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Series Summation RKHS-Method Applications for Radio-Physics Problems Simulations

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Introduction

The different types of series arise at solving a lot of boundary problems of mathematical physics, diffraction problems connected with propagation of electromagnetic waves: in the analysis of the scattering of a plane wave on a cylindrical screen structure, for related waveguide problems, for radiation from the flanged parallel plate waveguide and so on. For such problems Series Summation Method in RKHS can be useful [ ].

The purpose of the investigation is to demonstrate some important applications of the RKHS Series Summation Method.

The research problems are:

- Results applications at solving some problems of antenna theory;
- Results applications at solving electromagnetic close cavities problems;
- Results applications at solving problems concerns waveguide.

Application at Solving Radio-physics problem

Antenna theory. The problem of phased antenna array formed by parallel endless planes semi-bounded wave-guides is considered. The wave-guide planes are assumed extremely thin and ideally conducting, a is the distance between them. The incident field in every of wave-guides has the same amplitude and for every two contiguous of them is phase shifted by the same angle (in-phase excitation). Let θ_0 be the counted out from the x-axis angle of inclination of the antenna RP main lobe. Then a phase of the incident field in the m-th wave-guide should be described by multiplier $\exp(imu)$, where $u = ka\sin\theta_0$. Suppose that the incident field in wave-guide area consists only of a main wave of TEM type. For each wave-guide the field components are presented as follows:

$$H_y = \Psi = e^{ikx} e^{imu}, \; E_x = \frac{1}{i\omega\varepsilon}\frac{\partial}{\partial z}\Psi, \; E_z = \frac{1}{\omega\varepsilon}\frac{\partial}{\partial x}\Psi,$$

where $x < 0$ and $ma < z < (m+1)a$.

It is necessary to obtain the field radiated in free space and reflected to wave-guides. The main goal of this investigation is to obtain analytically expressions for electromagnetic field amplitude coefficients by Series Summation Method in RKHS.

The formulated problem is solved by the method of partial areas with the use of the Floquet harmonics. The method is applied for investigations of volumetric complex structures, which are decomposed into two or more simple adjacent areas. For each of them it is possible to get a solution by variable division. The first step consists in presenting unknown fields for each partial area in the form of expansion into eigen functions. In the rectangular co-ordinate system, components of the electromagnetic field constitute solutions of the Helmholtz equation in the corresponding area. Building the periodical solution of the Helmholtz equation satisfying boundary conditions is the matter of the Floquet theorem. Orthonormal functions of transverse co-ordinates form the system of scalar spatial harmonics (Floquet modes). On their basis, full systems of vector harmonics are built [1]. The explicit representations for inter-orthogonal functions are known. Thus, the problem is reduced to obtaining amplitude coefficients to eigen functions for the field expansion in every partial area. For this purpose, it is necessary to satisfy boundary conditions of the field. As a result, one can obtain an infinite system of linear algebraic equations (SLAE) relatively to unknown eigen wave amplitudes. In general case, it is impossible to find the accurate solution of this infinite system. Usually it is confined by the rough solution obtained with the help of methods of reduction or sequential approximations [2]. But in particular cases, it is possible to obtain the accurate solution either with the functional-theoretical method (a method of residues [3], a modified method of residues [3]) or with the Wiener-Hopf method. In the method of residues, an integral over a special analytical function is introduced, and then residues of the sub-integral function are associated with unknown coefficients.

From the continuity condition for tangent field components on a surface of the area $x = 0$, the following system of functional equations holds

$$e^{imu} + \sum_{n=0}^{\infty} A_n^m \cos\frac{n\pi(z-ma)}{a} = \sum_{p=-\infty}^{\infty} B_p e^{i\alpha_p z}, \quad (1)$$

$$ike^{imu}+\sum_{n=0}^{\infty}\omega_n A_n^m \cos\frac{n\pi(z-ma)}{a}=$$

$$=\sum_{p=-\infty}^{\infty}(-\Omega_p)B_p e^{i\alpha_p z}\ , \qquad (2)$$

where $ma<z<(m+1)a$, $\alpha_p=(2p\pi+u)/a$, $\Omega_p=(\alpha_p^2-k^2)^{1/2}$, $\omega_n=[(n\pi/a)^2-k^2]^{1/2}$.

