity and the possibilities of using the fractal spectrum as an indicator for the brain activities specific to a certain task; the last one is committed to the conclusions.

The fractal dimension based on the Rényi entropy

Dealing with EEG signals classification and taking into account the nonlinear character of these signals there are just a few possibilities to choose from when characterization is needed. One of them is to determine the so-called correlation dimension. This measure belongs to a wider, in fact infinite, class of fractal dimensions and this is why it seems possible to gather more information when computing the whole fractal spectrum, not only the correlation dimension itself.

The definition for the generalized entropy based on the moments of order r of the probability p_i , according to Rényi, [1], is given by

$$S_r = \frac{1}{1-r} \log_2 \sum_{k=1}^m p_k^r, \quad (1)$$

with $r \in \mathbf{R} - \{1\}$ and $p_i \in [0,1]$.

When dealing with EEG signals, the above probability is not known and therefore an adapted algorithm must be employed: the signal is divided into m intervals and for each one the times m_k that the signal passes through it is counted. Consequently, the probability for a generic interval is

$$p_k = \frac{m_k}{m}. \quad (2)$$

Knowing the above probabilities, the generalized fractal dimension of order r may be written as follows:

$$D_r = \lim_{\Delta x \rightarrow 0} \left(\frac{1}{1-r} \cdot \frac{\log_2 \sum_{k=1}^m p_k^r}{\log_2 \Delta x} \right), \quad (3)$$

where Δx is, in a practical circumstance, the smallest value of the signal that may be evidenced by the instrumentation used to record the EEG signal.

For a given probability distribution, the generalized fractal dimension D_r is named fractal spectrum; it provides information concerning both the amplitudes and frequency of the analyzed time series and that is why it is considered a better way to characterize it.

It is worth noticing the following particular cases for the above definition: 1) the correlation dimension, mentioned earlier, is obtained when $r = 2$ and 2) there are two limit cases, for $r = -\infty$ and $r = \infty$, when the fractal dimension is

$$D_{-\infty} = \lim_{\Delta x \rightarrow 0} \left(\frac{\log_2 p_{\min}}{\log_2 \Delta x} \right), \quad (4)$$

$$D_{\infty} = \lim_{\Delta x \rightarrow 0} \left(\frac{\log_2 p_{\max}}{\log_2 \Delta x} \right), \quad (5)$$

where

$$p_{\max} = \max\{p_k\}, \quad k = \overline{1, m}; \quad (6)$$

$$p_{\min} = \min\{p_k\}, \quad k = \overline{1, m}. \quad (7)$$

These two cases define the ranges of fractal dimensions and their difference, $D_{-\infty} - D_{\infty}$, is a strong indicator of the chaotic behavior of the time series: the bigger the difference, the better evidenced are the chaotic properties.

The presence of less expected values of the signal is evidenced by larger values of the fractal dimensions of the same order.

The data sets

The EEG signal seen as a chaotic time series is not a well-established topic. Some authors even conjecture that the nonlinear dynamics governing the brain may be seen as “consciousness”, at least at the evolutionary scale, or in pathology, [2] - [4]. Nevertheless, due to the obvious nonlinearities of the signal, the methods suitable for such signals are supposed to provide more accurate information than the ones usually employed for linear ones (e.g. FFT).

There are two major possibilities to characterize chaotic (or chaotic-like) signals: the Lyapunov coefficients and the fractal analysis. In what follows we shall focus on the latter in order to suggest a method suitable to fulfill the need to predict epileptic seizures and to differentiate between the various points of the scalp where the EEG signals are taken from, when dealing with P300 evoked potentials as the ones in the Donchin paradigm.

Classification of the epileptiform EEG was a research topic since 1990 and the different papers show the various stages of development of the signal processing tools involved, [5]-[8].

The EEG signals used for testing our method were those described in [9] and downloaded from [10].

There are four types of data sets, each one containing 100 files with 4096 samples, taken at a rate of 173.61 Hz, which is in the 128-1024 Hz. range, as recommended in [11]. First type of data are taken from a healthy subject, the second from a subject that has the disease but is between crises, the third set refers to the epileptogenic zone and the last is recorded during the crisis. The first 1000 samples of a typical EEG recording from a healthy person, as described in [9], are presented in Fig. 1. The first 1000 samples were evidenced for clarity reasons; the signals look quite the same for the rest of the samples. This remark is also valid for the rest of the figures.

