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Cyclotron Frequency Oscillations in Semiconductor Plasma in Modulated Magnetic Field

Z. Jankauskas, V. Kvedaras
Department of Electrical Engineering, Vilnius Gediminas Technical University,
Saulėtekio av. 11, LT-10223 Vilnius, Lithuania. E-mail: Zigmantas.jankauskas@el.vtu.lt

Introduction

The plasmas that exist in solids offer an unique opportunity to observe plasma behaviour under well-defined and accurately known conditions. In a solid, one can usually determine quite precisely the number of charge carriers, their masses, their random heat energy and the boundaries of the plasma. For example, radio frequency magnetoplasmic waves known as helicons [1] will propagate in solid-state plasma of semiconductors when a strong magnetic field is applied. Helicons have an exact analogy with a electromagnetic whistler wave which is frequently propagated in the rarefied plasma of the Earth's ionosphere. In our experiments the modulated magnetic field is being used for excitation of helicons. It is shown that in the case of pulse modulated field along with the RF helicon waves the transient cyclotron frequency oscillations exist in the semiconductor plasma. For observation of the cyclotron radiation frequency ω_c the modulation depth about one percent is sufficient. The measurement of ω_c provides an opportunity to determine the charge carriers masses in the solid-state plasma of semiconductors.

Geometry of experimental device

The semiconductor sample (n-InSb) is in the form of plate [2]. The strong magnetic field *H* is directed along the axis *z* and is perpendicular to the sample plane *x-y*. The modulation depth of *H* varies from 0 to 10 percent. The sample dimensions in *x-y* directions are much larger than the sample thickness. In this case the electromagnetic wave (helicon) propagating in semiconductor is a circulary polarized plane wave of the form $\exp i(-kz+\omega t)$ with the components only *x* and *y*.

Equations of motion for one-component plasma

Assuming that the conductivity of the experimental plate is provided by the electrons with an isotropic mass we have the following equations of motion for the electrical current components j_x and j_y (Gauss units)

$$\frac{d}{dt} j_x + \gamma\, j_x + \frac{eH}{mc} j_y = \frac{Ne^2}{m} E_x, \qquad (1)$$

$$\frac{d}{dt} j_y + \gamma\, j_y - \frac{eH}{mc} j_x = \frac{Ne^2}{m} E_y, \qquad (2)$$

where N, e, m and γ are the density, charge, mass and reciprocal of collision time of the electrons. E_x and E_y are the components of varying electrical helicon field, and H is the strong magnetic field along the axis *z*, *c* – velocity of light.

Multiplying the eq. (2) by *i* and adding to the eq. (1) we obtain for the circulary polarised wave

$$j_x + i\, j_y = Ie^{i\omega t}, \qquad E_x + iE_y = Ee^{i\omega t}, \qquad (3)$$

$$\frac{d}{dt}\left(Ie^{i\omega t}\right) + \left(\gamma - i\omega_c\right)Ie^{i\omega t} = \frac{Ne^2}{m} Ee^{i\omega t} \qquad (4)$$

or

$$\frac{d}{dt} I + \left[\gamma + i(\omega - \omega_c)\right]I = \frac{Ne^2}{m} E, \qquad (5a)$$

$$\frac{d}{dt} I + \left[\gamma + i\Delta\omega\right]I = \frac{Ne^2}{m} E, \qquad \Delta\omega = \omega - \omega_c\,, \qquad (5b)$$

where

$$\omega_c = \frac{eH}{mc} \qquad (6)$$

is a cyclotron frequency of the electron, and *I* and *E* are the amplitudes of circulary polarized wave, ω – RF frequency electrical field.

The influence of pulse modulated magnetic field

Pulse modulated strong magnetic field H may be written in the form

$$H = H_0 + h \cdot u(t), \quad (7)$$

where time function $u(t)$ represents a square wave of rectangular pulses

$$u(t) = 1 \quad \text{for} \quad 2nT < t < (2n+1)T, \quad (8)$$

$$u(t) = 0 \quad \text{for} \quad (2n+1)T < t < 2(n+1)T, \quad (9)$$
$$n = 0; 1; 2; 3; ...$$

For simplicity the pulse duration and the interval between pulses are assumed equal to T.

Waveforms like (8) and (9) are very common in digital circuits [3].

