

## Estimation of Frequency Characteristics of Super-Narrow Band Digital Filters

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### Introduction

The modern digital signal processing makes rigid demands to the narrowity of bands and to the speed of filters' response. Earlier in [1-3] the so-called biline structures providing the bands of the orders  $\tilde{\Omega} \approx 10^{-13} - 10^{-15}$  were offered.

However, the verification of frequency characteristics of biline structures is rather problematic. The traditional way of an estimation of dynamic frequency characteristics by the impulse response (IR) is inconvenient in this case because of its excessively big demanded length. It does not allow investigating the frequency characteristics with necessary accuracy by means of fast Fourier transformation (FFT). The last one brings practically unpredictable errors because of insufficient precision of machine arithmetic and excessively big number of demanded arithmetic operations.

In work the new method of quality check of recursive narrow-band systems' synthesis is offered. It's based on an estimation of a deviation degree of the real system impulse response from the ideal one. The question of a choice of necessary duration of the impulse response and criteria of its maximum deviation from ideal is being discussed.

### Description of Research

It is obvious, that the calculation of the impulse response of the supernarrow-band filters is the problem demanding significant time. According to a principle of uncertainty, the length of the impulse response which has the spectrum that meets the frequency requirements increases at reduction of the digital filter's cut-off frequency. For an acceptable calculation of amplitude-frequency response (AFR) values of the digital filter with a passband of the order  $\tilde{\Omega}_1 = 1 \cdot 10^{-5}$  at least 4 million points of the impulse response are required.

Calculation of all these IR values takes approximately 55 seconds of the machine time and 1 GB of the operative memory. It is logical to assume that for analytical calculation of the filter's AFR with a passband of the order

$\tilde{\Omega} \approx 10^{-13} - 10^{-15}$  there can appear problems, both connected with a time character, and a machine realization.

In Table 1 the time of calculation of the impulse response of digital elliptic biline filter at various cut-off frequencies and at other invariable requirements is resulted.

**Table 1.** The dependence of the calculations time from minimal estimated length of the impulse response and cut-off frequency<sup>1</sup>.

Normalized Cut-off Frequency	IR Length, samples	IR Evaluation Time, sec
$1 \cdot 10^{-03}$	$2^{18} (\sim 2.6 \cdot 10^5)$	3.41
$1 \cdot 10^{-04}$	$2^{19} (\sim 5.2 \cdot 10^5)$	6.82
$1 \cdot 10^{-04}$	$2^{20} (\sim 1.0 \cdot 10^6)$	14.20
$1 \cdot 10^{-04}$	$2^{21} (\sim 2.1 \cdot 10^6)$	28.54
$1 \cdot 10^{-04}$	$2^{22} (\sim 4.2 \cdot 10^6)$	54.54
$1 \cdot 10^{-04}$	$2^{23} (\sim 8.4 \cdot 10^6)$	109.40

Moreover, processing a very long "tail" part of the IR by means of FFT can bring an unpredictable error in an estimation of dynamic frequency characteristics. This complex of problems forces to search for the alternative decisions of an estimation method of the digital filters' dynamic frequency characteristics.

As a base, let's consider the transfer function of a digital biline filter in Z-area, of the odd order in general

$$H(z^{-1}) = H_0 \frac{1 - z_0 z^{-1}}{1 - p_0 z^{-1}} \prod_{i=1}^{N-1/2} \frac{(z_i - z^{-1})(z_i^* - z^{-1})}{(p_i - z^{-1})(p_i^* - z^{-1})}. \quad (1)$$

We shall consider the even order of the filter for simplicity. For calculation of the ideal impulse response we shall present the expression (1) without the first co-factor in the form of the sum of first-order transfer characteristics, using residues:

<sup>1</sup> The modeling of all processes is carried out under AMD Athlon Duo Core 3.0 GHz Processor, 1 GB RAM, and environment: Matlab 6.5 Release 13.

