Methodology of Operation Efficiency Prognosis

V. Bagdonas, J. Daunoras, A. Dervinienė
Department of Control Technology, Kaunas University of Technology,
Studentų str. 48-320, LT-51367 Kaunas, Lithuania, phone: +370 37 300294, e-mail: alma.derviniene@ktu.lt

Introduction

Sometimes we have situations where decision-making, based on classical optimisation methods, is impossible. It happens, when making of the analytical objective function is complicated, when variables are physically unmeasured, and etc. Such problems associated with objects, action and processes fuzzily described. For example: estimation of reforms and reorganizations expedience. In similar case, methods based on expert estimations are used, that is, human intellect is being used as a measuring instrument.

The easiest way is to process estimations of the single expert. But to appeal to a single expert opinion could be too risky, because influence of subjective extraneous factor is quite distinct. Therefore methods of collection and processing group experts’ estimations are described.

The well-known and widely used group estimation methods are:
1) based on premise, that competence of all experts is equal;
2) based on robust decisions search.

Both methods have limitations. The fact that different experts have different competence is blinked in the first case. The “extreme” estimations (that necessarily are not incorrect) are usually eliminated in the second case (this is too risky when number of experts is small).

In this paper we analyse a methodology of processing group expert estimations (the methodology is based on search and use of experts’ competence rates). The experts competence rates can be get applying excess of the experts offered information.

Such methods and algorithms are described in literature [1-4], but they are not universal and applicable just in that case, when importance of goals independent from goals consumption. Therefore, our improved methodology (without noted limitation) of processing group expert estimations is discussed in this paper.

Criterions of Estimation

For estimation of an object, action or process $A$ the following procedures are executed:

The set $S_i = 1, n$ of $A$ goals (that specify objectives of creative object, executable action or realizable process) is formed. The set $S$ has to be full, but minimal, the goals must not overlap.

After interviewing of each expert such information is obtainable:

a) for estimation of each $A$ (there $A$ – object, action or process) goal $S_i$ its subjective importance rate $g_i \in [0,1]$ is suggested.

b) the consumption rate $h_i$ (further $h_i$) of each goal $S_i$ is prognosticated.

c) the complex rate (criterion) $e_A$ for aggregation of each $A$ goal importance’s and consummation’s estimations is offered.

Three typical variants of importance $g_i$ and consumption $h_i$ estimations aggregation into one complex rate (criterion) $e_A$ are suggested:

1. When importance of goals is independent from goals consumption;

2. When goals with high consumption rate $h$ are meaning (the goals with low consumption rate can be “contributed” under the goals with higher consumption rate);

3. When goals with low consumption rate $h$ are meaning.

Rates $h_i [0,1]$ and $e_A$ have to be quantitative, that is, measurable, calculable or subjectively rateable. They mostly are presented in normal numbers, that is, $e_A, h_i \in [0,1]$.

In the simple cases, when expert direct, that “importance of goals is independent from goals consumption rate $h$”, can be used formula [1-4]:

$$e_A = \sum_{i=1}^{n} g_i h_i / \sum_{i=1}^{n} g_i.$$  

(1)

Symbol “/” is sign of division. If fuzzy numbers are dividend, then instead ordinary division its adjective division is executed.
In another case (when aggregation formula (1) unacceptable for expert) the additional test for identification of \( g_i \) and \( h_i \) estimations aggregation variant into one complex rate \( e_A \) is suggested. In this case the test can be formulated as follows:

Let’s note value of complex rate \( e_A \), when is given:

<table>
<thead>
<tr>
<th>Importance rate ( g_i ) of each goal</th>
<th>0,6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption rate ( h_i ) of each goal</td>
<td>0,8</td>
</tr>
</tbody>
</table>

Such expert’s answers are usually met:
1) \( e_A = 0,6 \);
2) \( e_A = 0,48 \);
3) \( e_A = 0,3 \);
4) \( e_A = 0,14 \).

All these (“typical”) aggregation variants we can express applying parameter \( g_A \) and fuzzy integrals based on it [5].

The point of method is such:

When fuzzy set is given:

\[
B = h_1 / g_1 + h_2 / g_2 + \ldots + h_n / g_n, \quad (2)
\]

where \( 0 \leq g_i \leq 1 \),

it is possible to define for it (for set \( B \)) parameter \( g_A \),

whose rating parameter \( \lambda \) must supply condition

\[
-1 < \lambda < \infty; \quad (3)
\]

and can be finding as follows:

\[
\frac{1}{\lambda} \prod_{i=1}^{n} \left(1 + \lambda g_i \right) - 1 = 1; \quad (4a)
\]

or

\[
\frac{1}{\lambda} \prod_{i=1}^{n} \left(1 + \lambda g_i \right) - 1 = 1. \quad (4b)
\]

We have note, the sets of \( g_i \) and \( h_i \) estimations have no priority, that is, parameter \( g_A \) can be definite for both \( g_i \) and \( h_i \) sets or for its product.