In accordance with eq. (7) – (9) and (6)

$$H = (H_0 + h) = H_1 \quad \text{for} \quad 2nT < t < (2n+1)T, \quad (10)$$

$$H = H_0 = H_2 \quad \text{for} \quad (2n+1)T < t < 2(n+1)T, \quad (11)$$

$$\omega_c = \omega_{c1} = \frac{eH_1}{mc} \quad \text{for} \quad 2nT < t < (2n+1)T, \quad (12)$$

$$\omega_c = \omega_{c2} = \frac{eH_2}{mc} \quad \text{for} \quad (2n+1)T < t < 2(n+1)T. \quad (13)$$

The solution of eq. (5b) for current amplitude I are correspondingly

$$I = \frac{Ne^2}{m} E \cdot \frac{1}{\gamma + i\Delta\omega_1} \left(1 - A_1 \exp[-(\gamma + i\Delta\omega_1)t]\right) \quad (14)$$
$$\text{for} \quad 2nT < t < (2n+1)T,$$

$$I = \frac{Ne^2}{m} E \cdot \frac{1}{\gamma + i\Delta\omega_2} \left(1 - A_2 \exp[-(\gamma + i\Delta\omega_2)t]\right) \quad (15)$$
$$\text{for} \quad (2n+1)T < t < 2(n+1)T,$$

$$\Delta\omega_1 = \omega - \omega_{c1}; \quad \Delta\omega_2 = \omega - \omega_{c2}, \quad (16)$$

where A_1 and A_2 are functions of $\Delta\omega_1$; $\Delta\omega_2$; γ and T.

In the case, when

$$T \geq 3\gamma^{-1} \quad (17)$$

we have

$$A_1 = \frac{i(\Delta\omega_2 - \Delta\omega_1)}{\gamma + i\Delta\omega_2}, \quad A_2 = \frac{i(\Delta\omega_1 - \Delta\omega_2)}{\gamma + i\Delta\omega_1}. \quad (18)$$

Thus, the current amplitude I alongside with the constant part contains the oscillatory transients of the frequencies $\Delta\omega_1 = \omega - \Delta\omega_{c1}$ and $\Delta\omega_2 = \omega - \Delta\omega_{c2}$. The helicon frequency ω is of RF range and ω_{c1} and ω_{c2} are of infrared range. Therefore, practically $\Delta\omega_1$ and $\Delta\omega_2$ coincides with the corresponding cyclotron frequencies, eq. (16). If $\Delta\omega_2 = 0$ and $\Delta\omega_1 >> \gamma$, then for the time $t = \pi / \Delta\omega_{c1}$ the current I has the value of opposit sign in comparison to its initial value for $t = 0$. Consequently the eq. (5b) and its solutions (14), (15) describe also the well known effect of density inversion when conductivity becomes negative.

It should be noted that the eqs. (5a, b) contain only the difference of the frequencies ω and ω_c, e.g. the frequency modulation of helicon wave and modulation of magnetic field are equivalent.

The transient oscillations of the semiconductor plasma's response with the cyclotron frequency experimentally were observed [4] for the modulation depths of the magnetic field about one percent, e.g. for $h / H_0 = 0.01$, see eq. (7).

The observation of cyclotron radiation especially is effective if the detected response signal is proportional to the time derivative of I when the signal becomes much stronger (proportional to ω_c).

The real part of the ratio I/E is equal to the conductivity of semiconductor plasma and the imaginary part is responsible for helicon wave propagation (effective dielectrical constant). We have from eqs. (14), (15)

$$\mathrm{Re}\frac{I}{E} = \frac{Ne^2}{m} \frac{1}{\gamma^2 + \Delta\omega_1^2} \left\{ \gamma + \frac{\Delta\omega_2 - \Delta\omega_1}{\gamma + \Delta\omega_2^2} \times \right.$$
$$\times \left[(\gamma^2 - \Delta\omega_1 \cdot \Delta\omega_2) \sin \Delta\omega_1 t - \gamma(\Delta\omega_1 + \Delta\omega_2) \cos \Delta\omega_1 t \right]$$
$$\left. \exp(-\gamma t) \right\}, \quad (19)$$