$$\prod_{i=1}^{N/2} \frac{(z_i - z^{-1})(z_i^* - z^{-1})}{(p_i - z^{-1})(p_i^* - z^{-1})} = \sum_{i=1}^{N/2} \left( \frac{K_i z_i}{p_i - z^{-1}} + \frac{K_i^* z_i^*}{p_i^* - z^{-1}} \right) + 1, \quad (2)$$

or

$$\frac{B(z^{-1})}{A(z^{-1})} = \sum_{i=1}^{N/2} \left( \frac{K_i z_i}{p_i - z^{-1}} + \frac{K_i^* z_i^*}{p_i^* - z^{-1}} \right) + 1. \quad (3)$$

Then the nominators of the transfer functions of the bilines, that forms the last expression, can be defined the next way:

$$K_i z_i = \left. \frac{B(z^{-1})}{(p_i - z^{-1})} \right|_{z^{-1}=p_i} ; K_i z_i = \frac{B(p_i)}{2 \cdot \text{Im}(p_i)}. \quad (4)$$

It is obvious, that the impulse response of a separate digital unit (truncated biline), the transfer function of which is a part of the sum (3) is a power function:

$$y_i(n) = K_i \cdot z_i \cdot p_i^n = [K'_i = K_i \cdot z_i] = K'_i \cdot p_i^n, \quad (5)$$

where  $n = 1 : \infty$ .

Then the general ideal impulse response of the system is described by the expression (6):

$$y(n) = \delta(0) - \sum_{i=1}^{N/2} (K'_i \cdot p_i^n + K_i^* \cdot p_i^{*n}), \quad (6)$$

$$y(n) = \delta(0) - 2 \sum_{i=1}^{N/2} \left( |K'_i| |p_i|^n \cos(\arg(K'_i) + \arg(p_i) \cdot n) \right), \quad (7)$$

As an example let's consider the quality verification of time and frequency responses of the elliptic biline low-pass filter (LPF) at the following requirements: order  $N=12$ ; cut-off frequency  $\tilde{\Omega}_1 = 1 \cdot 10^{-3}$ ; pass-band ripple  $a_{\max} = 0.1 \text{ dB}$ .

We shall limit the length of the impulse response equal to  $2^{18}$  points. The differences between real and ideal time and frequency characteristics are shown in Fig. 1.

For a quantitative estimation of the distinction degree of dynamic characteristics we shall define the following relation:

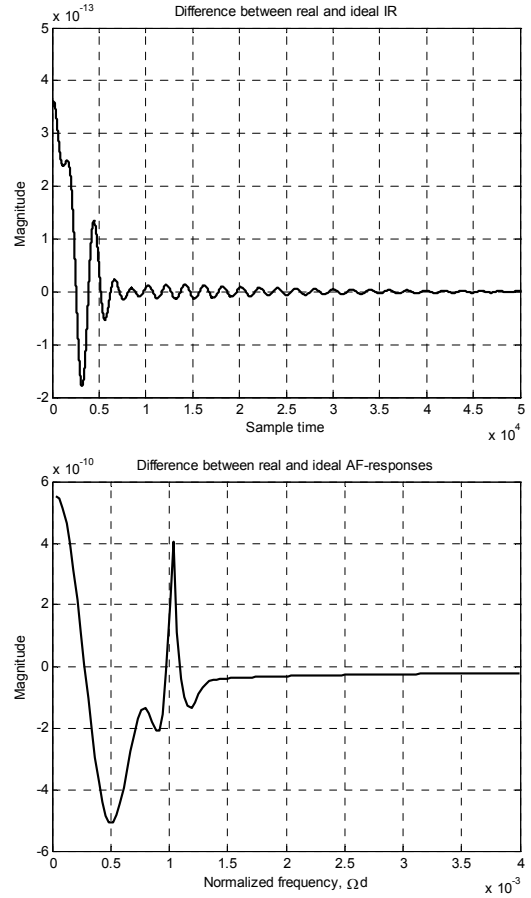
$$R = \frac{MSE_{\Delta}}{E_{\text{signal}}}, \quad (8)$$

where  $MSE_{\Delta}$  is a root-mean-square value of the difference between real and ideal impulse responses;  $E_{\text{signal}}$  – is the energy of the ideal response. In this case:  $MSE_{\Delta} = 7.781 \cdot 10^{-28} [V^2]$ ,

$$E_{\text{signal}} = 1.007 \cdot 10^{-3} [V^2]; R = \frac{7.781 \cdot 10^{-28}}{1.007 \cdot 10^{-3}} \cdot 100\% = 6.122 \cdot 10^{-18} \%$$

