

## Investigation of Normal Modes in Microstrip Multiconductor Line Using the MoM

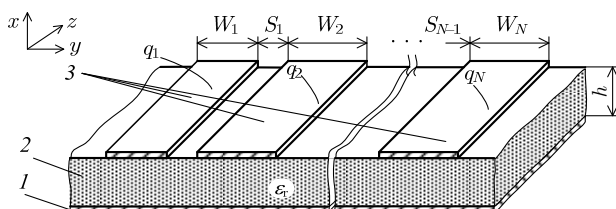
Š. Mikučionis, V. Urbanavičius

Department of Electronic Systems, Vilnius Gediminas Technical University,

Naugarduko str. 41-422, LT-03227 Vilnius, Lithuania, phone: +370 5 2744756, e-mail: vytautas.urbanavicius@el.vgtu.lt

### Introduction

The generalized design of the microstrip multiconductor line (MMCL) is shown in Fig. 1. The MMCL consists of a dielectric substrate with a conducting layer (shield) on one side and microstrip conductors on the other side. The number of the microstrip conductors, their width and space between them are specified independently. Due to their planar design, these lines are broadly used in many electronics areas [1–3]. MMCL are also successfully used as physical models designing microwave devices [4–9].



**Fig. 1.** Generalized design of the microstrip multiconductor line: 1 – shield; 2 – dielectric substrate; 3 – microstrips conductors

It is known, that, generally, in an MMCL consisting of  $N$  conductors,  $N$  normal waves (or normal modes), which are characterized by  $N$  propagation factors  $\beta_i$  and  $N^2$  characteristic impedances  $Z_{ki}$  (where  $k$  is type of normal mode;  $i$  is conductor number), can propagate.

Phase and group velocities of different types of normal waves differ from each other in the MMCL. Therefore it is desirable to use only normal waves for information transmission.

Conditions for propagation of the certain type of the normal wave in MMCL, generally, is determined by selection of amplitude and a sign of voltages, which are sent into MMCL conductors.

Various methods of analysis are used for calculation of main characteristics of MMCL: some researchers assume that the number of conductors in MMCL is infinite [4–6]. The line in this case is periodic structure and only two

types of normal waves can propagate in it: *even* and *odd* (if width of microstrip conductors is equal), or *c-wave* and  $\pi$ -*wave* (if the period of MMCL contains two conductors of different widths). Such approach in analysis of MMCL saves computing resources (characteristics of only two adjacent conductors of MMCL are actually calculated), and it is correct enough if the structure of the researched device is close to periodic, i.e. the number of conductors in the corresponding MMCL is great, suppose, more than 50. If the number of conductors of MMCL is small, heterogeneity of the modelled device should be taken into account. Many methods of analysis of MMCL may be used in this case. For example, the quasi-TEM approximation of MMCL in isotropic media was studied from a theoretical point of view in [10]. It was shown that, in the quasi-TEM approach, such a multiconductor line can be represented in circuit terms by a set of coupled transmission lines described by the classical telegraphers' equations. Numerical analysis, using integral equations, can be found, for example, in [11, 12].

In the present contribution we will first concentrate on analysis of MMCLs in the quasi-TEM approach using the method of moments (MoM).

Quasi-TEM analysis is often regarded as an approximation for low frequencies. Low frequency for the quasi-TEM means that the dimensions of the cross section of the structure are small compared to the wavelength [10], [13]. Really, the quasi-TEM analysis neglects wave propagation effects in the cross section of the structure and only takes longitudinal propagation into account [13].

Further it has been shown that in the isotropic case [14] the quasi-TEM analysis remains useful, at least qualitatively, to high frequencies for the fundamental modes. Really, the quasi-TEM analysis may be surely used as an approximation for the fullwave analysis where the longitudinal field components have been neglected.

### Normal Modes Definition

The concept of normal waves is rather wide and has some interpretations. According to encyclopedic definition

[15] normal waves (natural waves) are harmonic waves of specific physical nature (electromagnetic, elastic, etc.), supporting constant cross structure of the field and (or) polarization at the rectilinear propagation. According to this attribute, normal waves differ from all other waves, capable to propagate in the given system. For example, at propagation of normal electromagnetic waves between parallel conducting planes, the cross structure (in relation to the direction of propagation) of the electric field of a normal wave is the same in all cross-sections. The cross structure of any other wave, which is distinct from normal waves, it is not maintained at propagation.

The physical essence of normal wave in the MMCL is determined by that in the multiconductor line, which is free from sources, the any propagated wave can be submitted as superposition of normal waves, and the resulting flow of electromagnetic energy is equal to the sum of flows of all normal waves.

