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Calculation of the Mathematical Model of the Linear Motor

M. Molis, E. Matkevičius, L. Radzevičius
Department of Automation, Vilnius Gediminas Technical University,
Naugarduko str. 41, LT-03227 Vilnius, Lithuania, phone: +370 2745063,
e-mail: marijanas.molis@el.vtu.lt, edvardas.matkevicius@el.vtu.lt, lionginas.radzevicius@el.vtu.lt

Introduction

Designing the electric drives with the linear electromagnetic motors (LEM), it is necessary to have a mathematical model of the motor which allows calculating of the static and dynamic operating modes of the drive. A mathematical model takes into account the static load of the motor and load components, which are dependent on the shift along the coordinate x and speed. Obtained mathematical model of the LEM allows calculating of the transitional processes of the current, speed, secondary element position, and dynamic traction force, therefore, a mathematical model of the motor can be used to analyze the drives with linear motors [1].

The development of the mathematical model of a linear electromagnetic motor is a complex task requiring special physical and mathematical knowledge.

In order to check the correspondence of the mathematical model of the LEM with the obtained experimental data, an idealized dynamic mathematical model of the linear electromagnetic motor was developed. On the basis of the obtained dependencies [1] of the inductance $L(x)$ and traction forces $f_x(x)$ on the position of secondary element (SE), the transitional processes of the mathematical model for the current $i(t)$, speed $v(t)$, SE position $x(t)$, and dynamic traction forces $f_x(t)$ were calculated.

Table 1. Parameters of the motor's physical model

Item	Parameter name	Symbol	Value	Unit
1.	Pole height	a_c	15	mm
2.	Pole width	b_c	14.5	mm
3.	Tooth height	a_z	15	mm
4.	Tooth width	b_z	15	mm
5.	Tooth thickness	2Δ	16	mm
6.	Tooth pitch	b_t	15	mm
7.	Air gap	δ	1.1	mm
8.	Number of induction coils	S_R	3	vnt.
9.	Number of coils	w	2200	-
10.	Wire diameter	d	0.04	mm
11.	Coil resistance	R	32.7	Ω

Mathematical model of the linear electromagnetic motor is calculated using a physical model of the motor, parameters of which are specified in Table 1.

A goal of this article is to develop an idealized mathematical model of the linear electromagnetic motor and calculate transitional processes of the mathematical model for the current, speed, secondary element (SE) position, and traction forces, using obtained dependencies of the inductance and traction forces on the SE position.

Dependence of the inductance on the position

Expression of useful motor traction force along the coordinate x can be obtained using the electromechanical energy conversion theory [2–5]. According to this theory, it is necessary to know the function of changing of the motor inductance with respect to the coordinate x. After experimental tests with a physical model of the linear motor, the main parameters of which are specified in Table 1, the change of the motor excitation coil inductance shown in Fig. 1 was determined.

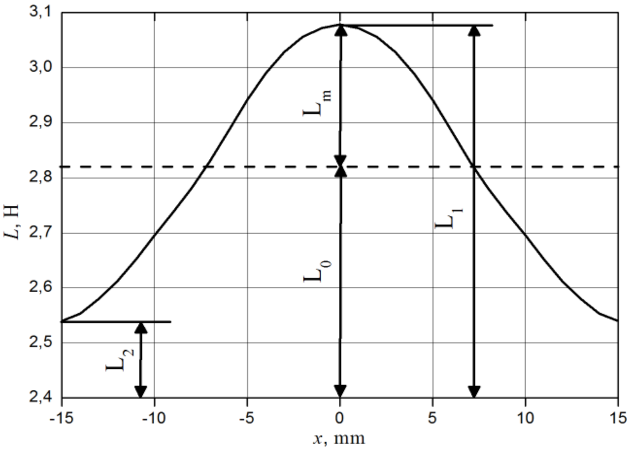

Fig. 1. Graph of the inductance change, at 100 V DC supply voltage

Experimental curve can be approximated by this formula with the accuracy sufficient for engineering calculations:

$$L(x) = L_0 + L_m \cdot \cos\frac{2\pi}{\tau}x\ ,\ -b \le x \le b\ ; \tag{1}$$

$$L_0 = \frac{L_1 + L_2}{2}; \tag{2}$$

$$L_m = \frac{L_1 - L_2}{2}; \tag{3}$$

here L_0, L_m – amplitude of fixed and variable component of the excitation coil inductance; L_1, L_2 – values of the excitation coil inductances when the coil pole is located in the middle of the tooth and the middle of the tooth pitch; $\tau = 2 \cdot b$ – pole pitch of the toothed secondary element; b – the width between the similar tooth and spans of the secondary element.

