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An Engineering Approach to Time-Frequency Uncertainty Criteria

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Introduction

The uncertainty principle or duration-bandwidth theorem is a well known fundamental result in signal analysis and in quantum mechanics, see, e.g. reviews [1, 2] and references therein. In the information and communication technology a large number of theoretical and practical problems exists where estimation for time-frequency spreading product are needed to assess, for example, the width of spectral peaks by the duration of the measurement window [3, 4]. Often the principal limitations are not set separately to the time or frequency domain but for the time-frequency spreading product $\Delta t \Delta \omega$ (below "uncertainty product"). In quantum mechanics, this kind of uncertainty criteria are of fundamental importance allowing to relate, e.g. uncertainty of particle momentum to the spatial spreading of wavefunction [1, 2].

In present study we will rely on the traditional complex Fourier transform $s(t) \leftrightarrow S(\omega)$ [1–6]:

$$S(\omega) = \int_{-\infty}^{+\infty} s(t) \exp(-\mathrm{j}\omega t) dt , \tag{1}$$

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega) \exp(+\mathrm{j}\omega t) d\omega , \tag{2}$$

but at that we will compare two different methodologies for evaluation of time-limited signals:

1) The conventional theoretical approach based on standard deviations $\Delta t, \Delta \omega$ of time and frequency [1, 2, 5–8] (see Appendix A) that yield the traditional inequality of the Fourier Uncertainty Principle (FUP)

$$\Delta t \Delta \omega \geq 1/2 . \tag{3}$$

2) An engineering approach that detects signal widths on certain relative levels and yields the uncertainty relations in the limit value form

$$\Delta' t \Delta' \omega \to \text{const} , \tag{4}$$

separately for any harmonic of signal if the number of signal oscillation periods within the time window increases. In particular, our pre-work [9] yielded an approximate result $\text{const} \approx 7.582$ for the special case of rectangular time window and 50% detection height.

The problematic tasks for the conventional approach

The lower limit of inequality (3) characterizes exactly the single Gaussian pulses [1–6] and yields also rather acceptable uncertainty product estimation for other single pulses [9]. However, already in the case of the most common tasks of the pulse train type, the conventional standard deviations yield an unreasonably growing uncertainty product $\Delta t \Delta \omega$ with the increasing number of pulses N [9]. Fig. 1 summarizes the typical problematic tasks for what the conventional standard deviations of frequency are not related to the widths of spectral peaks.

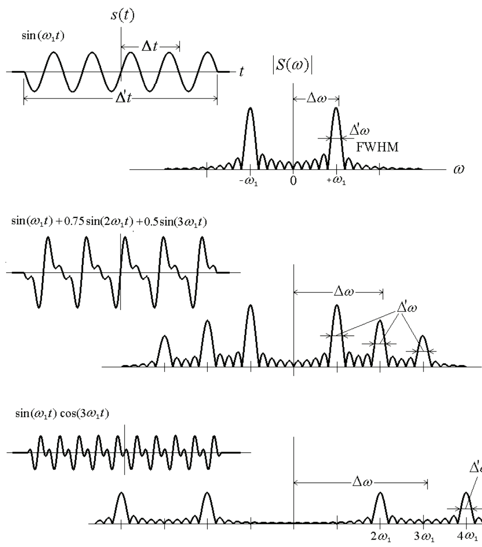

Fig. 1. The problematic tasks of pulse train signal (left) to Fourier spectrum (right) transforms for what the conventional methodology yields artificially increased uncertainty product values. The standard deviations $\Delta t, \Delta \omega$ are shown in comparison with one possible alternative definition: full pulse train duration in time domain $\Delta' t$ and the full width at half maximum (FWHM) of main spectral peaks $\Delta' \omega$

The first case in Fig. 1 considers the time-limited train of sine-pulses. The Fourier transform (1) yields at that two main spectral peaks at $\pm\omega_1 = \pm 2\pi / T_1$ where T_1 is the single pulse repetition period. The standard deviation methodology (see Appendix A) that integrates the square of frequency deviation from the average value yields here a large frequency uncertainty $\Delta\omega \approx \omega_1$ that is impractical for most of applications. In contrast, a relative definition $\Delta'\omega$, e.g., full width at half maximum (FWHM) of any of spectral main peaks, may offer a much more reasonable measure.