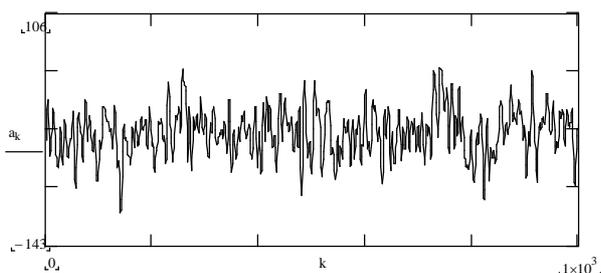


Fig. 1. Typical EEG signal from a healthy subject

The same type of signal, for an epileptic subject between the crises, which is slightly different from the one in Fig. 1, is presented in Fig. 2.

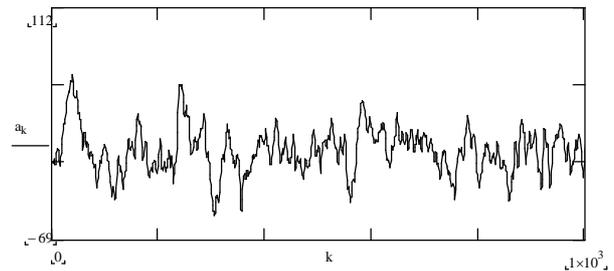


Fig. 2. The EEG from an epileptic subject between crises

The shape of the signal changes significantly in the epileptogenic region, as shown in Fig. 3, also for the first 1000 samples of the signal.

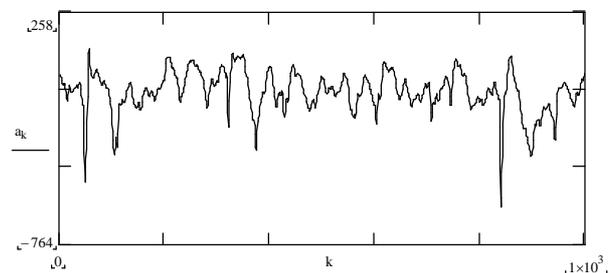


Fig. 3. The EEG of the epileptogenic zone

The last type of data was the one taken during epileptic seizure and looks like the one in Fig. 4.

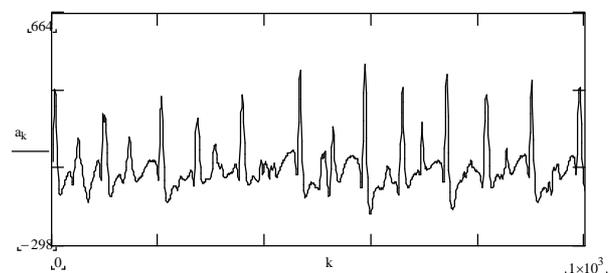


Fig. 4. The EEG signal during epileptic seizure

From the above figures, it may be clearly seen that there are strong differences between the four types of EEG signals. We will show that the shape of these signals are evidenced in the values of the fractal dimensions of the signals and that the difference $D_{-\infty} - D_{\infty}$ is significantly changed for the signals from a healthy person but also for the three different types for the ill subjects: between seizures, in the epileptogenic zone and during the seizure.

The same treatment was applied to signals taken according to the “10-20” system, [12], used in a Donchin paradigm, [13]. It assumes evidencing lines or columns on a 6x6 matrix with letters and numbers appearing on the screen of the computer; the subject is instructed to think of a symbol and to react every time the line/column that contains it is evidenced. This type of stimulus is an infrequent one, and it usually generates a “spike” (evoked potential) in the EEG signal, after approximately 300

milliseconds from its occurrence, hence its name: P300. This study aims to individualize the place on the scalp that has EEG signals with significantly higher difference $D_{-\infty} - D_{\infty}$ and in this way to suggest another way to choose the appropriate electrode for signal acquiring when dealing with feature translation problems in brain computer interfaces.

The fractal spectrum of the EEG signals as a measure of their chaotic properties

The algorithm for computing the Rényi entropy was programmed and running the program clearly showed the differences between the four types of signals, as follows: in Fig. 5 the fractal spectrum of a healthy subject is shown.