To define unknown coefficients, rewrite (1), (2) as follows

$$1+\sum_{n=0}^{\infty}A_n^m e^{-imu}\cos\frac{n\pi(z-ma)}{a}=\sum_{p=-\infty}^{\infty}B_p e^{i\alpha_p(z-ma)}\ , \quad (3)$$

$$ik+\sum_{n=0}^{\infty}\omega_n A_n^m e^{-imu}\cos\frac{n\pi(z-ma)}{a}=$$

$$=\sum_{p=-\infty}^{\infty}(-\Omega_p)B_p e^{i\alpha_p(z-ma)}\ ,\ ma<z<(m+1)a\,. \quad (4)$$

Let us introduce designation: $z'=z-ma$. Then from the given equations there follows a relation for amplitudes of eigen waves in different wave-guides:

$$A_n^m=(-1)^n A_n^0 e^{imu}\ , \qquad (5)$$

where $n=0,1,2,...;$ $m=0,\pm1,\pm2,...$. It is easy to show that with $m=0$ formulas (3), (4) can be given as:

$$1+\sum_{n=0}^{\infty}(-1)^n A_n^0\cos\frac{n\pi}{a}z=\sum_{p=-\infty}^{\infty}B_p e^{i\alpha_p z}\ , \qquad (6)$$

$$ik+\sum_{n=0}^{\infty}(-1)^n A_n^0\omega_n\cos\frac{n\pi}{a}z=\sum_{p=-\infty}^{\infty}(-\Omega_p)B_p e^{i\alpha_p z}\ ,$$
$$0<z<a\,. \qquad (7)$$

Thus, the problem is reduced to consideration of one period $m=0$. From (6), (7) one can obtain the system of algebraic equations with respect to unknown coefficients $\{A_n\}$. For this, let us multiply both parts by $\exp(-i\alpha_q z)$ and integrate the result with respect to z, $z\in(0,a)$. This yields

$$\sum_{n=0}^{\infty}(-1)^n A_n^0\frac{1-(-1)^n e^{-iu}}{\omega_n^2-\Omega_q^2}+\frac{1-e^{-iu}}{\omega_0^2-\Omega_q^2}=\frac{a}{i\alpha_q}B_q\ , \quad (8)$$

$$\sum_{n=0}^{\infty}(-1)^n A_n^0\omega_n\frac{1-(-1)^n e^{-iu}}{\omega_n^2-\Omega_q^2}-\omega_0\frac{1-e^{-iu}}{\omega_0^2-\Omega_q^2}=$$

$$=\frac{a}{i\alpha_q}B_q(-\Omega_q)\ ,\quad q=0,\pm1,\pm2,...\ , \qquad (9)$$

where $\omega_0=-ik$. Further, equation (8) is multiplied by Ω_q and added with (9), what gives

$$\sum_{n=0}^{\infty}(-1)^n\frac{1-(-1)^n e^{-iu}}{\omega_n-\Omega_q}A_n^0=\frac{1-e^{-iu}}{\omega_0+\Omega_q}\ , \qquad (10)$$

$$\sum_{n=0}^{\infty}(-1)^n\frac{1-(-1)^n e^{-iu}}{\omega_n+\Omega_q}A_n^0-\frac{1-e^{-iu}}{\omega_0-\Omega_q}=$$

$$=\frac{i2a\Omega_q}{\alpha_q}B_q\ ,\quad q=0,\pm1,\pm2,...\,. \qquad (11)$$

From the problem statement it follows that $\{A_n^0\}$ are independent of q. Thus, from subsystem (10) for $q=0$ one can obtain

$$\sum_{n=0}^{\infty}(-1)^n\frac{1-e^{-iu}\cos\pi n}{\omega_n-\Omega_0}A_n^0=\frac{1-e^{-iu}}{\omega_0+\Omega_0}\,. \qquad (12)$$

Let's search unknown amplitude coefficients $\{A_n^0\}$ in the following form

$$A_n^0=\frac{A\varepsilon_n}{a(\omega_n+\Omega_0)(1-e^{-iu}\cos\pi n)}\ , \qquad (13)$$

where A is some constant, which is independent of n. Substituting (13) in (12), obtain

$$A=\frac{u(e^{-iu}-1)\sin u}{a(\omega_0+\Omega_0)}\,.$$

Thus, from subsystem (10) unknown amplitudes $\{A_n^0\}$ have been obtained in the form

$$A_n^0=\frac{(e^{-iu}-1)\varepsilon_n u\sin u}{a^2(\omega_0+\Omega_0)(\omega_n+\Omega_0)(1-e^{-iu}\cos\pi n)}\ , \quad (14)$$

ε_n is the Neumann number. It should be noted that representation (14) transforms (10) into identity [4, 5.139, p. 350]. After this, the coefficient B_q is defined from (11) and (14). For this purpose it is necessary to define sum of series in (11). Application of reproducing summing operator yields

$$B_q=\frac{\alpha_q}{2ia\Omega_q}\frac{e^{-iu}-1}{\omega_0-\Omega_q}\ ,\ q=0,\pm1,\pm2,...\,. \qquad (15)$$