The second case in Fig.1 presents the polyharmonic signal that may also considered as an approximation for the arbitrary repeating pulse. As one can see, the conventional uncertainty of frequency $\Delta\omega$ tends to include the full band of peaks corresponding to different harmonics and not the width of individual main peaks that remain nearly constant if the relative definition at 50% of height would be used.

The third case in Fig.1 shows the carrier wave with a modulating signal. At that the conventional $\Delta\omega$ will be predominantly defined by the frequency of carrier wave in contrast to the modified definition $\Delta'\omega$ that is not influenced by the carrier wave.

In conclusion, due to artificially increased frequency spreading that follows the distances between spectral peaks and not the widths of individual peaks, the conventional methodology yields also the increasing uncertainty product $\Delta t \Delta\omega$ if the number of pulses increases [9]. An additional problem is that the signals with discontinuities on the time window border yield not enough rapidly vanishing spectra of the $1/\omega$ type. At that the standard deviation calculation methodology (Appendix A) fails to give a finite result.

Due to the abovementioned problems, the straightforward use of FUP (3) has been rather limited for signal processing tasks. Instead of that in engineering the rough time-frequency uncertainty estimations of the form

$$\Delta t \Delta f \approx 1 \,, \tag{5}$$

where $f = \omega / 2\pi$ is the ordinary frequency, have been often employed, e.g. [10-12]. The background of uncertainty relation (5) relies on Fourier transform of the rectangular pulse signal [3,4] (see below Figs. 2 and 3).

In terms of angular frequency $\omega = f / 2\pi$, the uncertainty relation (5) must be written as

$$\Delta t \Delta\omega \approx 2\pi \approx 6.28 \,, \tag{6}$$

that offers over 12 times larger uncertainty product than the lower limit of conventional FUP (3). The results for separate spectral peaks in Fig. 1 support clearly relation (6) rather than lower limit of (3). Below we will focus on the criteria in the form of (5) and (6).

It should be mentioned that to overcome the problem of double peaks (the first case in Fig. 1), earlier Kharkhevich has proposed a modified conventional methodology [13] that uses only positive frequencies for calculation of $\Delta\omega$. However, as the numerical calculations show [9], this truncated approach also does not yield the expected saturation of the uncertainty product at a certain level if the number of pulses increases. Additionally, the approach of Kharkhevich is not solving the problem of over increased $\Delta\omega$ for the cases of polyharmonic and the carrier wave shifted signals (the second and third example in Fig. 1).

The engineering approach for time-frequency uncertainty

To overcome the abovementioned problems, associated with the conventional methodology of standard deviations (Appendix A), the more practical approaches may be developed on the basis of the next principles:
1) Considering separately the different harmonics of signal (assumption that frequencies of the possible coexisting harmonic signals are separated enough compared to characteristic frequency $1/T$ of the time window);
2) Discussing only one half of spectrum (assumption that harmonic frequency is enough high $f >> 1/T$ that the influence of other spectrum half is negligible). This simplification relies on the fact the Fourier transform of time-windowed harmonic signal with frequency ω_1 creates two halves of spectra that are shifted by $\pm\omega_1$ (see Appendix B and Fig. 2).

Next, in contrast to the conventional theoretical approach that uses *the single Gaussian pulse as a primary model task*, the practical engineering approach is reasonable to develop on the basis of *the model task of rectangular pulse with a harmonic carrier wave*. This task is explained below in Fig. 2 and Fig. 3 and by the Appendix B.