However, even without considering the opportunities of FFT, the time of DSP calculation of the whole impulse response of the digital filter with a passband of the

$\tilde{\Omega} \approx 10^{-13} - 10^{-15}$  order can reach the order of 1 million seconds at digitization frequency on an input of 1 GHz. In this case the fast verification of the dynamic frequency characteristics is practically impossible.



**Fig. 1.** The differences between real and ideal time and frequency characteristics of LPF

Let's consider the following approach. As it is known, any IIR-filter can be described by the system of finite number of difference equations, corresponding to the order of structure N. It means that the behavior of the IR of stable filter is completely defined at least by the first N samples. Therefore there's no necessity to count all of the IR values and then to process received hundred billions samples by means of FFT for comparison the real IR with an analytical signal.

Actually, it's enough to limit the lengths of estimated real and analytical IRs to a window in 2N-3N numbers of samples and compare them, for example, using least-squares method. It is possible to show that the attitude R remains, generally, constant or decreases with increase in lengths of calculated real and ideal impulse responses.

As an example let's consider the verification of time and frequency responses of the elliptic biline filter at the following requirements: order  $N=16$ ; cut-off frequency  $\tilde{\Omega}_1 = 1 \cdot 10^{-3}$ ; pass-band ripple  $a_{\max} = 0.05 \text{ dB}$ .

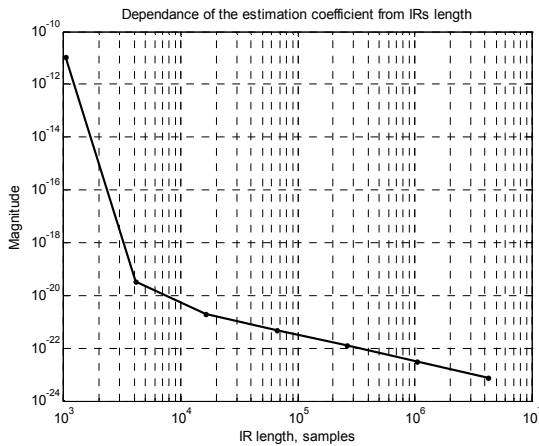
Let's show that at different requirements to the filter the relation R decreases with increase in length of the estimated real IR, otherwise, the speed of increase of a root-mean-square (RMS) mistake of estimation is less than

the speed of increase of cumulative signal energy. It gives the basis to suppose, that the considerable growth of IR length practically does not influence the quality of its estimation.

**Table 2.** The dependence of the coefficient of the estimation degree R on the duration of the real and analytical impulse responses

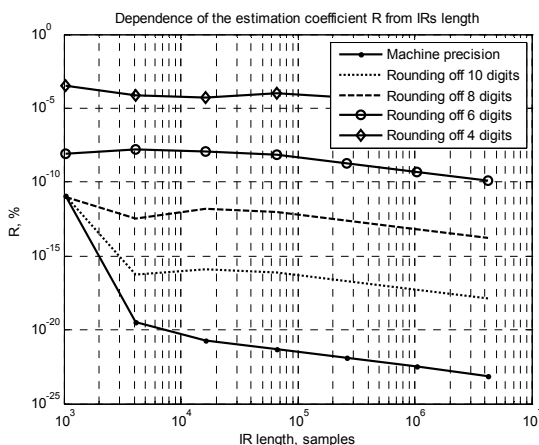
IR Length, samples	Coefficient R, %
$2^{10}$ (1024)	$1.11 \cdot 10^{-11}$
$2^{12}$ (4096)	$1.13 \cdot 10^{-16}$
$2^{14}$ (16392)	$2.55 \cdot 10^{-16}$
$2^{16}$ ( $\sim 6.5 \cdot 10^4$ )	$2.47 \cdot 10^{-16}$
$2^{18}$ ( $\sim 2.6 \cdot 10^5$ )	$3.67 \cdot 10^{-17}$
$2^{20}$ ( $\sim 1.1 \cdot 10^6$ )	$5.80 \cdot 10^{-17}$
$2^{22}$ ( $\sim 4.2 \cdot 10^6$ )	$2.61 \cdot 10^{-18}$

Graphically this dependence is presented in the Fig. 2.



