New Hybrid EEM/BEM Method for Ellipsoidal Electrets Determination

I. M. Novakovic¹, S. R. Aleksic², N. B. Raicevic²

¹Business School of Applied Studies,

Kralja Petra 70, 18420 Blace, Serbia

²Department of Theoretical Electrical Engineering, Faculty of Electronic Engineering, University of

Nis,

Aleksandra Medvedeva 14, P. O. Box 73, 18000 Niš, Serbia,

nebojsa.raicevic@elfak.ni.ac.rs

Abstract-Precise numerical calculations are essential in the modeling and design of the electrets, and in choosing the electret manufacturing technology. It is necessary to optimize various parameters, including arrangement of the high voltage needles and the grids. In combination with Equivalent Electrodes Method, one novel numerical method, so called Hybrid Boundary Elements Method, is proposed in this paper. An arbitrary shaped boundary between two dielectrics can be replaced by equivalent charges, located at the dielectrics boundary. It is possible, by using boundary conditions, to form a system of equations. By solving this system, the unknown polarized charges can be determined. Electrostatic field distribution both inside and outside the electrets were determined by using our method. The obtained results were compared with a commercial software (FEM) results and some analytical data. The results are in a good agreement.

Index Terms—Electret, electric field distribution, hybrid method, polarization vector.

I. INTRODUCTION

Although it has been more than a century since the discovery and the first analysis of an electret, they are still a challenge to analyse [1], [2].

The name "electret" was suggested by Heaviside in the end of the 19th century, inspired by the name of the magnet with permanent value of magnetic moment density (the permanent magnet). The first idea of forming an electret came from Michael Faraday in 1839.

Modern electret research started in Japan. For 83 years the electret was only a theoretical concept, until year 1922, when it was physically created. The main contribution to this came from Professor of Physics at the Higher Naval College of Tokyo, Mototaro Eguchi [3].

Permanent electrets, or just electrets, are solid dielectric objects (although there are those in liquid form), which are polarized under the influence of a strong electric field. After switching off the external electric fields, the electric dipoles, characterized by electric dipole moments, are still present in the dielectric object.

Manuscript received February 13, 2012; accepted December 30, 2012. The work presented here was partly supported by the Serbian Ministry of Education and Science in the frame of the project TR 33008. Electrets are used for making dust collecting filters (electric forces are widely used in air filtering), miniature microphones [4], micro-electro-mechanical microphone systems, motors and generators [5]–[8], organic transistors, regular and piezoelectric transducers, dosimeters, in an electro-acoustic transducers, biomedicine, electrostatic power-generators (proposed by Jefimenko and Tada), telecommunications, non destructive testing, medical imaging, automation, environment, advertising, safety, biological sciences etc.

Electrets are also used in production of capacitors and electrodes to increase the accuracy of their design [9]. It is known that around a semi-spherical bump there is three times stronger electric field than expected (when the electrode is perfectly smooth) and it doesn't depend on the radius of the bump. Also, chemical surface modification and low density films affect electrets properties.

The electret effects and charge storage capability continue to be the focus of research for many investigators [10], [11].

Nowadays there are many ways to create and manufacture the electrets. The best known way of making electrets is to produce an extremely strong electric field, using a large number of pin-point of a needle above a grounded plate, in order to form corona discharges. The corona charging [12] is the most widely used method to form electrets. In [13] authors describe a high speed method of charging electrets, using vacuum ultraviolet photo ionization.

Natural electrets exist as well, such as quartz and electrets made of biomaterial. Artificial electrets are made using the polarization process, the ion and electron penetration, or the incorporation of "corona charges".

All electrets can be classified into two groups. The first group consists of those that are made by injecting or incorporating space charges. They are called space charge electrets. The second group consists of dipolar electrets, which are made in the process of dipole orientation in dielectric.