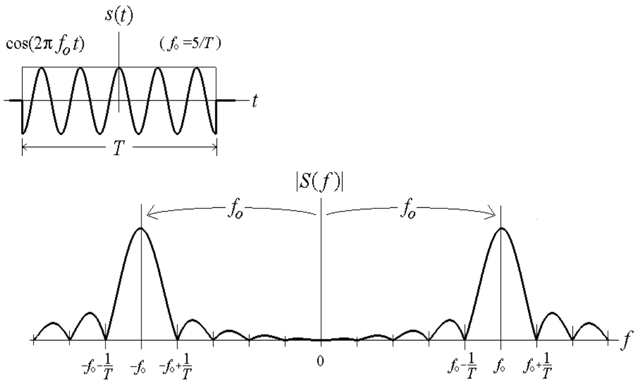

Fig. 2. The primary model task for engineering approach to time-frequency uncertainty: the harmonic signal in rectangular time-window (above) and the two halves of spectra that follow the well-known $\sin(\pi fT)/\pi fT$ function (below). If the harmonic signal is missing (has zero frequency) then the two spectral halves merge together near the zero frequency. In contrast, if the harmonic frequency is high compared to the characteristic window frequency $1/T$ then two spectral parts do not influence each other. Both limit situations allow us to analyze the uncertainty problem on basis of the window signal only

Fig. 2 explains one general property of Fourier transform: the harmonic signal within a time-window splits the window spectrum into two halves that are shifted by the signal frequency (formulae in Appendix B). If this frequency f is remarkably higher than the characteristic window frequency $1/T$ then the two halves of spectrum do not interfere and the analysis may focus only on the spectrum of the window function.

In the case of rectangular window model task

$$s(t) = A, \quad -T/2 \le t \le T/2, \tag{7}$$

the spectrum is the well-known sinc-function [3–6]

$$S(f) = AT\,\frac{\sin(\pi f T)}{\pi f T}. \tag{8}$$

Properties of function (8) are explained by the Fig.3 below.

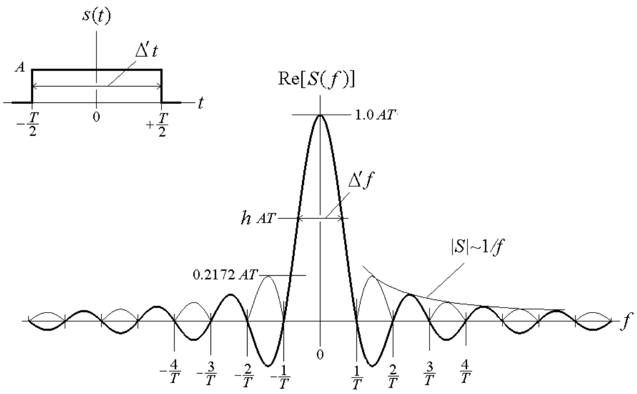

Fig. 3. The rectangular pulse (above) and its spectrum (below). By choosing the relative detection height parameter h is possible to obtain different spectral peak widths between 0 and $2/T$ and correspondingly different time-frequency uncertainty product values. Note that the relatively slow $1/f$ type decreasing rate of side-bands does not allow to obtain a finite standard deviation for frequency and thereby this very widely used signal cannot be analyzed by the conventional uncertainty methodology

Fig. 3 explains the important features of rectangular pulse that cannot be analyzed by the conventional approach as the slow $1/f$ decreasing rate of spectral side-bands does not allow to obtain a finite standard deviation for frequency. Important feature of this particular spectrum is that the zero points are separated by the characteristic window frequency $f_1 = 1/T$. This property is widely used in measurement applications [3, 4] and may be interpreted as a duration-resolution product relation $\Delta t \Delta f = 1$. From engineering viewpoint, the time duration (uncertainty) of this signal is well defined and equals to $\Delta' t = T$. As one can see, different widths $\Delta' f$ of the main spectral peak between 0 and $2/T$ may be obtained by varying the relative detection height parameter h. This corresponds to the range of uncertainty product $\Delta' t \Delta' f$ between 0 and 2. However, the detection heights below the value 0.2172 that is defined by the maxima of side lobes may be considered impractical in the presence of noise.

It is not difficult to conclude from the analysis of the spectrum (8) that for the rectangular pulse (window) the equation for definition of uncertainty products reads

$$\sin(\pi c_u / 2) = h \pi c_u / 4, \tag{9}$$

where the uncertainty product constant

$$c_u = \Delta' t \Delta' f. \tag{10}$$

In particular, the detection height $h = 0.5$ yields $c_u \approx 1.206709$. Multiplication of this number by 2π gives the constant 7.581977 that agrees with the approximate results in [9].