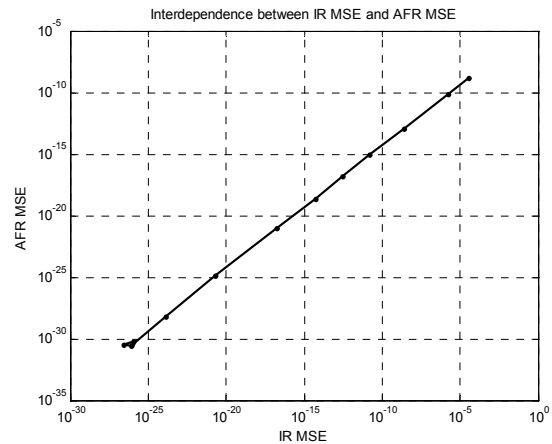
**Fig. 2.** The dependence of the coefficient of estimation degree R on the duration of the real and analytical impulse responses

Let's consider the estimation of the time response with inexact parameters, as imitation of restriction of precision of machine arithmetic. For this purpose we shall enter the additive noise in the form of a rounding off of coefficients of the transfer function (multipliers of the filter) and we will similarly consider the dependence of estimation criterion R on duration of the estimated IR



**Fig. 3.** The dependencies of the coefficient of estimation degree R on the duration of the real and analytical impulse responses

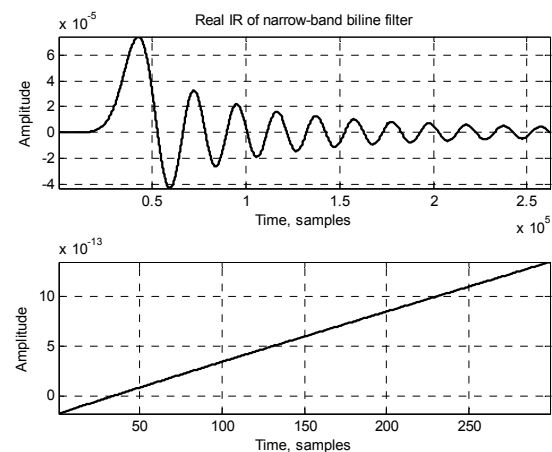
Let's consider the dependences of distinction degrees of the real and ideal impulse and frequency responses from of precision of machine arithmetic. The practically linear interdependence between root-mean-square errors at an estimation of time and frequency characteristics is shown on Fig. 4.



**Fig. 4.** The interdependence between mean-square-errors of IR and AFR of the narrow-band filter

It is necessary to notice that as the conversation is about the narrow-band and supernarrow-band digital filtration, it's possible to apply the Kotelnikov-Shannon theorem for the narrow-band signals. It has been proved that the minimal value of the digitization frequency  $F_d$ , where the exact reconstruction of the signal is possible, is defined not by upper signal frequency  $F_{up}$ , but by its passband width  $\Delta F$ . However, in this case it's not possible anymore to be limited by the samples of instant signal values.

It is obvious that the impulse characteristic of the narrow-band filter is enough smooth function of frequency, therefore for an estimation of a deviation degree of a real IR from the ideal one it's possible to decimate both characteristics with the decimation period in hundreds and thousand times more than the frequencies of samples following. As discussed above, it's enough to have only first  $2N-3N$  of samples of the real IR to compare it with an ideal one, but the order of magnitude in this range is too small relatively to the local extremums of the signal.



