# True Six-Phase Space Vector Modulation Scheme with Reduced Low Order Harmonics 

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#### Abstract

A novel space vector pulse width modulation (PWM) scheme based on the employment of medium magnitude space vectors for six-phase voltage source inverter has been developed. The proposed PWM scheme allows generating the six-phase voltage with the reduced level of low order harmonics. It is suitable for inverters driving six-phase AC induction motors with sinusoidal distribution of windings, with a single neutral point.


Index Terms-Six-phase inverter, single neutral, multiple d$q$ planes, even phase number, space vector modulation.

## I. Introduction

Multi-phase systems have gained interest over past decade due to some advantages over their three-phase counterparts. Some of those advantages are increased reliability because of the higher phase number, decreased torque pulsations, decreased acoustic noise, and lower current per phase. These advantages make multi-phase machines suitable for propulsion of electric cars, locomotives, ships, and aircraft [1]-[3]. However, to control multiphase machines, complex modulation techniques have to be implemented.
Most common modulation scheme used in Voltage Source Inverters (VSI) for AC machines is Space Vector PWM (SVPWM). Three-phase SVPWM was investigated many years ago, and now is industry standard for driving threephase AC machines, however extension of the three phase SVPWM theory to multiphase operation is far from trivial. This is because of increasing number of space vectors ( $2^{n}$, where $n$ is the number of phases), and increasing number of sub-spaces where these vectors are mapped $((n-1) / 2$, when the phase number is odd, and ( $n-2) / 2$, when even) [4].

Increased number of subspaces is the basis for new features of multiphase machines, such as torque enhancement for concentrated winding machines by injecting harmonics, or independent driving AC machines connected in series to one VSI [5].

The new SVPWM scheme using medium space vectors

[^0]developed for six-phase VSI connected to a symmetrical load with a one neutral point (Fig. 1) is proposed in this paper.

## II. Problem Formulation

In this paper, a six-phase VSI with a load, with a single neutral point presented in Fig. 1 [6] is considered. The switching function for this type of system can be defined as $m=1$ (where $i=a, b, c, d, e, f$ ) when the inverter's upper switch is on, and lower switch is off, and $m_{i}=0$ when inverter switch states are inverted respectively. Then, the voltage output value for each phase in every inverter state can be calculated by the expression

$$
\begin{equation*}
v_{i}=V_{d c}\left[m_{i}-1 / 6\left(m_{a}+m_{b}+m_{c}+m_{d}+m_{e}+m_{f}\right)\right] \tag{1}
\end{equation*}
$$



Fig. 1. Diagram of a six-phase VSI with a six-phase symmetrical load with one neutral point.

There are 64 inverter switch combinations in total, which correspond to 64 space vectors. To acquire space vector diagrams for a six-phase VSI, it is needed to transform all the phase voltages in all possible inverter switch combinations with Clarke's transformation matrix [6]:

$$
C=\frac{2}{6}\left[\begin{array}{cccccc}
1 & \cos (\alpha) & \cos (2 \alpha) & \cos (3 \alpha) & \cos (4 \alpha) & \cos (5 \alpha)  \tag{2}\\
0 & \sin (\alpha) & \sin (2 \alpha) & \sin (3 \alpha) & \sin (4 \alpha) & \sin (5 \alpha) \\
1 & \cos (2 \alpha) & \cos (4 \alpha) & \cos (6 \alpha) & \cos (8 \alpha) & \cos (10 \alpha) \\
0 & \sin (2 \alpha) & \sin (4 \alpha) & \sin (6 \alpha) & \sin (8 \alpha) & \sin (10 \alpha) \\
1 / 2 & 1 / 2 & 1 / 2 & 1 / 2 & 1 / 2 & 1 / 2 \\
1 / 2 & -1 / 2 & 1 / 2 & -1 / 2 & 1 / 2 & -1 / 2
\end{array}\right],
$$