$$\mathrm{Im}\frac{I}{E} = -\frac{Ne^2}{m} \frac{1}{\gamma^2 + \Delta\omega_1^2} \left\{ \Delta\omega_1 + \frac{\Delta\omega_1 - \Delta\omega_2}{\gamma + \Delta\omega_2^2} \right.$$
$$\left[-\gamma(\Delta\omega_1 + \Delta\omega_2) \sin \Delta\omega_1 t + (\Delta\omega_1 \cdot \Delta\omega_2 - \gamma^2) \times \right.$$
$$\left. \times \cos \Delta\omega_1 t \right] \exp(-\gamma t) \Big\} \text{ for } \quad 2nT < t < (2n+1)T, \quad (20)$$

and

$$\mathrm{Re}\frac{I}{E} = \frac{Ne^2}{m} \frac{1}{\gamma^2 + \Delta\omega_2^2} \left\{ \gamma + \frac{\Delta\omega_1 - \Delta\omega_2}{\gamma^2 + \Delta\omega_1^2} \left[(\gamma - \Delta\omega_1 \cdot \Delta\omega_2) \times \right. \right.$$
$$\left. \sin \Delta\omega_2 t - -\gamma(\Delta\omega_1 + \Delta\omega_2) \cos \Delta\omega_2 t \right] \exp(-\gamma t) \Big\}, \quad (21)$$

$$\mathrm{Im}\frac{I}{E} = -\frac{Ne^2}{m} \frac{1}{\gamma^2 + \Delta\omega_2^2} \left\{ \Delta\omega_2 + \frac{\Delta\omega_2 - \Delta\omega_1}{\gamma^2 + \Delta\omega_1^2} \times \right.$$
$$\left[-\gamma(\Delta\omega_1 + \Delta\omega_2) \sin \Delta\omega_2 t + + (\Delta\omega_1 \cdot \Delta\omega_2 - \gamma^2) \cos \Delta\omega_2 t \right]$$
$$\exp(-\gamma t) \Big\} \text{ for } \quad (2n+1)T < t < 2(n+1)T. \quad (22)$$

Eqs. (19–22) show that semiconductor plasma's conductivity and effective dielectrical constant are modulated by the cyclotron frequency.

Sinusoidal modulation

The above stated method may used also in the case of the sinusoidal modulation of the magnetic field

$$H = H_0 + h\cos\Omega t. \qquad (23)$$

Substituting (23) into (5) we obtain the steady state solution for current I in the form

$$\frac{I}{E} = \frac{Ne^2}{m}\exp\left[-\gamma t + i\omega_{co}t + i\frac{\omega_c^{`}}{\Omega}\sin\Omega t\right]\times$$
$$\times\int\exp\left[\gamma t - i\omega_{co}t - i\frac{\omega_c^{'}}{\Omega}\sin\Omega t\right]dt, \qquad (24)$$

here $\omega_c = \frac{eH_0}{mc}$ and $\omega_c^{'} = \frac{eh}{mc}$.

On the basis of formula for Bessel functions $I_l(\beta)$ [5]

$$\exp(-i\beta\sin x) = \sum_{l=-\infty}^{\infty} I_l(\beta)\exp(-ilx). \qquad (25)$$

We can write instead of (24)

$$\frac{I}{E} = \frac{Ne^2}{m}\sum_{l=-\infty}^{\infty} I_l\left(\frac{\omega_c^{'}}{\Omega}\right)e^{il\Omega t}\sum_{s=-\infty}^{\infty}\frac{I_s\left(\frac{\omega_c^{'}}{\Omega}\right)}{\gamma - i(s\Omega+\omega_c)}e^{-is\Omega t} =$$
$$= \sum_{l=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} I_l\left(\frac{\omega_c^{'}}{\Omega}\right)I_s\left(\frac{\omega_c^{'}}{\Omega}\right)e^{-i(l-s)\Omega t}\frac{\gamma + i(s\Omega+\omega_c)}{\gamma^2 + (s\Omega+\omega_c)^2} =$$
$$= \sum_{s,k=-\infty}^{\infty} I_s\left(\frac{\omega_c^{'}}{\Omega}\right)I_{s+k}\left(\frac{\omega_c^{'}}{\Omega}\right)e^{-ik\Omega t}\frac{\gamma + i(s\Omega+\omega_c)}{\gamma^2 + (s\Omega+\omega_c)^2}. \qquad (26)$$

By separation of the real and imaginary parts of (26) the plasma's conductivity and dielectrical constant may be determined.