When the electric dipole is placed in a homogeneous electric field, it is only influenced by a force couple that tends to rotate the dipole. The resulting force is zero, thus there is no translation. In the inhomogeneous electrostatic field, except rotation, there is also translation, caused by the resulting electric force.

High quality numerical calculation is crucial in the electret design and in choosing the electret manufacturing technology. It is necessary to optimize various parameters for corona charging, including arrangement of the high voltage needles and the grid. Both electrets and permanent magnets can be numerically modelled in a similar fway, so that our proposed method can be applied not only to the electrets, but also in the design of permanent magnets.

In the practice is often necessary to make permanent magnets having rod or oblate spheroid form. These shapes are special cases of ellipsoidal ferromagnetic body. Using theoretical analysis form, external magnetic field orientation and expression for demagnetization curve, one accurate method for magnets design will be given in a separate paper.

In recent decades, meshless methods have attracted a growing attention from mathematicians and engineering communities. It is an alternative to the domain methods of analysis in electromagnetics, such as the finite difference method (FDM) or the FEM. In combination with Equivalent Electrodes Method (EEM), the authors propose one new numerical method [14], [15], so called Hybrid Boundary Elements Method (HBEM). The basic idea of the theory is that an arbitrary shaped boundary between two dielectrics can be replaced by equivalent charges (ECH), where ECH are located at the dielectrics boundary [15], [16]. It is possible, by using condition for the normal component of the polarization vector, to form a system of linear equations, where equivalent polarized charges are unknown. By solving this system, the unknown charges can be determined.

For arbitrary shaped perfect electric conductor, electrodes are replaced by finite system of Equivalent Electrodes (EEs) [15]. In contrast to the Charge Simulation Method, where the fictitious sources are placed inside the electrodes volume, the EEs are located on the body surface. The radius of EEs is equal to the equivalent radius of the electrode part, which is substituted.

Similarly, an arbitrary shaped boundary between two magnetic materials can be replaced by a finite system of equivalent currents (EC) [17]. The ECs are located on the boundary surface of the magnetic layers, having different permeability. It is possible, using condition that the tangential components of the magnetization vector are different on the boundary of the two magnetic materials to form a system of linear equations, where the surface densities of the Ampere's microscopic currents are unknown. The unknown currents can be determined.

HBEM is possible to apply on cable terminations determination [18], metamaterial structures [19], grounding systems [20], anisotropic strip lines [21], anisotropic micro strip lines [22], nonlinear corona design [23] etc.

II. POLARIZATION OF ELECTRETS AND ELECTRIC FIELD DETERMINATION

An electret needle, approximated by the upper half of an ellipsoidal dielectric body ($z \ge 0$), with semi axis a, b and c is placed in external homogeneous electric field (Fig. 1)

$$\boldsymbol{E}_{0} = E_{0} \left(\cos \psi_{0} \sin \theta_{0} \, \hat{\mathbf{x}} + \sin \psi_{0} \sin \theta_{0} \, \hat{\mathbf{y}} + \cos \theta_{0} \, \hat{\mathbf{z}} \right), \quad (1)$$

where θ_0 and ψ_0 are spherical coordinates, vector quantities are signified by bold-italic letters, and $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ represent unit vectors. The ellipsoid is placed at conducting surface of large dimensions.



Fig. 1. Ellipsoidal dielectric body in external electric field E_0 .

The electrets filters consist of a number of electrets fibers or needles. In their surrounding an inhomogeneous field exists, under whose influence dust particles are polarized and attracted into the most inhomogeneous field area. It is an area near the needle surface.

Particles movement is translational (under the force $F = \nabla(pE)$ influence, where p is the electric dipole moment) and rotational (under the moment $T = p \times E$ influence).

Constitutive relations and Maxwell's equations, inside the electret, are:

$$\boldsymbol{D}_i = \boldsymbol{\varepsilon}_0 \boldsymbol{E}_i + \boldsymbol{P}, \ \nabla \mathbf{x} \boldsymbol{E}_i = 0 \text{ and } \nabla \boldsymbol{D}_i = 0,$$
 (2)

where D_i is electric displacement, E_i is electric field and P is polarization vector.