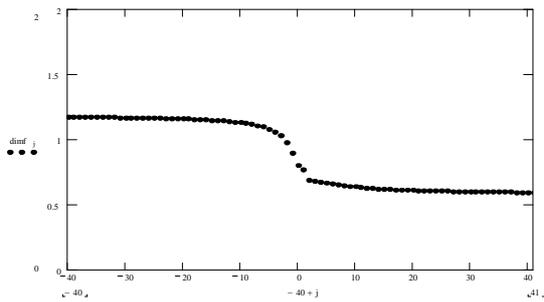


Fig. 5. The typical fractal spectrum for a healthy subject

It is worth noticing the fact that Fig. 5 presents the average values for all fractal spectrums that belong to the data set for healthy persons. This average was found to be in the same range with the one characterizing the periods between seizures for ill persons, as shown in Table 1, where only the maximum values are slightly higher in the case of an ill person. It is also worth noticing the fact that the average values for $D_{-\infty} - D_{\infty}$ are, in both cases, closer to those give by the maximum ones.

Table 1. Average values for all fractal spectrums

	healthy			ill		
	average	min	max	average	min	max
D_{∞}	0.640	0.517	0.714	0.575	0.348	0.714
$D_{-\infty}$	1.201	1.144	1.276	1.186	1.112	1.362
$D_{-\infty} - D_{\infty}$	0.560	0.626	0.561	0.610	0.763	0.647

Fig. 6 presents the fractal spectrum of the EEG signals in the case of the epileptogene zone.

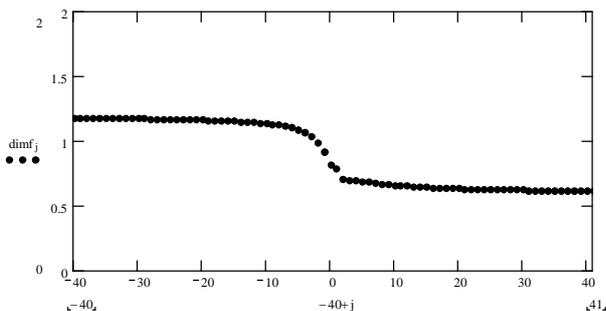


Fig. 6. The typical fractal spectrum for the epileptogene zone

Even if the average values are near the ones in Fig. 5, noteworthy differences may be seen in Table 2 that presents a comparison between the values for the fractal spectrum of the EEG signals from healthy persons and the one from the epileptogene zone of an ill subject.

Table 2. Comparison between the values for the fractal spectrum of the EEG signals

	healthy			ill		
	average	min	max	average	min	max
D_{∞}	0.640	0.517	0.714	0.551	0.155	1.070
$D_{-\infty}$	1.201	1.144	1.276	1.380	1.118	3.260
$D_{-\infty} - D_{\infty}$	0.560	0.626	0.561	0.829	0.963	2.190

In fact, all the values are higher in this case, but the differences are obvious, especially in the case of the maximum values, which are significantly greater.

The real difference may be perceived during seizures, when the values characterizing the fractal spectrum are significantly higher, even compared to the epileptogene zone, as it can be seen from Table 3.

Table 3. Comparison when the values characterizing the fractal spectrum are significantly higher

	healthy			ill		
	average	min	max	average	min	max
D_{∞}	0.640	0.517	0.714	0.730	0.168	1.195
$D_{-\infty}$	1.201	1.144	1.276	2.654	1.342	5.042
$D_{-\infty} - D_{\infty}$	0.560	0.626	0.561	1.923	1.174	3.846

As a rule, the value of $D_{-\infty} - D_{\infty}$ during seizures is on the average twice as high as the one for a healthy subject and this is a clear indicator of the state of illness.

The second part of the research concerning the fractal spectrum was dedicated to the way in which it can be used to choose which electrode is most suitable for signal acquiring in a “10-20” system employed in a Donchin paradigm. The signals were those obtained from Wadsworth BCI Dataset, [14].