Dependencies of the traction forces of the LEM on the secondary element position

Supposing that the magnetic system is not saturated, in accordance with the theory of electromechanical energy transformation, the traction force created by one coil of the motor can be expressed by the following equation:

$$f_x = -\frac{\pi}{\tau} \cdot L_m \cdot i^2 \cdot \sin\frac{2\pi}{\tau}x, -b \le x \le b. \tag{4}$$

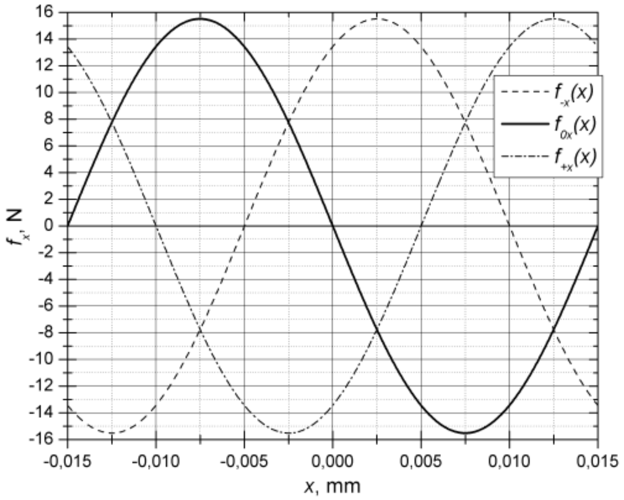

Fig. 2. The diagrams of the traction force components, at 24 V supply voltage

With the simultaneous power supply of the three motor coils, it is necessary to take into account the mutual layout of the coils with respect to the secondary element [6]. In this case, the traction forces created by separate coils are expressed by the following equation:

$$\begin{aligned} f_{-x} &= -\frac{\pi}{\tau}L_m i^2 \sin\left[\frac{2\pi}{\tau}\left(x - \frac{\tau}{3}\right)\right], -b \le x \le b; \\ f_{0x} &= -\frac{\pi}{\tau}L_m i^2 \sin\frac{2\pi}{\tau}x, -b \le x \le b; \\ f_{+x} &= -\frac{\pi}{\tau}L_m i^2 \sin\left[\frac{2\pi}{\tau}\left(x + \frac{\tau}{3}\right)\right], -b \le x \le b. \end{aligned} \tag{5}$$

here f_{-x}, f_{0x}, f_{+x} – traction forces of the appropriate coil the pole of which is shifted in the direction of (– x) coordinate, of the middle coil, and the coil the pole of which is shifted in the direction of (+ x) coordinate.

The diagrams of the traction force components are shown in Fig. 2. It is seen from it that the amplitude of the traction force component of each LEM coil amounts to about 15.5 N.

Development of the mathematical model of LEM

Mathematical model of LEM is developed for one middle coil of the linear electromagnetic motor, with respect to which the following initial equations are used:

$$U = iR + \frac{d\Psi}{dt}; \tag{6}$$

$$m\frac{dv}{dt} = f - f_s - k_x x - k_v v; \tag{7}$$

$$\frac{dx}{dt} = v; \tag{8}$$

$$\Psi = L(x) \cdot i\ ; \tag{9}$$

$$\frac{d\Psi}{dt} = \frac{dL(x)}{dx} \cdot \frac{dx}{dt} i + L(x)\frac{di}{dt}\ . \tag{10}$$

Mathematical model of the linear electromagnetic motor is calculated using a software package MathCad 2001 Professional. Mathematical model is solved by the Runge-Kutta method, using *rkfixed* function of the software package. Therefore, the initial equations (6–10) are written in a normal form:

$$\frac{di}{dt} = \frac{1}{L(x)}\left(U - iR - \frac{dL(x)}{dx}v \cdot i\right); \tag{11}$$

$$\frac{dv}{dt} = \frac{f - f_s - k_x x - k_v v}{m}; \tag{12}$$

$$\frac{dx}{dt} = v; \tag{13}$$

$$\frac{dL(x)}{dx} = -\frac{2\pi}{\tau}L_m \sin\left(\frac{2\pi \cdot x}{\tau}\right); \tag{14}$$

$$f = -\frac{\pi}{\tau}L_m i^2 \sin\left(\frac{2\pi \cdot x}{\tau}\right); \tag{15}$$

here U – supply voltage, R – coil resistance, b – tooth width, τ – SE pole pitch, m – motor mass, $f_s = 1$ N – motor static load, $k_x = 10$ – coefficient of the load depending on the distance x, $k_v = 20$ – coefficient of the load depending on the speed v.

Transitional processes of the current, speed, SE position, and traction force of the LEM

The solution of the system of differential equations allows determination of the transitional processes of the current, speed, and SE position, shown in the figures 3, 4 and 5 accordingly.