Using expression for electric scalar potential ϕ , Poison's equation is obtained as

$$\Delta \boldsymbol{\varphi} = \nabla \boldsymbol{P} \ . \tag{3}$$

Assuming that the electret is homogenously polarized, $\nabla P = 0$, the electric scalar potential satisfies the Laplace's equation. The homogeneous polarization of the needle is done under the external field influence. That polarization exists also after the field removal.

The procedure for the calculation is based on Laplace's equation integration, taking into account the electret's material nonlinear effects.

In the external region it is:

$$\boldsymbol{D}_e = \boldsymbol{\varepsilon}_0 \, \boldsymbol{E}_e \,, \, \boldsymbol{E}_e = \nabla \boldsymbol{\varphi}_e \, \text{and} \, \Delta \boldsymbol{\varphi}_e = 0 \,.$$
 (4)

In order to determine the electric field and potential distribution, the Laplace's equation in the elliptic coordinate system should be integrated. It is necessary to satisfy boundary conditions so that the potential and the normal component of the electric displacement vector are continuous, and the conducting plane potential is equal to zero.

It should be noticed that the potential at the large distance from the needle, when solving Laplace's equation, is

$$\varphi \cong \varphi_0 = -E_{0x}x - E_{0y}y - E_{0z}z.$$
 (5)

The connections between the coordinates of a curved coordinated system (u,v,w) and the Cartesian coordinate system are given by expressions [2]:

$$\begin{cases} x^{2} = \frac{(a^{2} + u)(a^{2} + v)(a^{2} + w)}{(a^{2} - b^{2})(a^{2} - c^{2})}, \\ y^{2} = \frac{(b^{2} + u)(b^{2} + v)(b^{2} + w)}{(b^{2} - a^{2})(b^{2} - c^{2})}, \\ z^{2} = \frac{(c^{2} + u)(c^{2} + v)(c^{2} + w)}{(c^{2} - b^{2})(c^{2} - a^{2})}. \end{cases}$$
(6)

Applying the method of variables separation, the solution for the potential can be presented in the form

$$\varphi = U(u) V(v) W(w) . \tag{7}$$

For determination of the function U(u) we solve

$$4R_{\rm u}^2U'' + 4R_{\rm u}R_{\rm u}U' - (b^2 + c^2 + 2u)U = 0, \qquad (8)$$

where

$$R_{\rm u} = \sqrt{(a^2 + u)(b^2 + u)(c^2 + u)}.$$
 (9)

This equation has two linear independent solutions. One of them is denoted with U_0 , and the other one is

$$U_1 = F(u)U_0. (10)$$

Substituting (10) into (8), the following function F(u) is obtained

$$F(u) = \int_{0}^{u} \frac{\mathrm{d}u}{(a^{2} + u)R_{u}} \,. \tag{11}$$

The second form for this integral is

$$F(u) = \frac{2[K(\alpha_0, p) - K(\alpha, p) + E(\alpha, p) - E(\alpha_0, p)]}{(a^2 - b^2)\sqrt{a^2 - c^2}},$$
 (12)

when using elliptic integrals of the first ($K(\alpha, p)$) and the second ($E(\alpha, p)$) kind

$$K(\alpha, p) = \int_{0}^{\alpha} \frac{\mathrm{d}x}{\sqrt{1 - p^2 \sin^2 x}}, E(\alpha, p) = \int_{0}^{\alpha} \sqrt{1 - p^2 \sin^2 x} \,\mathrm{d}x, \quad (13)$$

where

$$\alpha = \arcsin \sqrt{\frac{a^2 - c^2}{a^2 + u}}, \ p = \sqrt{\frac{a^2 - b^2}{a^2 - c^2}}.$$
 (14)