where $\alpha=2 \pi / n$, and $n=6$ for six phases.
After performing Clarke's transformation, a new matrix of $d-q$ values is created. The first column of the matrix is the $d$ values, the second $-q$ values of the $d_{1}-q_{1}$ plane. The $3^{\text {rd }}$ and $4^{\text {th }}$ columns of the matrix are $d-q$ values for $\mathrm{d}_{2}-\mathrm{q}_{2}$ plane; the $5^{\text {th }}$ and $6^{\text {th }}$ columns are $0_{+}$and 0 . components respectively. It should be noted, that so called zero sequence components $0_{+}$ and 0 _ do not form a two dimensional plane like d-q components. If we look at Fig 2(a), it can be seen that there are four kinds of vectors: large magnitude $\left(0.66 V_{d c}\right)$, e.g. vector no. 36, medium magnitude $\left(0.577 V_{d c}\right)$, e.g. 4 , small magnitude $\left(0.33 V_{d c}\right)$, e.g. 2 , and zero magnitude, e.g. 1 .


Fig. 2. Space vectors of a six-phase VI: (a) $-d_{1-} q_{1}$ (main) plane, (b) $-d_{2-}$ $\mathrm{q}_{2}$ plane.

It should be pointed out, that the vector distribution pattern in both planes is identical, but vectors are placed
differently in different planes, with the exception 4 vectors: $1,22,43$ and 64 . Vectors 1 and 64 are null vectors, which generate zero voltage in the output. Vectors 22 and 43 could be called pseudo-null vectors, because each of the vectors create an opposite voltage for the half of the load, but since the load is symmetrical, the sum of output voltage is zero.


Fig. 3. Zero sequence component 0 - magnitudes for all space vectors.
Also the magnitudes and phases of some vectors, e.g. 4 and 40 in Fig. 2(a), are the same, therefore these vectors are redundant in the $d_{1}-q_{1}$ plane, however their phases are different in $\mathrm{d}_{2}-\mathrm{q}_{2}$ plane. This means, that vectors can be redundant in a local, one plane's scale, however on a global scale, the vectors are not redundant. This fact should be always taken into account, when generating a reference voltage vector for the inverters output, otherwise, unwanted output voltage distortions might appear.

All odd phase number vector diagrams have also one feature - they can be divided into $2 n$ symmetrical sectors in angle increments of $\pi / n$ [5], [7]. This is not exactly true for a six-phase VSI, it can be divided into $2 n=12$ sectors, however then, the sectors would be asymmetrical.

There are also two single dimension zero sequence components. The zero sequence component $0_{+}$will not exist in a star connected load with one isolated neutral, and therefore can be ignored. But if the phase number is even, like in this case, the component 0 - will exist, and could not be ignored [4]. The magnitude of each vector 0 - component is shown in Fig. 3.

## III. Medium Space Vector PWM

If one has ever investigated odd phase number VSI's, the most intuitive choice for implementing space vector modulation would be dividing the $\mathrm{d}_{1}-\mathrm{q}_{1}$ plane into sectors, and use various vectors, definitely including largest ones, to form the output voltage. This method was implemented in author's earlier work [6], and was proven to be fairly successful. The only drawback of this method is that there is a fair amount of third harmonic in the output, so this scheme is not efficient in driving AC motors with sinusoidal distribution of windings, because third harmonic would produce reverse torque.

After further analysis of the space vector planes, it was found that harmonic appears from the 0 - plane. It may be seen from Fig. 2 that all largest magnitude vectors in the main plane ( $\mathrm{d}_{1}-\mathrm{q}_{1}$ ), e.g. vector 36 , are zero magnitude in $\mathrm{d}_{2}-$ $\mathrm{q}_{2}$, however in 0. plane (Fig. 3) they are not zero. Third
harmonic is generated, if, for example, we are trying to build a reference voltage vector from vectors 36 and 8 , it seems that in $\mathrm{d}_{2}-\mathrm{q}_{2}$ plane they are zero, and no harmonics should be generated, however in 0 . plane they are of different magnitudes, which do not sum up to zero, therefore an output vector of some magnitude is formed, which leads to the third harmonic being generated. The solution to this problem would be to find vectors in the same sector of $d_{1}-q_{1}$ plane, which cancel each other out in $\mathrm{d}_{2}-\mathrm{q}_{2}$ plane, and cancel the large vectors out in 0 . plane, but further investigation showed that there are no such vectors available.