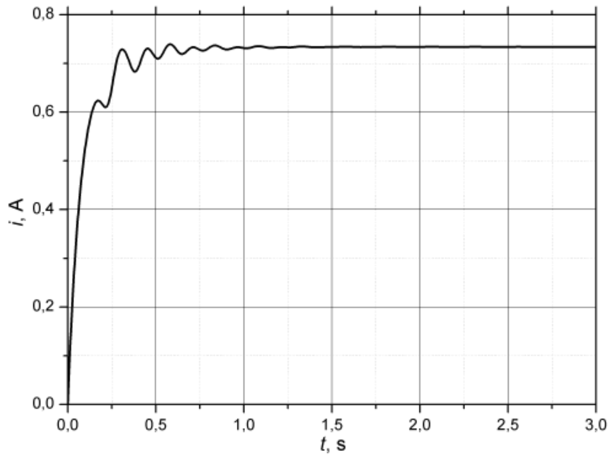

Fig. 3. The current *i* transitional process of the LEM, at 24 V supply voltage

The diagram of the Fig. 3 shows that the current transitional process takes 1.5 sec to reach 0.73 A stable current.

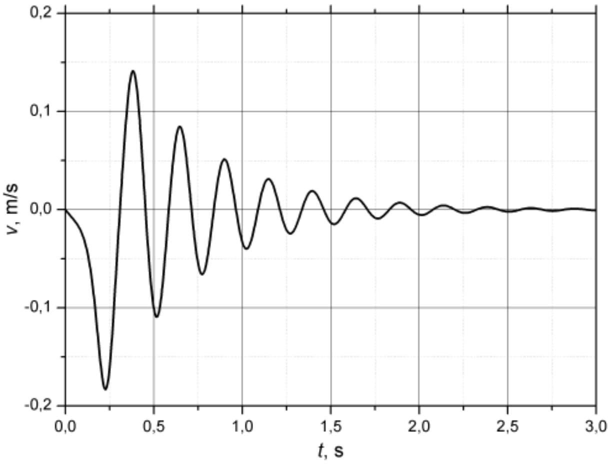

Fig. 4. The speed *v* transitional process of the LEM, at 24 V supply voltage

The diagram of the Fig. 4 shows that the speed *v* transitional process has an oscillating character and takes 3 sec to stabilize.

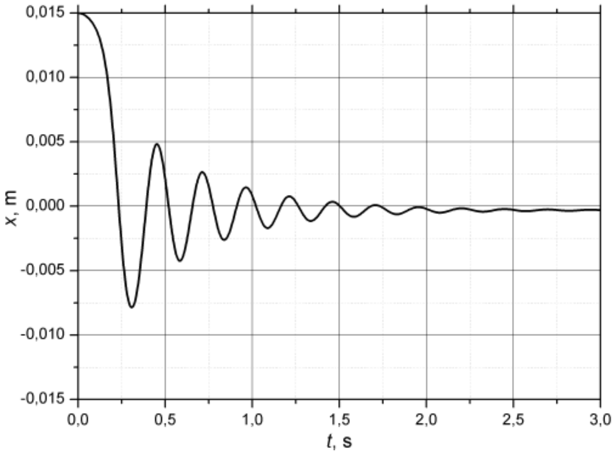

Fig. 5. The secondary element position transitional process of the LEM, at 24 V supply voltage

The diagram of the Fig. 5 shows that the secondary element (SE) position transitional process becomes stabilized after 3 sec.

The transitional processes of the dynamic traction force components of the linear electromagnetic motor are obtained with the solution of the system of differential equations. These transitional processes are shown in Fig. 6.

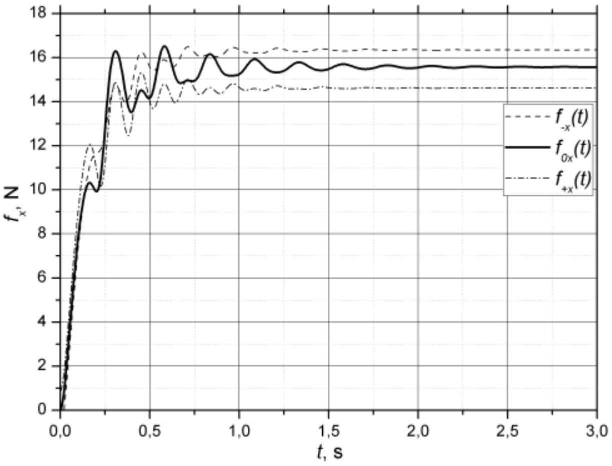

Fig. 6. The transitional processes of the dynamic traction force components of the LEM, at 24 V supply voltage

The diagram of the Fig. 6 shows that the traction force developed by the linear electromagnetic motor, at 24 V supply voltage, after the end of the transitional process, is about 15.5 N. The traction force changes, if the supply voltage of the linear electromagnetic motor is changed; therefore, by selecting certain values of the supply voltages for the individual motor coils it is possible to form a desirable law of changes of the total motor traction force meeting specific requirements of the technological process.