Solving the Laplace's equation and satisfying the boundary conditions, the potential in the needle vicinity can be presented in the form:

$$\varphi = \begin{cases} x \left(-E_0 + \frac{abc}{2\varepsilon_0} \sum_{u,v,w} \int_{u,v,w}^{\infty} \frac{\mathrm{d}t}{(a^2 + t)R_t} \cdot P \right), & u,v,w \ge 0, \\ x \left(-E_0 + \frac{abc}{2\varepsilon_0} \int_{0}^{\infty} \frac{\mathrm{d}t}{(a^2 + t)R_t} \cdot P \right), & -c^2 \le u, v, w \le 0. \end{cases}$$
(15)

The strength of the electric field inside the electret is obtained as

$$\boldsymbol{E}_{i} = \boldsymbol{E}_{0} - \boldsymbol{L}_{x} \boldsymbol{P}_{x} \, \hat{\mathbf{x}} - \boldsymbol{L}_{y} \boldsymbol{P}_{y} \, \hat{\mathbf{y}} - \boldsymbol{L}_{z} \boldsymbol{P}_{z} \, \hat{\mathbf{z}} \,, \tag{16}$$

where

$$L_{u} = \frac{abc}{2} \int_{0}^{\infty} \frac{\mathrm{d} p}{\left(d_{u}^{2} + p\right) \sqrt{\left(a^{2} + p\right)\left(b^{2} + p\right)\left(c^{2} + p\right)}}$$
(17)

are depolarization coefficients, $u = x, y, z, d_u = a, b, c$, and

$$P = P_x \hat{\mathbf{x}} + P_y \hat{\mathbf{y}} + P_z \hat{\mathbf{z}} =$$

= $P(\cos\psi \sin\theta \hat{\mathbf{x}} + \sin\psi \sin\theta \hat{\mathbf{y}} + \cos\theta \hat{\mathbf{z}}).$ (18)

The intensity of the electric polarization vector is

$$P = E_i \sqrt{\left(\frac{\cos\psi_0 \sin\theta_0}{L_x}\right)^2 + \left(\frac{\sin\psi_0 \sin\theta_0}{L_y}\right)^2 + \left(\frac{\cos\theta_0}{L_z}\right)^2} \quad (19)$$

and the spherical coordinates obey

$$\tan \theta = L_z \tan \theta_0 \sqrt{\left(\frac{\cos \psi_0}{L_x}\right)^2 + \left(\frac{\sin \psi_0}{L_y}\right)^2} , \ \tan \psi = \frac{L_x}{L_y} \tan \psi_0.$$
 (20)

III. WORKING LINE AND WORKING POINT

Electric field distribution along the electret needle is

$$\frac{E_{x0}}{E_s} = \frac{\frac{I(x/a) - \frac{1}{o(x)\sqrt{(o(x)^2 + o(b)^2 - 1)(o(x)^2 + o(c)^2 - 1)}}}{I(0, \infty) - \frac{1}{o(b)o(c)}}, (21)$$

where o(x) = x/a, and

$$I(x/a) = \int_{x/a}^{\infty} \frac{dt}{t^2 \sqrt{(t^2 + (b/a)^2 - 1)(t^2 + (c/a)^2 - 1)}} .$$
 (22)

The electric field at the needle's top, according to (11), is:

$$E_s = 1 - \frac{abc}{2} P_0 F(\infty) . \tag{23}$$

The electrets needle remains polarized, even when the external excitation field is removed $(E_0 = 0)$, i.e. $P_0(E_0 = 0) \neq 0$.

Thus the equation of the load line, shown in Fig. 2, can be determined

$$\beta = -\frac{E_{\rm c}}{D_{\rm r}} \varepsilon_0 \left(\frac{2}{F(\infty)} - 1\right) \kappa , \qquad (24)$$

for
$$\beta = \frac{D}{D_{\rm r}}$$
 and $\kappa = \frac{E}{E_{\rm c}}$.



Fig. 2. Depolarization curve and hysteresis loop.