**Fig. 5.** The real impulse response of the biline narrow-band LPF

As an example we shall consider the verification of time and frequency characteristics of elliptic biline filter at following requirements: order  $N=16$ ; cut-off frequency  $\tilde{\Omega}_1 = 1 \cdot 10^{-4}$ ; passband ripple  $a_{\max} = 0.01 \text{ dB}$ .

The real impulse characteristic of the biline LPF is shown in Fig. 5.

As it's shown above, the magnitude of the first hundreds of samples has the order of  $10^{-13}$ , that is closer to the rounding errors, while first extremums has the order of  $10^{-5}$ . Thus the second suggested more nice estimation consists of the next steps: using the Kotelnikov theorem for narrow-band signals, take the samples of the first sequential local extremums and compare them with the values of the analytical signal. As the different experiment show, the results of an estimation of decimated time and frequency dynamic characteristics has the same order as shown above.

## Conclusions

The new method of verification of frequency characteristics of recursive narrow-band systems is offered.

The problem of a choice of necessary duration of the corresponding impulse response and criteria of its maximum deviation from ideal is being discussed.

The examples of correct characteristics verification of narrow-band digital filters with inexact parameters are shown.

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### T. Mamirov. Estimation of Frequency Characteristics of Super-Narrow Band Digital Filters // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 5(77). – P. 69–72.

The verification of frequency characteristics of narrow-band biline structures is rather problematic. The traditional way of an estimation of dynamic frequency characteristics by the impulse response (IR) is inconvenient in this case because of its excessively big demanded length. It does not allow investigating the frequency characteristics with necessary accuracy by means of fast Fourier transformation (FFT). The last one brings practically unpredictable errors because of insufficient precision of machine arithmetic and excessively big number of demanded arithmetic operations. The new method of verification of frequency characteristics of recursive narrow-band systems is offered. The problem of a choice of necessary duration of the corresponding impulse response and criteria of its maximum deviation from ideal is being discussed. The examples of correct verification of dynamic characteristics are shown. Ill. 5, bibl. 9 (in English; summaries in English, Russian and Lithuanian).

### T. Мамиров. Оценка частотных характеристик сверхузкополосных цифровых фильтров // Электроника и электротехника. – Каунас: Технология, 2007. – № 5(77). – С. 69–72.

Верификация частотных характеристик узкополосных билайновых структур весьма проблематична. Традиционный способ оценки динамических частотных характеристик по импульсной характеристике в данном случае затруднителен из-за чрезмерно большой требуемой ее длины. Это не позволяет исследовать частотные характеристики с необходимой точностью с помощью дискретного преобразования Фурье. Последнее вносит практически непредсказуемые погрешности из-за недостаточной разрядности машинной арифметики и чрезмерно большого числа требуемых арифметических операций. Предложен новый метод проверки качества синтеза рекурсивных узкополосных систем. Он основан на оценке степени отклонения реальной импульсной характеристики системы от идеальной. Обсуждается вопрос выбора необходимой длительности импульсной характеристики и критерии ее допустимого отклонения от идеальной. Ил. 5, библи. 9 (на английском языке; рефераты на английском, русском и литовском яз.).

### T. Mamirov. Upač siaurajuosčių skaitmeninių filtrų dažninių charakteristikų vertinimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 5(77). – P. 69–72.

Siaurajuosčių dvitiesių struktūrų dažninių charakteristikų patikra yra palyginti sudėtinga. Tradiciškai taikoma dinaminių dažninių charakteristikų analizė paremta impulsine reakcija (IR) yra nepatogi, dėl gana ilgos šios reakcijos trukmės. Dažninių charakteristikų taip pat negalima pakankamai tiksliai ištirti taikant greitąją Furjė transformaciją, nes atsiranda beveik neprognozuojamų klaidų dėl mašinų aritmetikos ir per didelio aritmetinių operacijų skaičiaus. Pasiūlytas naujas dažninių charakteristikų vertinimo metodas. Aptariama, kaip pasirinkti reikiamą atitinkamos impulsinės reakcijos trukmę ir jos maksimalaus nuokrypio nuo idealios kriterijų. Pateikta taisyklingos dinaminių charakteristikų patikros pavyzdžiai. Il. 5, bibl. 9 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).