Conclusions

1. The law of changes of the linear motor inductance was analyzed and the mathematical expression approximating this law was obtained.
2. Using the theory of electromechanical energy transformation, mathematical expressions of the dynamic traction force components of the linear electromagnetic motor were formulated and the dependencies of the traction force components on the secondary element position were obtained.
3. An idealized dynamic mathematical model of the linear electromagnetic motor was developed.
4. After calculation of the mathematical model of the linear electromagnetic motor, the transitional processes diagrams for the current, speed, secondary element position, and dynamic traction force components were obtained.
5. Selecting certain values of the supply voltages for the individual motor coils, it is possible to form a desirable law of changes of the total motor traction force meeting specific requirements of the technological process.

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Development of the mathematical model of the linear electromagnetic motor and the dependencies of the inductance and traction force on the secondary element position expressed by mathematical equations, are presented. The dependency of the inductance on the secondary element position was obtained, approximating the inductance change diagram obtained experimentally. Also, using the theory of electromechanical energy transformation, mathematical expressions of the dependency of the traction force on the secondary element position were obtained. Mathematical model of the linear electromagnetic motor is composed of the system of differential equations. The Runge-Kutta calculation method was used to solve these equations. The transitional processes of the current, speed, secondary element position, and dynamic traction force components were obtained with the solution of the system of differential equations. All obtained results of the dependencies and transitional processes of the mathematical model are presented in the graphic form. In accordance with the obtained results of the mathematical model the conclusions were formulated, specifying electromagnetic properties of the linear electromagnetic motor. Ill. 6, bibl. 7 (in English; the summaries in English, Russian and Lithuanian).

М. Молис, Э. Маткявичюс, Л. Радзявичюс. Расчёт математической модели линейного двигателя // Электроника и электротехника. – Каунас: Технология, 2009. – № 7(95). – С. 91–94.

Представлен расчёт математической модели линейного электромагнитного двигателя, а также расчёт индуктивности и силы тяги в зависимости от положения вторичного элемента, которые выражены математическими уравнениями. Аппроксимируя кривую индуктивности, полученную экспериментным путём, получена зависимость индуктивности от положения вторичного элемента. А также, применив теорию преобразования электромеханической энергии, получены математические выражения зависимости силы тяги от положения вторичного элемента. Математическая модель линейного электромагнитного двигателя составлена, используя систему дифференциальных уравнений. Для решения этих уравнений использован вычислительный метод Рунге-Кутта. Решив систему дифференциальных уравнений, получены переходные процессы тока, скорости, положения вторичного элемента и составляющих динамической силы тяги. Все по математической модели полученные результаты зависимостей и переходных процессов представлены в графическом виде. По результатам исследования математической модели сформулированы выводы, характеризующие электромагнитные свойства линейного электромагнитного двигателя. Ил. 6, библ. 7 (на английском языке; рефераты на английском, русском и литовском яз.).

M. Molis, E. Matkevičius, L. Radzevičius. Tiesiaeigio variklio matematinio modelio skaičiavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 7(95). – P. 91–94.

Pateiktas tiesiaeigio elektromagnetinio variklio matematinio modelio sudarymas bei matematinėmis lygtimis išreikštos induktyvumo ir traukos jėgos priklausomybės nuo antrinio elemento padėties. Induktyvumo priklausomybė nuo antrinio elemento padėties gauta aproksimuojant eksperimentiškai nustatytą induktyvumo kitimo kreivę. Naudojant elektromechaninės energijos keitimo teoriją, gautos traukos jėgos priklausomybės nuo antrinio elemento padėties matematinės išraiškos. Tiesiaeigio elektromagnetinio variklio matematinį modelį sudaro diferencialinių lygčių sistema. Šioms lygtims spręsti panaudotas Rungės ir Kutos metodas. Išsprendus diferencialinių lygčių sistemą gauti srovės, greičio, antrinio elemento padėties bei dinaminės traukos jėgos dedamųjų pereinamieji procesai. Visi gauti matematinio modelio priklausomybių bei pereinamųjų procesų rezultatai pateikti grafikų pavidalu. Pagal gautus rezultatus suformuluotos išvados, apibūdinančios tiesiaeigio elektromagnetinio variklio elektromagnetines savybes. Il. 6, bibl. 7 (anglų kalba; santraukos, anglų, rusų ir lietuvių k.).