Thus, the accurate solution of the problem of the field in a phased antenna array made of semi-bounded flat wave-guides at in-phase excitation is obtained. The formulated problem is solved with the use of the linking method with application of Floquet harmonics in co-operation with the Functional Expansion Method over Selective Values in Hilbert space with reproducing kernel. The latter one allows deriving alternative representations for unknown amplitude coefficients in explicit form in terms of more elementary functions. The considered problem was solved in [3] by method of residues. Amplitude coefficients have the following form

$$A_n^0=\frac{\operatorname{Res}f(\omega_n)}{1-(-1)^n e^{-iu}}\ ,\quad n=1,2,3,...\ , \qquad (16)$$

$$B_q = \frac{\alpha_q}{i2a\Omega_q} f(-\Omega_q)\,,\; q = 0, \pm 1, \pm 2, ... \qquad (17)$$

where $f(w)$ is the analytical function:

$$f(w) = \frac{1-e^{-iu}}{w+\omega_0} \exp\left[\frac{(w+\omega_0)a}{\pi} \ln 2\right] \frac{2}{1-w/\omega_0} \times$$

$$\times \prod_{m=1}^{\infty} \frac{(1+\omega_0/\omega_m)(1-w/\Omega_0)}{(1-w/\omega_m)(1+\omega_0/\Omega_0)} e^{-\frac{(w+\omega_0)a}{m\pi}} \times$$

$$\times \prod_{m=1}^{\infty} \frac{(1-w/\Omega_m)(1-w/\Omega_{-m})}{(1+\omega_0/\Omega_m)(1+\omega_0/\Omega_{-m})} e^{\frac{(w+\omega_0)a}{m\pi}}\,.$$

Apparently, there are infinite products in equations for amplitude coefficients (16), (17). Therefore, it is possible to obtain only approximate results for A_n^0 and B_q. The comparison of expressions (14), (15) and (16), (17) yields the following formula for infinite products

$$\operatorname{Res} f(\omega_n) = \varepsilon_n \frac{u \sin u}{a(\omega_n + \Omega_0)} \frac{e^{-iu}-1}{a(\omega_0+\Omega_0)}\,,$$

$$f(-\Omega_q) = \frac{e^{-iu}-1}{\omega_0 - \Omega_q}\,.$$

The obtained solutions are accurate. It is applicable for the numerical analysis of wave-guides of different sizes.

The exact analytical solution of electromagnetic field problem in flat waveguides array with a dielectric lamina is proposed [5]. The summation series method on selective values is applied to definition of the field amplitude factors. The accurate analytical solution of excitation problem by the phased antenna array of a composite frame is considered [6]. The problem is solved by a matching method with application of Floquet harmonics [7], but in combination with series summation method on selective values in RKHS. The method is used for obtaining amplitude field factors. Representations in analytical form without series have been obtained while in [7] the amplitudes are determined from infinite systems of linear algebraic equations including multiple bilateral series. The obtained results can be applied in practice for simulation of the indicated frame.

A wide range of diffraction wave problem by semi-infinite equidistant succession of identical obstacles with axial symmetry has been considered in [8], where the integral equation of kind (9) [See Part 1 of this paper] has been used [8, formula (5), p. 313]. Thus we can solve this solution by Series Summation Method in RKHS.

Electromagnetic Close Cavities

The electromagnetic cavity with co-axial plunger used in electro-optical modulators is considered. The given approach will be shown on an example of an interior boundary problem of electrodynamics about an electromagnetic field in the cylindrical cavity of coaxial type. Alternative representation for dispersion equation as analytical form without series is derived.

The problem about searching of eigen frequencies of electrical type oscillations in a cylindrical cavity with a coaxial turning plunger is considered. Suppose that b − radius and l − length of resonator, a − radius of a rod, d − interval from the left-hand resonator's wall to the left-hand rod's wall, that is the partial areas is determined by following intervals: *I*: $0 \le z \le d$, $0 \le r \le a$; *II*: $0 \le z \le l$, $a \le r \le b$.

It is necessary to derive a dispersion equation of described structure. Obtaining the dispersion equation is main goal of this investigation. It will be further obtained in analytical form by much-studied method of matching together Series Summation Method in RKHS.