An intersection of the depolarization curve and the load line determines the working point. The working point can be determined analytically.

The depolarization curve is a part of the hysteresis loop between the points $(0, D_r)$ and $(-E_c, 0)$, for p = 1 (Fig. 2).

Also, *s* is a parameter that represents the width of the hysteresis loop.

The new expression for the depolarization curve is

$$\frac{D}{D_m} = \frac{2}{\pi} \arctan\left[\tan\left(\frac{\pi}{2} \frac{D_r}{D_m}\right) \frac{g_1^{\ p} - g_2^{\ p}}{g_3^{\ p} - g_4^{\ p}} E_m^{\ p} \right], \quad (25)$$

where $g_1 = \frac{E_m + s E_c}{E_m - E}$, $g_2 = \frac{E_m - s E_c}{E_m + E}$, $g_3 = E_m + E_c$ and

 $g_4 = E_m - E_c \; .$

The expressions are proposed by the authors and will be used for this calculation.

IV. NUMERICAL RESULTS FOR ELECTRETS OBTAINED USING HYBRID EEM/BEM METHOD

In order to demonstrate the application of the new hybrid EEM/BEM on electrets design and determination, several examples are presented.

Applying the hybrid EEM/BEM, the whole system was divided into small parts, replaced by equivalent charges, according to the procedure described in the previous section.

The ECHs are placed at the electret's surface. The depolarization coefficients, L_x , L_y , and L_z , are used for numerical calculations (Fig. 3). It can be noticed that $L_x = L_y = L_z = \frac{1}{3}$ for a sphere, where a = b = c.



Fig. 4 shows electric field distribution, E_{x0}/E_s , versus x/a, with b/a and c/a as parameters.



b = 0.5a, c = 0.3a; 3 - b = c = 0.5a; 4 - b = 0.5a, c = 0.3a; 5 - a = b = c.

TABLE I. RESULTS OF COMPARISON FOR b = 0.5a, c = 0.3a.

Applied method	E_{x0} / E_s
Analytical	0.2396785
Hybrid EEM/BEM	0.2396701
Finite element method (FEM)	0.2394329

Table II. Normalized electrostatic field strength, $\,E_{x0}\,/\,E_s$, at the observation point (x = $1.5\,a\,$) for a different number of the

EQUIVALENT CHARGES (EE-S AND BOUNDARY ELEMENTS).

$N_{\rm ech}$	b = 0.5a, c = 0.1a	b = 0.5a, c = 0.3a
5	0.10086798	0.19994753
10	0.13090387	0.22045645

$N_{\rm ech}$	b = 0.5 a, c = 0.1 a	b = 0.5a, c = 0.3a
20	0.16898974	0.22186577
30	0.17909775	0.23009005
50	0.18144450	0.23883422
100	0.18178093	0.23909213
200	0.18190879	0.23943290

In Table I, the hybrid EEM/FEM results for the electric field at the observation point (x=1.5a) have been compared with the results obtained by the analytical method and results obtained from a commercial FEM program.

The results were found to be in a very good agreement (Relative error was less than 0.1%).

The calculated electric field values are grouped around the analytical values. The normalized ratio does not depend on the material properties, but only on the needle shape.

The 3D electric field distribution in the vicinity of the system of eight electrets, having prolate spheroid shape, is presented in Fig. 5. The electrets is placed along the z axis, in the external electric field, $\boldsymbol{E}_0 = E_0 \left(\cos 32^0 \sin 27^0 \, \hat{\mathbf{x}} + \sin 32^0 \sin 27^0 \, \hat{\mathbf{y}} + \cos 27^0 \, \hat{\mathbf{z}}\right)$, where $E_0 = 5 \,\text{MV}/m$.



Fig. 5. Electric field in the vicinity of eight spheroid electrets.

The electric field intensity, after removing external field, along directions r = 2a, r = 3a and r = 4a is shown in Fig. 6.