If \( \lambda = 0 \), parameters \( g_A \) is converted into stochastic parameter. But solution \( \lambda = 0 \) (in equation (4)) is obtainable, when

\[
\sum_{i=1}^{n} g_i = 1 \quad \text{or} \quad \sum_{i=1}^{n} g_i h_i = 1. \quad (5)
\]

In other cases are available sub additive mates

\[
\left( \sum_{i=1}^{n} g_i > 1 \quad \text{or} \quad \sum_{i=1}^{n} g_i h_i > 1; \quad -1 < \lambda < 0 \right);
\]

or super additive mates

\[
\left( \sum_{i=1}^{n} g_i < 1 \quad \text{or} \quad \sum_{i=1}^{n} g_i h_i < 1; \quad \lambda > 0 \right).
\]

The fuzzy set (2) integral is expressed:

\[
S = \sup_{\alpha \in [0,1]} \min \{ \alpha; g_A \}. \quad (6)
\]

Using parameter \( g_A \), \( g_A \) in (6) is as follows:

\[
\hat{g}_A = \frac{1}{\lambda} \prod_{i=1}^{n} \left(1 + \lambda g_i \right) - 1; \quad (7a)
\]

or

\[
\hat{g}_A = \frac{1}{\lambda} \prod_{i=1}^{n} \left(1 + \lambda g_i h_i \right) - 1. \quad (7b)
\]

Formulas (4a), (6), (7a) are applied, when expert gives the first answer of test mentioned above.

When expert gives the second answer, formulas (4b), (6), (7b) are applied.

In all cases the complex rate \( e_A \) is expressed:

\[
e_A \to S. \quad (8)
\]

Here \( S \) calculated by (6) and (7a) or (7b)

That is fitted for a variant, when goals with high consummation rate \( h \) are meaning.

If goals with low consummation rate \( h \) are meaning, so it is enough to use some changed formulas (6), (7a) or (7b) as follows:

\[
S = 1 - \sup_{\alpha \in [0,1]} \min \{ \alpha; g_A \}; \quad (9)
\]

or

\[
\hat{g}_A = \frac{1}{\lambda} \prod_{i=1}^{n} \left(1 + \lambda g_i \right) - 1; \quad (10a)
\]

or

\[
\hat{g}_A = \frac{1}{\lambda} \prod_{i=1}^{n} \left(1 + \lambda g_i h_i \right) - 1. \quad (10b)
\]

Formulas (4a), (9), (10a) are applied, when expert gives the third answer of test mentioned above.

When expert gives the fourth answer, formulas (4b), (9), (10b) are applied.

If expert’s answers are at variance with 4 “typical” answers, the additional researches to define subjective low of \( g_i \) and \( h_i \) aggregation are required.

**Aggregation of estimations**

As we have already mentioned, for estimation of object, action or process \( A \) goal an expert is invited to suggest its importance rate \( g_i \) and consumption rate \( h_i \) together with the variant of theirs aggregation into one complex rate (criterion) \( e_A \) (or answer to the additional question mentioned above).

If we want to find and use expert’s competence rate \( \alpha_j \), we must ask expert to give \( e_A \) value by intuition (but not calculated by applying methodology mentioned above), what further is called \( h_A \).

We have note, that \( e_A \) and \( h_A \) are not the same: the \( e_A \) can be calculated by applying methodology mentioned above, while \( h_A \) the expert specifies as the overall effect.
Submitting $h_A$ value, the expert by intuition must aggregate (generalize) those data, which he suggests before ($h_i, g_i$ and aggregation variant).

Submitting $h_A$ value the expert specifies a Pareto set of object, action or processes $A$ estimations, while submitting $h_i$ and $g_i$ – an expert specifies a certain item of this set.

If expert can’t aggregate information right, his suggested value $h_A$ and calculated value $e_A$ will differ. The degree of their difference can be the rate of an expert competence. Naturally, such rate has meaning just in these cases, if estimation of $A$ is executed by the more than one expert.

Let’s say, that estimation of the object, action or processes $A$ is executed by $k$ experts.

To put the case that from experts we get all necessary information:

a) estimations $g_{ij}, h_{ij}$ or $h_{jA}$, ($j = 1, 2, \ldots, k$);

b) founded (calculated) complex estimations $e_{jA}$.

For finding $j$-th expert’s (we have $k$ experts) competence rate $\alpha_j$ ($j = 1, 2, \ldots, k$), we can use methods suggested in literature [1-4].

In that cases, when $h_{jA}(x), h_i(x)$ are the triangle membership functions, $j$-th expert’s competence rate is characterized by:

$$\beta_j = \max_{0 \leq \alpha \leq 1} \{ \alpha \| (h_{jA}(x) \cap H_{eA}(x)) \neq \varnothing \} \quad \text{(11)}$$

where $H_{\alpha}(x) = \{ x \| h(x) \geq \alpha \}$, $H_{\alpha}(h_{jA}(x))$, $H_{\alpha}(e_{jA}(x))$ – $\alpha$-level sets of fuzzy numbers $h_{jA}(x)$ and $e_{jA}(x)$.