Electromagnetic field of symmetrical modes of electrical kind can be expressed through one-component Hertz vector $\vec{\Pi} = \vec{z}_0 \Pi(r,z)$. Potential function can be written down as follows:

$$\Pi(r,z) = \begin{cases} \Pi_1(r,z) = \\ = \sum\limits_{n=0}^{\infty} \varepsilon_n A_n R_n(p_n r) \cos(\frac{\pi n}{l} z)\,, \\ \quad a \le r \le b,\; 0 \le z \le l\,; \\ \Pi_2(r,z) = \\ = \sum\limits_{m=0}^{\infty} \varepsilon_m B_m J_0(q_m r) \cos(\frac{\pi m}{d} z), \\ \quad 0 \le r \le a,\; 0 \le z \le d\,, \end{cases} \qquad (18)$$

where

$$R_n(p_n r) = J_0(p_n r) N_0(p_n b) - J_0(p_n b) N_0(p_n r)\,,$$

$$p_n = \sqrt{k^2 - (\pi n/l)^2}\,,\; q_m = \sqrt{k^2 - (\pi m/d)^2}\,, \qquad (19)$$

$\varepsilon_j, j = n, m$ – the Neumann number; J_0, N_0 – the functions of Bessel and of Neumann; A_n, B_m – the unknown numerical coefficients. The field components are determined by the formulas

$$E_z = \frac{\partial^2 \Pi(r,z)}{\partial z^2} + k^2 \Pi(r,z)\,,\; H_\varphi = ik \frac{\partial \Pi(r,z)}{\partial r}\,, \qquad (20)$$

and must satisfy following boundary conditions: 1) tangential components of the electrical field vector is equal to zero on interior cavity surface; 2) electrical and magnetic fields must be convenient on the bound of partial cavity areas.

Subjecting the tangent components of the electromagnetic field to the boundary conditions on the cavity surface one can obtain a direct formula for the coefficients B_m and a homogeneous system of linear algebraic equations in regard to unknown coefficients A_n:

$$p_n^2 A_n R_n(p_n a) -$$

$$-\theta\sum_{m=0}^{\infty}\varepsilon_m q_m K_{mn}\frac{J_0(q_m a)}{J_0'(q_m a)}\sum_{s=0}^{\infty}\varepsilon_s p_s A_s R_s'(p_s a)K_{ms}=0, \quad (21)$$

where $K_{ms}=\frac{1}{2}\left(\frac{\sin\pi(m-s\theta)}{\pi(m-s\theta)}+\frac{\sin\pi(m+s\theta)}{\pi(m+s\theta)}\right)$.

To derive dispersion equation rewrite (21) as follows

$$p_n^2 A_n R_n(p_n a)-$$

$$-\theta\sum_{s=0}^{\infty}\varepsilon_s p_s A_s R_s'(p_s a)\sum_{m=0}^{\infty}\varepsilon_m q_m K_{mn}\frac{J_0(q_m a)}{J_0'(q_m a)}K_{ms}=0 \quad (22)$$

Application of (4) [See Part 1 of this paper] the to series by index m in (22) yields

$$p_n^2 A_n R_n(p_n a)-\theta\sum_{s=0}^{\infty}\varepsilon_s A_s p_s^2 R_s'(p_s a)\times$$

$$\times\frac{J_0(q_s a)}{J_0'(q_s a)}\frac{n}{n+s}\frac{\sin\pi\theta(n-s)}{\pi\theta(n-s)}=0. \quad (23)$$

A condition of existence and uniqueness of its solution is vanishing of its determinant what results in the dispersion equation with respect to the eigenvalues k to be sought for, namely:

$$\det\{p_s^2[\delta_{sn}R_s(p_s a)-\varepsilon_s\theta R_s'(p_s a)\times$$

$$\times\frac{J_0(q_s a)}{J_0'(q_s a)}\frac{n}{n+s}\frac{\sin\alpha(n-s)}{\alpha(n-s)}]\}=0, \quad (24)$$

where k is involved in p_s and q_m, see (19); δ_{sn} – Kroneker's delta; $\alpha=\pi\theta$, $n=0,1,2,...$, $s=0,1,2,...$.

Thus in this problem the vector basis elements have been determined via solving Boundary Eigenvalue Problem for Laplacian in the same way as in Classical Electro-dynamics. The obtained by proposed method dispersion equation yields the transcendental form without series. Alternative representation for dispersion equation as analytical form without series is derived. Obtained solutions are more very exact.

Inhomogeneous waveguide structures

The problem about interaction of a moving ring current with inhomogeneous waveguide structure is solved by series summation method on selective values in a Reproducing Kernel Hilbert Space. This method allows analytically to obtain alternative representations for electromagnetic field coefficients without series.

The considered structure will consist of round wave guides with radiuses a and b, connected by a flange in a plane $z=0$. A source of a field is the ring current moving along an axis of structure with speed $v=\beta c$. The density of a current and its spectral amplitude are known.