Fig.4 shows the dependence of the uncertainty product $\Delta' t \Delta' f$ on the selection of the detection height h. Additionally, the relative signal energy content $\int |S|^2\, df$ within frequency interval $\Delta' f$ in the vicinity of main spectral peak is evaluated. As one can see, $h = 50\%$ corresponds to the energy content 84.1 %.

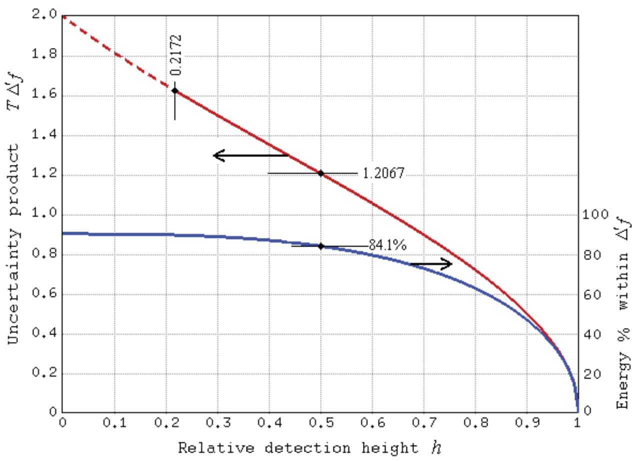

Fig. 4. The time-frequency uncertainty (duration-bandwidth spreading) product $T\Delta' f$ dependence on the selection of the relative height parameter h for spectral peak width detection in the case of the rectangular pulse (see Fig. 3). The corresponding energy content of the frequency interval $\Delta' f$ near the main spectral peak is displayed by the right y-axis

Thus, the full width at half maximum (FWHM) criterion yields here the uncertainty product $\Delta' t \Delta' f = 1.206709$. This may be a rather practical estimation for asynchronous (analog) spectral measurements where unknown but periodic sum of different harmonic signals is analyzed within a time window with abrupt borders. Use of the exact number $\Delta' t \Delta' f = 2$ that corresponds to the full width of spectral main peak at zero level may be questionable in the presence of noise.

Different time windows for the theoretical and engineering approaches

We proposed above that the engineering approach to uncertainty product criteria should rely on the model task of rectangular pulse that cannot be analyzed by the conventional approach. Now the questions arise what happens if the rectangular pulse is not ideal and, additionally, what results gives the FWHM criterion for the Gaussian pulse that is the primary model task of conventional theoretical approach.

To find answers for those questions we performed Fourier transform and uncertainty calculations moving from rectangular pulse through trapezoidal and triangle pulses towards more smooth cosine-signals and the

Gaussian pulse. The results are presented below in Fig. 5 and in Table 1.

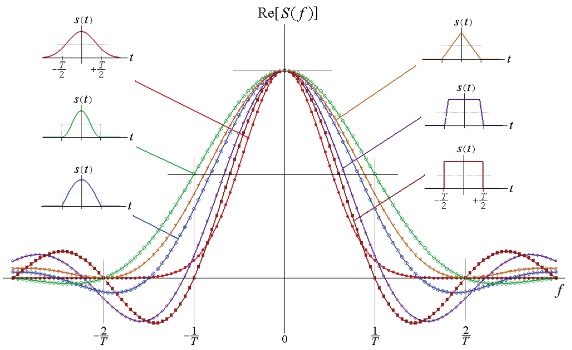

Fig. 5. The spectra of different time-window signals: Gaussian, raised cosine-fullwave, cosine-halfwave, triangle, trapezium, and rectangle pulse (all variants of the same window width except Gaussian; additional explanations in Table 1). Although the conventional approach based on standard deviations predicts remarkable differences in the uncertainty products, the FWHM definition for time and frequency yields rather close uncertainty product values. Note that the rectangular pulse that has no finite standard deviation of frequency has actually the narrowest main spectral peak if the same window width *T* could be applied

Table 1. Uncertainty products for different time-window signals, calculated by standard deviation methodology and by the 50% height methodology (*T* – width of time window)