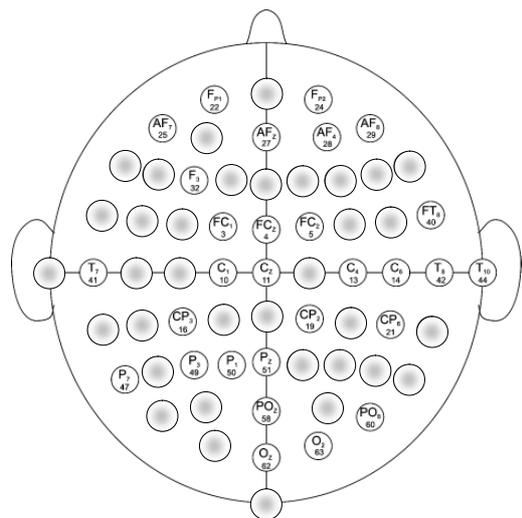


Fig. 7. The active electrodes in a P300 acquisition system from the point of view of the fractal spectrum

The available data sets were slightly modified so that only the P300 parts of the signal to be processed. The

fractal spectrum was computed for the signal of each electrode. The mean value (denoted by m) and the standard deviation (denoted by σ) of the difference $D_{-\infty} - D_{\infty}$ were also computed for all electrodes. Then, only those values outside the interval $[m-\sigma, m+\sigma]$ were considered as significant. A map of the scalp, in fact Fig. 2 from [15], in which only the active electrodes were left with a tag bearing their name, is presented in Fig. 7.

It is worth remembering that this result is in obvious concordance with other research in the field. However, the fractal spectrum, due to the amount of computing power needed is not suitable for applications that require high speed and, fortunately, the brain computer interfaces are still quite tolerant from this point of view.

Conclusions

Computing the fractal spectrum for the EEG signal evidenced the cases in which the condition of an epileptic subject is changing and therefore it may be used successfully as a detection algorithm for preictal states. In other applications involving brain-computer interfaces, like the one that evidences the active electrodes in a Donchin paradigm, the fractal spectra may be used as an initial setup or, in conjunction with other methods, to detect P300 evoked potentials, as an supplementary tool.

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A. M. Lazar, R. Ursulean. Further Applications of the Fractal Spectra of the EEG Signals // *Electronics and Electrical Engineering.* – Kaunas: *Technologija*, 2008. – No. 2(82). – P. 45–48.

The chaotic properties of the EEG signals are evidenced by means of fractal spectra. The method involved in computing is based on the so-called Rényi entropy. The fractal spectra are clearly changed according to the state of the subject and this proved to be a good indicator in preictal and ictal states of epileptic patients. Another possible application of the fractal spectrum is the one in which, in a Donchin paradigm, active electrodes may be evidenced based on P300 evoked potentials. Ill. 7, bibl. 15 (in English; summaries in English, Russian and Lithuanian).

A. M. Лазар, Р. Урсулеан. Исследования применения фрактального спектра сигнала EEG // *Электроника и электротехника.* – Каунас: *Технология*, 2008. – № 2(82). – С. 45–48.

Хаотические свойства сигналов электроэнцефалограммы свидетельствуются посредством фрактальных спектров. Метод основан на так называемой энтропии Rényi. Фрактальные спектры ясно изменены согласно состоянию предмета, и это хороший индикатор состояний эпилептических пациентов. Другое возможное применение фрактального спектра основан на парадигме Donchin'a, когда активные электроды характеризуется потенциалами P300. Ил 7, библи. 15 (на английском языке; рефераты на английском, русском и литовском яз.).

A. M. Lazar, R. Ursulean. EEG signalų fraktalinio spektro pritaikymo galimybės // *Elektronika ir elektrotechnika.* – Kaunas: *Technologija*, 2008. – Nr. 2(82). P. 41–44.

Chaotiškas EEG signalo savybes galima stebėti naudojant fraktalinį spektrą. Skaičiuojamasis metodas remiasi vadinamąja Rényi entropija. Fraktalinis spektras aiškiai kinta priklausomai nuo subjekto būsenos. Tai yra geras epilepsija sergančių pacientų būklės prieš priepuolį ir priepuolio metu indikatorius. Kitas galimas fraktalinio spektro pritaikymas – aktyvių elektrodų stebėsena remiantis P300 sužadintais potencialais ir Donchin'o paradigma. Il. 7, bibl. 15 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).