In the case of MMCLs, normal waves are not defined in terms of the cross-structure of fields, but rather in terms of the relation of amplitudes of oscillations on separate conductors. This relation is kept constant along conductors.

### Investigation of Normal Modes by the MoM

In order to excite and analyze normal modes in an MMCL, modal voltages sent to the line conducting microstrips, should be known. Normal mode in case of MMCL is achieved when phase velocity of the electromagnetic wave along each microstrip conductor  $i$  is the same:

$$v_{p1} = v_{p2} = \dots v_{pi} = \dots = v_{pN} = c_0 / \sqrt{\varepsilon_{\text{ref}}}, \quad (1)$$

where  $c_0$  – velocity of electromagnetic wave in free space;  $\varepsilon_{\text{ref}} = C_{1i} / C_{1i}^{(a)}$  – effective dielectric permittivity of the MMCL;  $C_{1i}$  – capacitance per unit length of  $i$ -th microstrip conductor of the MMCL;  $C_{1i}^{(a)}$  – capacitance per unit length of the same microstrip conductor, with free space (air) in place of the dielectric substrate.

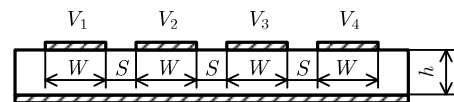
Since  $c_0$  is constant,  $\varepsilon_{\text{ref}}$  must be the same for all microstrip conductors in order to excite a normal mode. Modal voltages in an MMCL are calculated using partial capacitance matrices  $\mathbf{C}_1$  and  $\mathbf{C}_1^{(a)}$ . The matrix  $\mathbf{C}_1$  is a square  $N \times N$  matrix, where  $N$  is number of microstrip conductors of MMCL, and each element  $C_{ij}$  is capacitance per unit length of  $j$ -th microstrip conductor, when voltage of  $i$ -th microstrip conductor is 1 V, and voltage of other conductors is 0.  $\mathbf{C}_1^{(a)}$  is the similar matrix of the same MMCL with free space in place of the dielectric substrate. At known both matrices, we can find modal voltages by calculating eigenvalues of the relative effective dielectric permittivity vector. We propose to use the combination of the method of moments and partial images technique [12], [16] for accurate calculation of partial capacitance matrices.

Our proposed technique for investigation of normal modes in an MMCL has been verified by comparison of calculated parameters that were published in article [17]. In the mentioned work the three-conductor asymmetrically coupled MMCL have been investigated. Results of calculations of parameters of the investigated line are submitted in Table 1. It is obvious, that values in Table 1 agree within 3 % in most cases, and taking into account that in [17] other technique of investigation (spectral domain method) is used, it is possible to tell about good reliability of the received results and the proposed technique.

**Table 1.** Comparison parameters for three asymmetric coupled microstrip lines ( $W_1 = 0.3$  mm,  $W_2 = 0.6$  mm,  $W_3 = 1.2$  mm,  $S_1 = 0.2$  mm,  $S_2 = 0.4$  mm,  $h = 0.63$  mm,  $\varepsilon_r = 9.8$ ), obtained using the proposed technique (MoM), and in [17].

Conductor	Mode A		Mode B		Mode C	
	MoM	[17]	MoM	[17]	MoM	[17]
Modal Voltages, V						
#1	1	1	1	1	1	1
#2	-0.9	-0.875	0.61	0.6	1.16	1.15
#3	0.17	0.175	-0.66	-0.66	1.19	1.13
Relative Effective Dielectric Permittivity						
-	5.51	5.55	6.1	6.15	7.58	7.6
Characteristic Impedance, $\Omega$						
#1	45	46.5	73.6	76	106	107
#2	32.6	33	55.7	57	72.6	73.5
#3	20	19.5	29	30	39.6	40

Further, using the proposed technique we investigated the conditions of excitation and propagation of normal waves in the four-conductor symmetrically coupled MMCL. The cross section of the investigated MMCL and arrangement of modal voltages which are sent into the MMCL for excitation of normal modes, are shown in Fig. 2.



**Fig. 2.** Cross-section of investigated MMCL and modal voltages structure for four normal modes

In case of symmetric four-conductor MMCL (where widths of all conductors are equal, and the gaps between conductors are also equal), four modes of normal wave propagation are possible. Information about modal voltages is presented in Table 2.