Task	Theoretical approach (standard deviations)		Engineering approach (FWHM detection)	
	$\Delta t\Delta\omega$	$\Delta t\Delta f$	$\Delta' t\Delta' f$	$T\Delta' f$
Gaussian pulse	0.5	0.0796	0.8825	>2 ?
raised cos, one period	0.513	0.0816	1	2
cos-halfwave	0.567	0.0902	1.093	1.639
Triangle pulse	0.545	0.0867	0.8859	1.7718
Trapezium 50% plateau	0.694	0.1105	1.1592	1.5456
Trapezium 80% plateau	1.191	0.1896	1.2013	1.3348
Trapezium 90% plateau	1.719	0.2736	1.2055	1.2689
Rectangular pulse	∞	∞	1.2067	1.2067

The results in Table 1 support the usefulness of the offered here engineering approach. As already emphasized, the standard deviations based conventional approach cannot evaluate the rectangular pulse at all (Table 1, last row). Already the results for trapezoidal pulses start to increase rapidly if the side slopes of those pulses become more abrupt. Moreover, the disturbing one order of magnitude difference exists typically between conventional theoretical and practical uncertainty product values (difference between $\Delta t\Delta f$ and $\Delta' t\Delta' f$ columns). In contrast, the practical engineering approach yields very similar uncertainty product values around $\Delta' t\Delta' f \approx 1$ if for both – time and frequency domain the detection criterion at 50% level is used (column $\Delta' t\Delta' f$). Very interesting conclusion may be drawn by analyzing the last column in Table 1 where the time uncertainty is made equal to window width *T* (except for Gaussian pulse that has no a certain duration). In this comparison the rectangular pulse becomes the narrowest in terms of uncertainty and the obtained above numerical value 1.206709 marks the minimal limit. This result agrees with general concepts of spectral measurements that the rectangular pulse has the best resolution [3, 4, 14].

An application example – spectral resolution versus time-frequency uncertainty product

In the previous section we demonstrated that, for example, in the case of rectangular window of duration T the width of spectral peak as defined at 50% level should be $\Delta' f \approx 1.2067 / T$. The question arises, can this spectral spreading result for one signal employed also for resolution problem of two signals of close frequency within the same time window. This problem is analyzed below for two cos-signals of unit amplitude and different frequency f_1, f_2 in a unit window $T = 1$ [3, 4] (the frequencies must be high enough $f_1, f_2 >> 1/T$, for example, of 10 kHz range for millisecond window). Fig.6 shows the summary spectra for two opposite cases: the in-phase and reverse phase adding (phase difference in the centre of the time window).

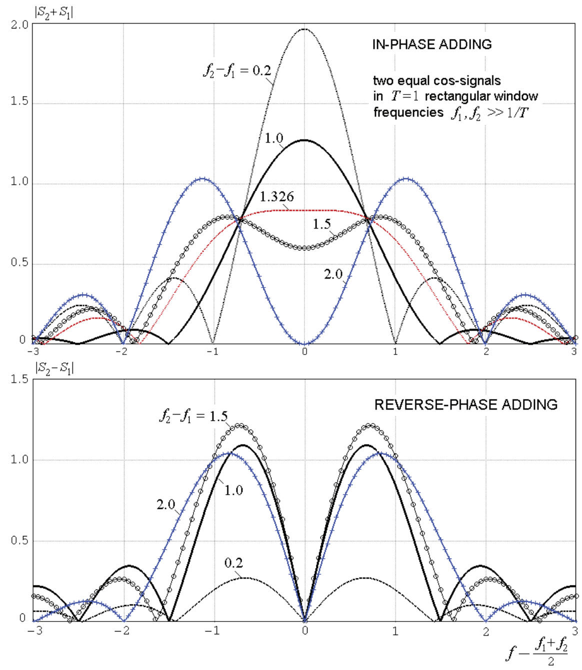

Fig. 6. The spectra for the sum of two time-limited cos-signals for in-phase adding (above) and reverse phase adding (below). The rectangular unit time window $T = 1$ is fixed and spectra are calculated for different frequency difference $f_2 - f_1$. It is assumed that the frequencies are high compared to $1/T$. In the case of in-phase adding a slight minimum between two peaks appears at $f_2 - f_1 \approx 1.326$ but detection of two separate peaks is possible starting approximately from $f_2 - f_1 > 1.5$. In the case of reverse phase adding the two peaks are separated by a difference ≈1.3 approximately up to $f_2 - f_1 < 1.5$