The average value of (26) for $k=0$ represents the DC component of electrical current I. Real and imaginary parts of I are given for this case by

$$\operatorname{Re} I_0 = \frac{Ne^2}{m}\sum_{s=-\infty}^{\infty}\frac{\gamma I_s^2\left(\frac{\omega_c^{'}}{\Omega}\right)}{\gamma^2 + (s\Omega+\omega_{c0})^2}, \qquad (27)$$

$$\operatorname{Im}\frac{I_0}{E} = \frac{Ne^2}{m}\sum_{s=-\infty}^{\infty}\frac{(s\Omega+\omega_{c0})I_s^2\left(\frac{\omega_c^{'}}{\Omega}\right)}{\gamma^2 + (s\Omega+\omega_{c0})^2}. \qquad (28)$$

The higher harmonics for k = 1, 2, 3, … also may be determined from (26).

$$\operatorname{Re}\frac{I_k}{E} = \frac{Ne^2}{m}\sum_{s=-\infty}^{\infty}\frac{\gamma I_s\left(I_{(s-k)} + I_{(s+k)}\right)}{\gamma^2 + (s\Omega+\omega_{c0})^2}\cos k\Omega t -$$
$$-\frac{Ne^2}{m}\sum_{s=-\infty}^{\infty}\frac{(s\Omega+\omega_{c0})I_s\left(I_{(s-k)} + I_{(s+k)}\right)}{\gamma^2 + (s\Omega+\omega_{c0})^2}\sin k\Omega t, \qquad (29)$$

$$\operatorname{Im}\frac{I_k}{E} = \frac{Ne^2}{m}\sum_{s=-\infty}^{\infty}\frac{(s\Omega+\omega_{c0})I_s\left(I_{(s-k)} + I_{(s+k)}\right)}{\gamma^2 + (s\Omega+\omega_{c0})^2}\cos k\Omega t -$$
$$-\frac{Ne^2}{m}\sum_{s=-\infty}^{\infty}\frac{\gamma I_s\left(I_{(s+k)} + I_{(s-k)}\right)}{\gamma^2 + (s\Omega+\omega_{c0})^2}\sin k\Omega t. \qquad (30)$$

The argument off all Bessel functions is $\omega_c^{'}/\Omega$.

Weak modulation

If the modulation depth ω'_c is much smaller than the modulation frequency Ω the exponentials (24) can be expanded as follows

$$I = E\frac{Ne^2}{m}\exp[-\gamma t + i\omega_{c0}t]\left(1 + i\frac{\omega'_c}{\Omega}\sin\Omega t\right)\times$$
$$\times\int\exp[\gamma t - i\omega_{c0}t]\left(1 - i\frac{\omega'_c}{\Omega}\sin\Omega t\right)dt. \qquad (31)$$

The real and imaginary parts of (31) can be written in the form

$$\operatorname{Re} I = E\frac{Ne^2}{m}\gamma\left(\gamma^2 + \omega_{c0}^2\right)^{-1} +$$
$$+E\frac{Ne^2}{m}\frac{\gamma\omega_{c0}\omega'_c}{\left(\gamma^2 - \omega_{c0}^2 + \Omega^2\right)^2 + 4\gamma^2\omega_{c0}^2}\left[\frac{\Omega}{\gamma}\frac{\omega_{c0}^2 - \Omega^2 - 3\gamma^2}{\gamma^2 + \omega_{c0}^2}\times\right.$$
$$\left.\times\sin\Omega t - 2\cos\Omega t\right]; \qquad (32)$$

$$\operatorname{Im} I = E\frac{Ne^2}{m}\omega_{c0}\left(\gamma^2 + \omega_{c0}^2\right)^{-1} -$$
$$-E\frac{Ne^2}{m}\frac{\omega'_c}{\left(\gamma^2 - \omega_{c0}^2 + \Omega^2\right)^2 + 4\gamma^2\omega_{c0}^2}\left[\gamma\Omega\frac{\Omega^2 + \gamma^2 - 3\omega_{c0}^2}{\gamma^2 + \omega_{c0}^2}\times\right.$$
$$\left.\times\sin\Omega t + \left(\gamma^2 - \omega_{c0}^2 + \Omega^2\right)\cos\Omega t\right] \qquad (33)$$

The Re*I* and Im*I*, as before, are responsible for the conductivity and effective dielectrical constant of the semiconductor plasma.