**Table 2.** Relations of modal voltages for four normal modes

Mode	Signs of modal voltages				Relations between modal voltages
	$V_1$	$V_2$	$V_3$	$V_4$	
A	+	+	+	+	$V_1 = V_4$ ; $V_2 = V_3$
B	+	+	-	-	$V_1 = -V_4$ ; $V_2 = -V_3$
C	+	-	-	+	$V_1 = V_4$ ; $V_2 = V_3$
D	+	-	+	-	$V_1 = -V_4$ ; $V_2 = -V_3$

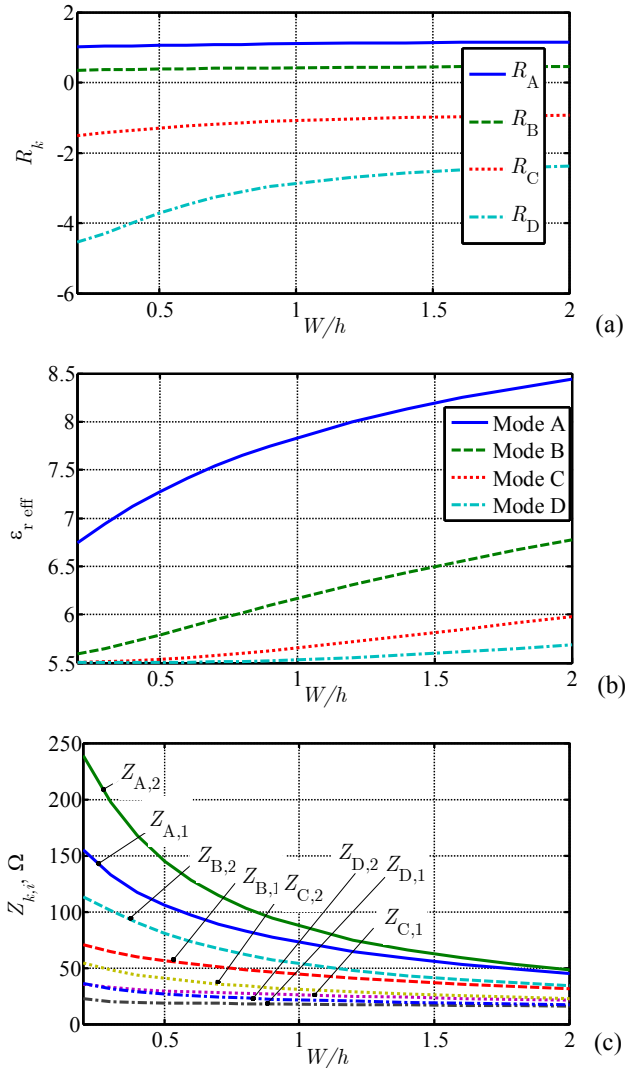
Plus and minus signs specify polarity in Table 2, but not magnitude of modal voltage. The calculation results of

normal modes parameters versus normalized microstrip width  $W/h$  are shown in Fig. 3.

It is seen in Fig. 3 (a) that in case of symmetrically coupled four-conductor MMCL, the value of voltages, sent into adjacent microstrips, ratio is only one for each mode:

$$R_k = V_2/V_1 = V_4/V_3, \quad (2)$$

where  $k$  denotes the mode index. Therefore, for each mode, only two voltage magnitudes are distinct:  $V_1 \neq V_2$ , and  $V_3 \neq V_4$ .



**Fig. 3.** Eigenvector elements of voltage ratio (a); effective dielectric permittivity (b); characteristic impedance (c) of four-conductor MMCL versus normalized microstrip width;  $\epsilon_r = 10$

At investigating dependences of normal waves parameters on the width of microstrips of the MMCL, we found that, in general, these dependences are similar to dependences for the microstrip coupled lines [18]. Sending voltages of identical sign into all microstrips of the MMCL (mode A, or c-normal wave), the voltages ratio, at increasing  $W/h$ , varies insignificantly (Fig. 3 (a)). In our example at increasing  $W/h$  by 10 times,  $R_A$  has increased only

by 10 %. Similar dependence is observed for mode B when voltages of identical signs also are sent (into the pairs of external microstrips). Nevertheless, at the same variations of  $W/h$  for mode D ( $\pi$ -normal wave),  $R_D$  increases almost 2 times.

Effective dielectric permittivity  $\epsilon_{r, \text{eff}}$  also increases (Fig. 3 (b)) with increasing  $W/h$  for all modes (because electric field concentrates in the dielectric substrate). But the degree of this increase is different for each mode. It is seen that  $\epsilon_{r, \text{eff}}$  for mode A varies most of all – by 24 %, and  $\epsilon_{r, \text{eff}}$  for mode D varies least – only by 4 %, correspondently  $\epsilon_{r, \text{eff}}$  for modes B and C vary by 20 % and 10 %.