This considered task is of the field of time-limited asynchronous spectral measurements. In digital (synchronized) measurements, if is possible to assure the integer number of oscillation periods within the time window, the resolution limit is defined as $\Delta f = 1/T$ [3, 4, 14]. The calculation results in Fig.6 show that the possibility of detection of two frequencies by two separate peaks arises approximately starting from $f_2 - f_1 > 1.5/T$. Quite reasonable detection with accuracy +13% (in-phase adding) and -16% (reverse phase adding) is obtained only starting from $f_2 - f_1 > 2/T$. Thus for the reliable resolution of two signals the greater values of numerical constants should be considered in comparison of 1.2067 that characterizes the spectral spreading of a single signal.

Applications in quantum mechanics

The present engineering approach may be extended also to quantum mechanics as the spatial frequency of oscillating wavefunctions represents the momentum of a particle [1, 2, 9, 15]. The standard coordinate-momentum Heisenberg's uncertainty relation reads

$$\Delta x \Delta p \ge 0.5\hbar\,, \qquad (11)$$

that is the direct counterpart of FUP inequality (3) [2, 9, 15].

The given lower limit of uncertainty product characterizes fairly the lower states in quantum wells. For example, the ground state in deep square quantum wells with the coordinate-dependent wavefunction of the form of the half period of cos-function [9, 15], the uncertainty product equals $\Delta x \Delta p \approx 0.567\hbar$ that may be also obtained from the column 2 of Table 1. However, the higher states in quantum wells that resemble the first case in Fig. 1, yield much higher uncertainty products if we follow the conventional methology. The latter accounts for distance between spectral peaks and not the width of individual main peaks (the first case in Fig. 1). To obtain more reasonable estimations for individual peaks in momentum domain (i.e. of the spatial spectrum), the developed here engineering approach with numerical constants from Table 1 may be applied. In particular, in the idealized model case of infinitely deep square quantum well where the coordinate-dependent wavefunctions of higher states consist of integer number of sine/cosine halfwaves [9, 15], the respective coordinate-momentum uncertainty relation, corresponding to the momentum peak width detection at 50% level, reads

$$\Delta' x \Delta' p \to 1.206709 \cdot 2\pi \cdot \hbar \approx 7.581977\,\hbar\,. \qquad (12)$$

Conclusions

In the present study we discussed the Fourier transform and Fourier Uncertainty Principle and pointed out several drawback of the conventional methodology based on standard deviations:

a) The artificial increasing of spectrum spreading due to account for positive and negative halves of Fourier spectra;

b) Accounting of distances between of all spectral peaks in the case of multi-harmonic signal.

c) Infinite standard deviations of spectra in the case of rectangular pulses.

In order to overcome the mentioned problems and to present the time-frequency uncertainty criteria in a more suitable form for practical signal processing tasks, we offered "an engineering approach" based on the following principles:

a) Separated analysis of different harmonic signals;

b) Consideration of only main spectral peaks of different harmonic signals;

c) Consideration of only positive frequencies half of Fourier spectra;

c) Width detection at certain level (predominantly FWHM) for the width of signal in time and in frequency domains;

d) Use of rectangular time window as the basic model task (in contrast to the Gaussian signal of the conventional approach).

Appendix A: calculation of standard deviations

Standard deviations of signal spreading in time domain and in frequency domain, associated with the conventional formulation (3) of Fourier Uncertainty Principle, should be calculated as the square roots of the respective mathematical variances [1, 2, 5–8]:

$$\Delta t = \sqrt{Var(t)}\,, \qquad (13)$$

$$Var(t) = \int_{-\infty}^{+\infty} (t - \langle t \rangle)^2 s^2(t)dt \Big/ \int_{-\infty}^{+\infty} s^2(t)dt\,, \qquad (14)$$

$$\langle t \rangle = \int_{-\infty}^{+\infty} t s^2(t)dt \Big/ \int_{-\infty}^{+\infty} s^2(t)dt\,, \qquad (15)$$

$$\Delta\omega = \sqrt{Var(\omega)}\,, \qquad (16)$$

$$Var(\omega) = \int_{-\infty}^{+\infty} (\omega - \langle \omega \rangle)^2 S^2(\omega)d\omega \Big/ \int_{-\infty}^{+\infty} S^2(\omega)d\omega\,, \qquad (17)$$

$$\langle \omega \rangle = \int_{-\infty}^{+\infty} \omega S^2(\omega)d\omega \Big/ \int_{-\infty}^{+\infty} S^2(\omega)d\omega\,. \qquad (18)$$