Fig. 6. Electric field [MV/m] distribution in the vicinity of prolate spheroid electrets along directions r = 2a, r = 3a and r = 4a.

Dependence of the polarization vector intensity, electric field and angle θ on orientation of the external electric field is shown in Fig. 7–Fig. 9.







-

Prolate spheroids represent electrets with semiaxis a=b=5c, 20c, and a=100c; b=5c, 20c.



Fig. 9. Orientation of polarization vector versus orientation of external electric field excitation ($\psi_0 = 0$).

V. CONCLUSIONS

In this paper, an application of the new hybrid EEM/BEM for calculation of the electric field at the ellipsoidal electret is presented.

During the formation of the electrets they were placed in

the homogeneous electrostatic field. Using experimental results, authors are proposing one new approximate formula for the hysteresis loop. Similar procedure can be applied to the permanent magnets design.

The obtained results show that the precision of the solution depends on the number of the equivalent electrodes and equivalent charges. Higher precision can be achieved by increasing the number of the ECHs. The number of the ECHs can't be too large because the system of the equations could be ill conditioned. In the examples presented here, a good convergence was obtained for 100÷200 ECHs per electret.

REFERENCES

- [1] G. M. Sessler, Electrets. *Laplacian Press–Morgan Hill*, California, 1998.
- [2] J. Lewiner, "Space Charge and Polarization in Insulators: A Long History with a Promising Future", *IEEE Transactions on Dielectrics* and Electrical Insulation, vol. 17, no. 4, pp. 1096–1105, 2010. [Online]. Available: http://dx.doi.org/10.1109/TDEI.2010.5539680
- [3] M. Eguchi, "Permanent Electret", *Proc. Phys. Math. Soc.*, Ser. 3, vol. iii, 1921.
- [4] E. G. Bakhoum, M. H. M Cheng, "Novel Electret Microphone", *IEEE Sensors Journal*, vol. 11, no. 4, pp. 988–994, 2011. [Online]. Available: http://dx.doi.org/10.1109/JSEN.2010.2077276
- [5] D. Miki, Y. Suzuki, and N. Kasagi, "Effect of Nonlinear External Circuit on Electrostatic Damping Force of Micro Electret Generator", in *Proc. of the Transducers 09*, Denver, USA, 2009, pp. 636–639.
- [6] C. B. Williams, C. Shearwood, M. A. Harradine, P. H. Mellor, T. S. Birch, and R. B. Yates, "Development of An Electromagnetic Micro-Generator", *IEE Circuits Devices and Systems*, vol. 148, no. 6, pp. 337–342, 2001. [Online]. Available: http://dx.doi.org/10.1049/ipcds:20010525
- [7] F. Lu, H. P. Lee, and S. P. Lim, "Modeling and Analysis of Micro Piezoelectric Power Generators for Micro-Electromechanical-Systems Applications", *Engineering Analysis with Boundary Elements – Smart Materials & Structures*, vol. 13, no. 1, pp. 57–63, 2004. [Online]. Available: http://dx.doi.org/10.1088/0964-1726/13/1/007
- [8] C. Marboutin, Y. Suzuki, and N. Kasagi, "Optimal Design of Micro Electret Generator for Energy Harvesting", in *Proc. of the PowerMEMS*, Freiburg, Germany, 2007, pp. 141–144.
- [9] T. Zhu, A. Winkleman, G.M. Whitesides, Z. Suo, "Mechanics of a Process to Assemble Microspheres on a Patterned Electrode", *Applied Physics Letters*, vol. 88, pp. 1–3, 2006. [Online]. Available: http://dx.doi.org/10.1063/1.2191743
- [10] R. M. McMeeking, C. M. Landis, "Electrostatic Forces and Stored Energy for Deformable Dielectric Materials", *Journal of Applied Mechanics*, vol. 72, pp. 581–590, 2005. [Online]. Available: http://dx.doi.org/10.1115/1.1940661
- [11] Z. Suo, X. Zhao, and W. H. Greene, "A Nonlinear Field Theory of Deformable Dielectrics", *Journal of the Mechanics and Physics of Solids*, vol. 56, pp. 467–486, 2008. [Online]. Available: http://dx.doi.org/10.1016/j.jmps.2007.05.021
- [12] G. A. Chen, "A New Model for Surface Potential Decay of Corona Charged Polymers", J. Phys D: Appl. Phys, vol. 43, pp. 055–405, 2010.
- [13] M. Honzumi, K. Hagiwara, Y. Iguchi, and Y. Suzuki, "High-Speed Electret Charging Using Vacuum UV Photoionization [Reprint]", *IEEE Electrical Insulation Magazine*, vol. 27, no. 5, pp. 62–64, 2011. [Online]. Available: http://dx.doi.org/10.1109/MEI.2011. 6025372
- [14] N. Raicevic, S. Aleksic, "One Method for Electric Field Determination in the Vicinity of Infinitely Thin Electrode Shells", *Engineering Analysis with Boundary Elements, Elsevier*, vol. 34, no. 2, pp. 97–104, 2010. [Online]. Available: http://dx.doi.org/10.1016/j.enganabound.2009.08.002
- [15] N. Raicevic, S. Aleksic, S. Ilic, "Hybrid Boundary Element Method for Multi-layer Electrostatic and Magnetostatic Problems", *Electromagnetics*, Taylor & Francis, vol. 30, no. 6, pp. 507–524, 2010. [Online]. Available: http://dx.doi.org/10.1080/ 02726343.2010.499067
- [16] N. Raicevic, "One Method For Multilayer Dielectric System Determination", in Proc. of the 8th International Conference on