Conclusions

In the pulse modulated magnetic field the semiconductor plasma response alongside with the traditional RF helicon wave contains the transients oscillating with cyclotron frequency, which can be observed experimentally.

Physically it means that the semiconductor conductivity and effective dielectrical constant are modulated with the same cyclotron frequency of infrared range.

The measurement of cyclotron frequency in semiconductor plasma provides an opportunity to determine the charge carriers' masses and mobility. The proposed method of solid state plasma investigation may be used in the cases of pulse and sinusoidal modulation of the magnetic field.

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Radio frequency magnetoplasmic waves known as helicons will propagate in solid-state plasma of semiconductors when a strong magnetic field is applied. Helicons have an exact analogy with a electromagnetic whistler wave which is frequently propagated in the rarefied plasma of the Earth's ionosphere. In our experiments the modulated magnetic field is being used for excitation of helicons. Physically it means that the semiconductor conductivity and effective dielectrical constant are modulated with the same cyclotron frequency of infrared range. The semiconductor sample is in the form of plae and magnetic field is perpendicular to the surface of the plate. It is shown that in the case of modulated field along with the RF helicon waves the transient cyclotron frequency oscillations exist in the semiconductor plasma. For observation of the cyclotron radiation frequency ω_c the modulation depth about one percent is sufficient. The measurement of ω_c provides an opportunity to determine the masses of electrons and holes in solid-state plasma of semiconductors. The proposed method can be used in the cases of pulse and sinusoidal modulation of the magnetic field. Bibl. 5 (in English; summaries in English, Russian and Lithuanian).

З. Янкаускас, В. Квядарас. Колебания циклотронной частоты в полупроводниковой плазме в модулированном магнитном поле // Электроника и электротехника. – Каунас: Технология, 2009. – № 5(93). – С. 83–86.

В твердотельной плазме полупроводников в сильном магнитном поле могут распространяться радиочастотные волны, известные как геликоны. Геликоны являются точной аналогией волн, возбуждаемых радиосигналом в атмосфере земного шара. Электромагнитные волны в намагниченной плазме твердого тела могут быть использованы для генерации колебаний циклотронной частоты. Для этой цели предлагается применение модулированного магнитного поля. Глубина модуляции не превышает одного процента. Новая методика применима как в случае импульсной, так и гармонической модуляции. Измеряя параметры циклотронной генерации можно определить массу, концентрацию и подвижность носителей тока в полупроводниковом материале. Физически это означает, что проводимость и эффективная диэлектрическая проницаемость полупроводника модулированы той же циклотронной частотой инфракрасного диапазона. Полупроводниковый образец имеет вид пластины, а магнитное поле перпендикулярно поверхности этой пластины. Библ. 5 (на английском языке, рефераты на английском, русском и литовском яз.).

Z. Jankauskas, V. Kvedaras. Ciklotroninio dažnio virpesiai puslaidininkinėje plazmoje moduliuotame magnetiniame lauke // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 5(93). – P. 83–86.

Puslaidininkinėje kietojo kūno plazmoje stipriame magnetiniame lauke gali sklisti radijo dažnio bangos, žinomos helikonų pavadinimu. Helikonai yra tiksli analogija bangų, kurias sužadina radijo signalai Žemės rutulio atmosferoje. Įmagnetintoje kietojo kūno plazmoje sklindančios elektromagnetinės bangos gali būti naudojamos ciklotroninio dažnio virpesiams generuoti. Tam tikslui plazmai įmagnetinti siūloma taikyti moduliuotą magnetinį lauką. Moduliavimo gylis neviršija vieno procento. Ši nauja metodika taikytina puslaidininkinės plazmos virpesiams sužadinti tiek impulsinės, tiek harmoninės moduliacijos atvejais. Matuojant ciklotroninio dažnio generacijos parametrus galima nustatyti puslaidininkinės medžiagos elektros srovės krūvininkų masę, koncentraciją ir judrumą. Fizikine prasme tai reiškia, kad puslaidininkio laidis ir defektinė dielektrinė skvarba moduliuojami tuo pačiu ciklotroninio dažniu infraraudonajame diapazone. Puslaidininkinis pavyzdys yra plokštelės pavidalo, o magnetinis laukas yra statmenas tos plokštelės paviršiui. Bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).