Note that if the decreasing rate of the spectrum is $1/|\omega|$ or slower then the final $\Delta\omega$ cannot be obtained.

Appendix B: harmonic signal within a time window

Let us consider a time-limited harmonic signal $\cos(\omega_0(t - t_0))$ within a time window $p(t)$

$$s(t) = p(t)\cos(\omega_0(t - t_0))\,, \qquad (19)$$

that is also

$$s(t) = \frac{1}{2} p(t)\left[\exp(\mathrm{j}\omega_0(t - t_0)) + \exp(-\mathrm{j}\omega_0(t - t_0))\right] =$$

$$= \frac{1}{2} p(t)\exp(-\mathrm{j}\omega_0 t_0)\exp(\mathrm{j}\omega_0 t) + \frac{1}{2} p(t)\exp(\mathrm{j}\omega_0 t_0)\exp(-\mathrm{j}\omega_0 t)\,. \qquad (20)$$

If we denote the spectrum of the window function as $P(\omega) = \int p(t)\exp(-\mathrm{j}\omega t)dt$ then the full spectrum of the time-limited harmonic signal within that window becomes

$$S(\omega)=\frac{1}{2}\exp(-j\omega_0 t_0)\int p(t)\exp(-j(\omega-\omega_0)t)dt +$$
$$+\frac{1}{2}\exp(+j\omega_0 t_0)\int p(t)\exp(-j(\omega+\omega_0)t)dt =$$
$$=\frac{1}{2}\exp(-j\omega_0 t_0)P(\omega-\omega_0)+\frac{1}{2}\exp(+j\omega_0 t_0)P(\omega+\omega_0). \quad (21)$$

This is the halved spectrum of the window function: one half shifted to $\omega=\omega_0$ with phase multiplier $\exp(-j\omega_0 t_0)$ and second half shifted to $\omega=-\omega_0$ with phase multiplier $\exp(+j\omega_0 t_0)$ (Fig. 2).

Acknowledgement

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The conventional formulation of the Fourier Uncertainty Principle associated with the standard deviations methodology has several drawbacks that makes this theoretical approach often useless for practical applications. Beside of that a more practical engineering approach that discusses separately the different harmonics in any time-windows and detects signal spreading in time- and frequency domains by the relative levels, may be formulated. In contrast to the single Gaussian pulse of conventional theoretical approach, the primary model task of this engineering approach is the harmonic signal in rectangular time window. The engineering approach gives possibility to formulate the simple time-frequency uncertainty product constants, for example, 1.206709 for the 50% level detection criterion in the case of rectangular time window. Ill. 6, bibl. 15, tabl. 1 (in English; abstracts in English and Lithuanian).

A. Udal, V. Kukk. Laiko ir dažnio neapibrėžtumo kriterijaus inžinerinis nustatymas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 1(117). – P. 3–8.

Tradicinė Furjė neapibrėžtumo principo su standartinių nuokrypių metodologija formuluotė turi keletą trūkumų, kurie dažnai daro ją bevertę naudoti praktikoje. Nepaisant to, gali būti suformuluotas praktiškesnis inžinerinis sprendimas, kuris atskirai įvertintų skirtingas harmonikas bet kuriuose laiko languose ir detektuotų signalo sklaidos laiko ir dažnio srityse pagal kiekį. Pirminis inžinerinio sprendimo modelio uždavinys yra harmoninis signalas stačiakampiame laiko lange. Šis sprendimas sudaro sąlygas suformuluoti paprastas laiko ir dažnio neapibrėžtumo konstantas, pvz., 1,206709 50 % detektavimo lygio kriterijaus, kai laiko langas yra stačiakampis. Il. 6, bibl. 15, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).