Applied Electromagnetics, PES 2007, Nis, Serbia, 2007, paper no. O3-7 (on CD).

- [17] N. Raicevic, S. Aleksic, "One Method For Magnetic Multilayer Systems Determination", Computer Field Models of Electromagnetic Devices Studies in Applied Electromagnetics and Mechanics, IOS Press, Amsterdam, vol. 34, pp. 220–227, 2010.
- [18] N. Raicevic, S. Aleksic, and S. Ilic, "One Numerical Method to Determine Improved Cable Terminations", *Electric Power Systems Research – Elsevier*, vol. 81, no. 4, pp. 942 – 948, 2011.
- [19] N. Raicevic, S. Ilic, "One Hybrid Method Application on Complex Media Strip Lines Determination", in Proc. of the 3rd International Congress on Advanced Electromagnetic Materials in Microwaves and Optics (METAMATERIALS 2009), London, United Kingdom, 2009, pp. 698–700.
- [20] S. Ilic, N. Raicevic, S. Aleksic, "Application of New Hybrid Boundary Element Method on the Grounding Systems", in *Proc. of* the 14th International IGTE Symposium on Numerical Field Calculation in Electrical Engineering (IGTE 2010), Graz, Austria, 2010, pp. 160–165.
- [21] S. Ilic, S. Aleksic, and N. Raicevic, "TEM analysis of vertical broadside symmetrically coupled strip lines with anisotropic substrate", *International Journal of Applied Electromagnetics and Mechanics, IOS Press*, vol. 37, no. 2–3, pp. 207–214, 2011.
- [22] N. Raicevic, S. Ilic, "Equivalent Electrode Method Application on Anisotropic Micro Strip Lines Calculations", in *Proc. of the Conference on Electromagnetics in Advanced Applications (ICEAA* 07), Torino, Italia, 2007, pp. 998–1001.
- [23] B. Petkovic, S. Ilic, S. Aleksic, N. Raicevic, D. Antic, "A Novel Approach to the Positive DC Nonlinear Corona Design", *Electromagnetics*, Taylor & Francis, vol. 31, no. 7, pp. 505–524, 2011. [Online]. Available: http://dx.doi.org/10.1080/ 02726343.2011.607106