Fig. 4. Vector diagram of SVPWM scheme, which uses medium vectors.
The solution is to use medium magnitude space vectors. If we look at the vectors that have been chosen in the solution (Fig. 4), it is possible to see, that it resembles the three phase system's d-q plane rotated by 30 degrees. If sinusoidal output voltage is desired, the reference vector should always be the same magnitude, thus draw a circle inside a hexagon, formed by the medium vector ends. Since the magnitude of medium vectors was already calculated, and it is known, that circle reaches hexagon border at $\alpha=\cos (\pi / 6)$ it is possible to find the maximum output voltage

$$
\begin{equation*}
V_{\max }=0,577 V_{d c} \cos (\pi / 6) \approx 0,5 V_{d c} \tag{3}
\end{equation*}
$$

The calculated maximum output voltage is $0.5 V_{d c}$, which corresponds to theoretical six-phase system analysis: in a six phase, or any other even phase number system, the output voltage cannot exceed $0.5 V_{d c}$ in linear operating region (when sinusoidal output voltage is desired), because there are two opposite phases, i.e. when one phase is at its maximum peak, the other will be at its minimum at the same moment in time, therefore the p-p voltage cannot exceed $V_{d c}$, thus peak amplitude cannot exceed $0.5 V_{d c}$.

The reference output voltage vector is built from the available vectors lying on the sector's borders. The turn on time for each border's vectors is calculated by:

$$
\begin{equation*}
t_{a}=\frac{\left|\overrightarrow{v_{s}}\right| \sin (k \pi / 3-\alpha)}{\left|\overrightarrow{v_{a}}\right| \sin (\pi / 3)} T_{s}, \tag{4}
\end{equation*}
$$

$$
\begin{gather*}
t_{b}=\frac{\left|\overrightarrow{v_{s}}\right| \sin (\alpha-(k-1) \pi / 3)}{\left|\overrightarrow{v_{b}}\right| \sin (\pi / 3)} T_{s},  \tag{5}\\
t_{0}=T_{s}-t_{a}-t_{b}, \tag{6}
\end{gather*}
$$

where $k$ is the sector number, $T_{\mathrm{s}}-$ switching period, $\alpha-$ angle of the output voltage reference vector, $t_{0}$ - null vector turn on time per switching period.

The voltage could be formed using only two vectors from each border, e.g. 4 and 7 (Fig. 2(a)), however these vectors would not cancel each other in $\mathrm{d}_{2}-\mathrm{q}_{2}$ plane (Fig. 2(b)), and output voltage will contain unwanted harmonics. This problem was solved by using all four vectors per sector. Vectors 4 and 40 are the same magnitude and direction in $\mathrm{d}_{1}-\mathrm{q}_{1}$ plane, but in $\mathrm{d}_{2}-\mathrm{q}_{2}$ plane, their magnitudes are the same, but their direction is opposite. For vectors to cancel out in $\mathrm{d}_{2}-\mathrm{q}_{2}$ plane, a constraint is added, i.e. the duty time of vectors 4 and $40\left(\overrightarrow{v_{a 1}}\right.$ and $\overrightarrow{v_{a 2}}$ in Fig. 4) should be equal at all times. Therefore vector duty times should be:

$$
\begin{align*}
& t_{a}\left|\overrightarrow{v_{a}}\right|=0.5 t_{a}\left|\overrightarrow{v_{a 1}}\right|+0.5 t_{a}\left|\overrightarrow{v_{a 2}}\right|,  \tag{7}\\
& t_{b}\left|\overrightarrow{v_{b}}\right|=0.5 t_{b}\left|\overrightarrow{v_{b 1}}\right|+0.5 t_{b}\left|\overrightarrow{v_{b 2}}\right| . \tag{8}
\end{align*}
$$

The investigation of 0 . component shows, that selected vectors from sector's 1 lower border, i. e. 40 and 4 , are of zero magnitude, and therefore will not cause low order harmonics.

Duty time $t_{0}$ is shared equally between two zero vectors: $0.5 t_{0}$ for vector no. 1 and $0.5 t_{0}$ for vector no. 64 .

According to the statements above, a switching table was prepared for a six-phase VSI (Table I). To lower switching noise, vectors in the switching table were aligned, so that when switching from one vector to other, less inverter legs would have to switch.