The characteristic impedance, in contrast, decreases with increasing  $W/h$ , for all modes (because the capacitances of conductors per unit length increase). For symmetrically coupled four-conductor MMCL we have found that  $Z_1 = Z_4$  and  $Z_2 = Z_3$ , where index denotes conductor number. Therefore it is enough to present only eight dependencies of characteristic impedance on  $W/h$ . In Fig. 3 (c)  $Z_{ki}$  denotes characteristic impedance of  $i$ -th conductor for mode  $k$ . It should be noted that for every mode internal microstrips of MMCL (# 2 and # 3) have the greater characteristic impedance, and external microstrips (# 1 and # 4) have smaller impedance. The impedance of internal microstrips also changes, changing  $W/h$ , in the greater degree. We found that as well as in case of  $\epsilon_{r, \text{eff}}(W/h)$ , mode A responds most sensitively to changing  $W/h$  – its impedances vary almost 5 times for internal microstrips and 3 times for external.

#### IV. Conclusions

A technique of investigation of normal waves in MMCLs, based on the method of moments (MoM) and partial images approach, is proposed in this paper.

The proposed technique was verified comparing calculated parameters of MMCL with published data, and calculated values agree with published ones within 3 %.

The four-conductor symmetrically coupled MMCL was investigated using the proposed technique. We revealed that changing MMCL conductor width, the voltages ratio for mode D, relative effective dielectric permittivity and characteristic impedance for mode A change in higher degree than for other modes. We also found that values of characteristic impedance of the inner conductors of MMCL are greater than impedance of the external conductors.

#### Acknowledgements

The authors would like gratefully acknowledge prof. S. Štaras of Vilnius Gediminas Technical University (VGTU) for valuable discussions. The authors also would like to acknowledge the reviewers of the manuscript for their suggestions.

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Received 2010 01 30

**Š. Mikučionis, V. Urbanavičius. Investigation of Normal Modes in Microstrip Multiconductor Line Using the MoM // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 4(100). – P. 91–94.**

The technique of investigation of normal waves excitation and propagation conditions in a microstrip multiconductor line (MMCL) in the quasi-TEM approximation is submitted in this article. The method of moments is used for calculation of capacitance matrixes of the MMCL and parameters of normal waves propagation. Values of voltages, which excite normal waves in the MMCL, are found calculating eigenvalues of relative effective dielectric permittivity vector. Using the software created by authors, and analyzing the three-conductor MMCL, good conformity of the parameters calculated by the proposed technique and parameters submitted in publications of other researchers is shown. Characteristics of normal waves excitation and conditions of their propagation in the four-conductor MMCL from its design parameters are investigated also. III. 3, bibl. 17, tabl. 2 (in English; abstracts in English, Russian and Lithuanian).

**III. Микучёнис, В. Урбанавичюс. Исследование нормальных волн в микрополосковой многопроводной линии с использованием метода моментов // Электроника и электротехника. – Каунас: Технология, 2010. – № 4(100). – С. 91–94.**

Представлена методика исследования условий возбуждения и распространения нормальных волн в микрополосковой многопроводной линии (ММЛ) в квази-ТЕМ приближении. Для расчёта емкостных матриц ММЛ и параметров распространения нормальных волн используется метод моментов. Величины напряжений, возбуждающие нормальные волны в ММЛ, находятся вычислением собственных значений вектора относительных эффективных диэлектрических проницаемостей. Используя программное обеспечение, созданное авторами, на примере трёхпроводной ММЛ показано хорошее соответствие параметров, рассчитанных по предлагаемой методике и представленных в публикациях других исследователей. Исследованы зависимости условий возбуждения и распространения нормальных волн в четырёхпроводной ММЛ от её конструктивных параметров. Ил. 3, библи. 17, табл. 2 (на английском языке; рефераты на английском, русском и литовском яз.).

**Š. Mikučionis, V. Urbanavičius. Mikrojuostelinėje daugialaidėje linijoje sklindančių normalių bangų tyrimas momentų metodu // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 4(100). – P. 91–94.**

Pateikta metodika normalių bangų žadinimo ir sklidimo mikrojuostelinėje daugialaidėje linijoje (MDL) sąlygoms tirti taikant kvazi-TEM artinį. MDL talpinėms matricoms ir normalių bangų sklidimo parametrams apskaičiuoti taikomas momentų metodas. Įtampų, žadinančių normaliąsias bangas MDL, dydžiai randami apskaičiuojant santykinės efektyviosios dielektrinės skvarbos vektorius savąsias vertes. Taikant autorių sukurtą programinę įrangą, ir nagrinėjant trijų laidininkų MDL pavyzdį, parodyta, kad parametrai apskaičiuoti pagal siūlomą metodiką ir pateikti kitų tyrėjų publikacijose, gerai sutampa. Ištirtos normalių bangų sužadavimo ir sklidimo sąlygų keturių laidininkų MDL priklausomybės nuo linijos konstrukcinių parametrų. II. 3, bibl. 17, lent. 2 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).