The best illustration of $\beta_j$ is from geometrical point of view – it is a magnitude of both triangles $h_{jA}$ and $e_{jA}$ intersection point (see Fig. 1).

Unfortunately, for the trapezoid membership case [6] formula (11) is unfitted. In this case we would like to propose the complex indicator:

$$\beta_j = \left( \frac{2 \int_{\chi} (e_{jA}(x) \wedge h_{jA}(x)) \, dx}{\int_{\chi} e_{jA}(x) \, dx + \int_{\chi} h_{jA}(x) \, dx} \right)^{0.5} \quad \text{(12)}$$

"$\wedge$" means "minimum".

From geometrical point of view, $\beta_j$ is a magnitude of both trapezoids $h_{jA}$ and $e_{jA}$ intersection point (see Fig. 2). Integrals mean areas limited by both triangles and trapezoids.

![Fig. 2. Illustration of formula (12)](image)

In the case, when $h_{jA}(x)$ and $e_{jA}(x)$ are the equal isosceles triangles membership functions, $\beta_j$ values calculated by (11) and (12) are the same.

Unfortunately, coefficient $\beta_j$ underestimates distinctness of expert prognosis: the bigger is the area limited by $h_{jA}(x)$ and $e_{jA}(x)$ or the bigger are values of $\int_{\chi} e_{jA}(x) \, dx$ and $\int_{\chi} h_{jA}(x) \, dx$, the more distinct is $j$-th expert prognosis (but that make the bigger $\beta_j$).

In this case, we would like to propose a weight coefficient, characterizing expert’s distinctness of estimations:

$$\gamma_j = \min_{v \leq 1} \left\{ \frac{\int_{\chi} e_{vA}(x) \, dx + \int_{\chi} h_{vA}(x) \, dx}{\int_{\chi} e_{jA}(x) \, dx + \int_{\chi} h_{jA}(x) \, dx} \right\} \quad \text{(13)}$$

The index $v$ is the same as $j$ ($v = 1, 2, \ldots, k$) – it is the index of the expert.

The complex expert’s weight coefficient is expressed by:

$$\alpha_j = \beta_j \gamma_j, \quad j = 1, 2, \ldots, k \quad \text{(14)}$$

and the complex indistinct estimation of an object, process or action $A$ is characterized by:

$$H_A(x) = \frac{1}{k} \sum_{j=1}^{k} \alpha_j h_{jA}(x) \quad \text{(15)}$$

Operating with $h_{jA}(x)$ and $e_{jA}(x)$ defuzzyfied values $h_{jA}$ and $e_{jA}$ the complex distinct group expert estimations of an object, process or action $A$ are available that are characterized by:
The methodology of operation efficiency estimations compilation and processing (according many rates) performed by group expert is discussed in this article. The methodology is based on search and use of experts’ competence rates, what can be get applying excess of the information offered by experts. Such methods and algorithms are described in literature, but they are not universal and applicable

partial estimations aggregation into one complex rate (criterion) is very important.

5. The aggregation variant of expert’s subjective partial estimations into one complex rate can be expressed using fuzzy set integrals.

References


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В статье анализируется методика экспертной оценки (по множеству критериев) эффективности планируемых операций, включая правила сбора и обработки экспертных данных. Методика предусматривает определение и учет индивидуальных показателей компетенции экспертов, получаемых на основе избывочности экспертных данных. Представленные в научной литературе аналогичные по назначению методы и алгоритмы не обладают достаточной универсальностью, так как не могут быть применены в тех случаях, когда важность целей меняется в зависимости от степени их достижения. Предлагаемая методика свободна от этого недостатка. Методика основана на применении нечетких мер и нечетких интегралов. Ил. 2, библ. 6 (на английском языке; рефераты на английском, русском и литовском яз.).


Straipsnyje nagrinėjama operacijų efektyvumo grupių ekspertų įvertinimą (pagal daugelių kriterijų) rinkimo ir apdorojimo metodika, grindžiama ekspertų kompetencijos rodiklių piaieska ir panaudojimo. Eksperų kompetencijos rodikliai gali būti gaunami panaudojant jų (eksperų) teikiamas informacijos perteklių (excess). Mokslineje literatūroje aprašyti šio tipo metodai ir algoritmai nėra pakankamai universalūs, kadangi tinka tik tai atvejai, kai tikslų svarbumas nepriklauso nuo jų pasiekiamo laipsnio (consummation). Čia siūloma universalesnė (neturi to trūkumo) grupių ekspertinių įvertinimų apdorojimo metodika, grindžiama Sugeno matų ir neraiškių integralų panaudojimu. II. 2, bibl.6 ( anglų kalba; santraukos anglų, rusų ir lietuvių k.).