The purpose of the investigation is to obtain direct formulas for calculation of amplitudes raised waveguide harmonics (unknown factors of an electromagnetic field) by method of summation of series on selective values in RKHS.

As is well known [See Part 1 of this paper] any function $f\in H$ in RKHS H is series expansion on selective values

$$f(s)=\sum_{k=1}^{\infty}f(t_k)\frac{2(st_k)^{1/2}}{\pi J_{\nu+1}(\pi t_k)}\frac{J_\nu(\pi s)}{(t_k^2-s^2)}, \; 0\le s<\infty, \quad (25)$$

where πt_k are positive zero points of cylindrical functions.

The formula (25) is equivalent

$$f(\frac{y_m}{\pi})=\sum_{s=1}^{\infty}f(\frac{\mu_s}{\pi})\frac{2(y_m\mu_s)^{1/2}}{J_{\nu+1}(\mu_s)}\frac{J_\nu(y_m)}{(\mu_s^2-y_m^2)}, 0\le y_m<\infty. \quad (26)$$

As shown in [9], unknown factors A_n are determined from follows infinite system of the linear algebraic equations:

$$\frac{h_n J_0(\mu_n)}{\mu_n}A_n+\sum_{s=1}^{\infty}\alpha_{ns}A_s=F_n, \; n=1,2,3,..., \quad (27)$$

where α_{ns}, F_n are described by known formulas.

The solution of infinite system (27) was spent in [9] by numerical ways. In this paper it is solved analytically by series summation method and yelds:

$$A_n=\frac{\mu_n F_n}{h_n J_0(\mu_n)}, \; n=1,2,3,.... \quad (28)$$

Taking into consideration (28) factors B_n are determined by the formula:

$$B_n=-\frac{J_1(y_n)}{J_0^2(\mu_n)}\frac{4iIk\rho}{\beta c^2}\frac{I_1(\Gamma\rho)}{I_1(\Gamma a)}(\Gamma^2 b^2+\mu_n^2)^{-1}. \quad (29)$$

Concluding remarks

The application of known results of RKHS-theory for solving the boundary electrodynamics problems gives possibility to simplify known methods and to receive on their basis the analytical solutions, that is represented as essential for the further numerical experiment. The proposed method is effective one and perspective mathematical vehicle for solution of the actual and practical problems. *The perspectives* of the following researches are: solving identities and theorems; solving summatory, integration, integration-summatory equations including multiple once and its systems and application to boundary radiophysics problems.

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S. Čiumačenko, V. Hahanov, O. Melnikova. Eilių sumavimo metodo taikymas radiofiziniams uždaviniams spręsti // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 7(63). – P. 5–9.

Analizuojami Žilberto erdvės metodai taikant RKHS branduolių metodus. Sumuojant šių metodų eiles, gautos analitinės formulės, kurios yra šių eilių baigtinės formos alternatyvos. Aprašomi nauji eilių teorijos rezultatai, kurie taikomi konkretiems uždaviniams spręsti. Pateikiami nauji eksperimentiniai rezultatai, kurie leidžia plačiau išanalizuoti teorinius teiginius, naudojamus RKHS eilėms sumuoti. Šie rezultatai panaudoti svarbiausių radijo fizinių uždavinių praktiniam pritaikymui. Bibl. 9 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

S. Chumachenko, V. Hahanov, O. Melnikova. Series Summation RKHS-Method Applications for Radio-Physics Problems Simulations // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 7(63). – P. 5–9.

Reproducing Kernel Hilbert Space (RKHS) Methods for Series Summation that allow analytically to obtain alternative representations for series in the finite form are considered. The new mathematical results of the series theory are used. The paper demonstrates the new results which develop theoretical statements of Series Summation Method in RKHS and application to the some radio-physics problems solving. Bibl. 9 (in English; summaries in Lithuanian, English and Russian).

С.В. Чумаченко, В.И. Хаханов, О.В. Мельникова. Применение RKHS метода суммирования при решении радиофизических задач // Электроника и электротехника. – Каунас: Технология, 2005. - № 7(63). – P. 5–9.

Рассматриваются методы гильбертовых пространств с воспроизводящими ядрами (ГПВЯ) для суммирования рядов, которые позволяют аналитически получать альтернативные представления рядов в конечной форме. Применяются новые результаты теории рядов к решению конкретных задач. Статья демонстрирует новые результаты, которые развивают теоретические положения метода суммирования рядов в ГПВЯ, и применение к решению некоторых практически важных задач радиофизики. Библ. 9 (на английском языке; рефераты на литовском, английском и русском яз.).