## TABLE I. SWITCHING TABLE FOR MEDIUM VECTOR SVPWM

 SCHEME.| Sector | $\mathbf{t}_{\mathbf{0}} / \mathbf{2}$ | $\mathbf{t}_{\mathbf{a}} / \mathbf{2}$ | $\mathbf{t}_{\mathbf{a}} / \mathbf{2}$ | $\mathbf{t}_{\mathbf{b}} / \mathbf{2}$ | $\mathbf{t}_{\mathbf{b}} / \mathbf{2}$ | $\mathbf{t}_{\mathbf{0}} / \mathbf{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 000000 | 110000 | 111001 | 111100 | 011000 | 11111 |
| 2 | 000000 | 011000 | 111100 | 011110 | 001100 | 111111 |
| 3 | 000000 | 001100 | 011110 | 001111 | 000110 | 111111 |
| 4 | 000000 | 000110 | 001111 | 100111 | 000011 | 111111 |
| 5 | 000000 | 000011 | 100111 | 110011 | 100001 | 111111 |
| 6 | 000000 | 100001 | 110011 | 111001 | 110000 | 111111 |

## IV. Simulation Results

All simulations were performed in Simulink, with identical parameters, only modulation schemes were different. Main simulation parameters were as follows: fundamental frequency of output voltage $f=50 \mathrm{~Hz}$, inverter switching frequency $f_{\mathrm{s}}=2 \mathrm{kHz}$. The 12 inverter output switches based on FETs were powered by a 400 V DC source. Inverter load model was created using six RL branches ( R and L connected in series), which were combined in to a star topology with an isolated neutral.

The voltage graphs in Fig. 5 is phase A to neutral voltage, filtered with a low pass filter. The abbreviations in Fig. 5 is as follows: SPWM - voltage generated using the simplest
sine-triangle method, SVPWML - voltage generated using only large space vector PWM, SVPWMM - voltage generated by a medium space vector PWM, proposed in this paper. The maximum achieved peak voltage by each PWM scheme is as follows: SPWM - 196 V, SVPWML - 204 V, SWPWMM - 199 V, which corresponds, accordingly, to $0.49 V_{d c}, 0.51 V_{d c}$ and $0.5 V_{d c}$.


Fig. 5. Phase A to neutral voltage using different PWM schemes.

a)

b)

Fig. 6. Spectrum of the unfiltered phase A to neutral voltage: (a) - using SPWM scheme, (b) - using SVPWMM scheme.

It is necessary to stress that the SVPWMM waveform is almost identical to SPWM waveform i.e. it is characterized by the low level of low order harmonics. This fact is confirmed by spectral analysis of the unfiltered phase A voltage (Fig. 6(a)). However, the high order harmonics with the frequency close to 2 kHz caused by the switching of inverter output transistors are more persistent in the SVPWMM voltage in comparison to SPWM one (Fig. 6(b)).

The control of amplitude of six-phase waveform generated using proposed SVPWMM scheme can be performed by employment of algorithm presented in [8].

## V. Conclusions

The proposed PWM scheme using medium vectors is characterized by the low level of low order harmonics and is suitable for driving a six-phase AC motor, with symmetrical sinusoidal distribution of windings, with a single isolated neutral point. The implementation of proposed PWM scheme is relatively simple; therefore it could be implemented in cheaper microcontrollers with less computational power. In addition, simulation results confirm, that the proposed SVPWM scheme has better DC bus utilization than conventional SPWM.

The drawback of proposed PWM scheme is that it generates more switching noise than simple SPWM. However, switching noise is fairly high frequency, and does not affect the AC motor operation in a significant way. Furthermore, there may be better methods to align vectors in the switching table to reduce switching noise, which could be adopted to the proposed SVPWM scheme, but they were beyond the scope of this work, and will be further investigated in future work.

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[^0]:    Manuscript received April 2, 2014; accepted September 15, 2014.
    This research is funded by the European Social Fund under the project "Microsensors, microactuators and controllers for mechatronic systems (Go-Smart)" (Agreement No VP1-3.1-SMM-08-K-01-015).

