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Bayesian-based MEDLL for the GPS Signal Tracking

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***Abstract*—In the satellite navigation systems, distortion of a received signal correlation function, due to the multipath propagation, can gravely degrade position estimation. The positioning accuracy is strongly affected by the quality of the received signal delay estimations. In the paper we propose a Bayesian-based extension of the MEDLL algorithm capable of estimating the parameters of the line-of-sight (LOS) signal and the multipath signals. The new algorithm takes into account statistical properties of the LOS signal and the reflected signals to find a Bayesian solution for the parameters.**

***Index Terms*—Global positioning system, global communication, electromagnetic reflection, Bayesian methods.**

I. INTRODUCTION

The most influential phenomenon that affects the GPS signals during propagation is multipath. Multipath degrades the correlation function in such a way that it is not possible to determine accurately the signal delay. Therefore, removal of multipath is very important for applications where high precision measurements are required (geodesy and surveying, instrument landing systems).

The Multipath Estimating Delay-Lock-Loop (MEDLL) is the well-established method for mitigating the effects of multipath by simultaneously estimating the parameters of Line-Of-Sight (LOS) and multipath signal components. The parameters of interest are the LOS signal delay and phase and they are estimated using the LOS correlation function. In order to determine correlation function of the direct signal, multipath contributions has to be reduced. The MEDLL calculates parameter values, according to maximum likelihood (ML) theory, by minimizing the mean square error between locally generated signal and received signal.

In the paper we present a particle filtering algorithm based on the MEDLL that tracks parameters of the LOS and the multipath signal components simultaneously. This algorithm (PF-MEDLL) takes advantage of some prior information about signal component delays.

II. RECEIVED SIGNAL MODEL

The received signal r(t) from a satellite, in the multipath environment, is composed of M paths, where one is the LOS signal and the others are reflected rays of the LOS signal. All additional sources of interference are set into a single additive Gaussian noise term, v(t). After carrier removal and filtering, received signal r(t) can be written as

$$r(t) = \sum_{m=1}^{M} \alpha_m \tilde{x}(t-\tau_m) e^{j\phi_m} + v(t)\,, \qquad (1)$$

where a_m is the amplitude of the m-th path, ϕ_m is the phase of the m-th path and τ_m is the channel delay introduced by the m-th path. $\tilde{x}(t)$ is the filtered spreading waveform of the satellite composed of a C/A (coarse/acquisition) code sequence {c(·)} embedded into signal as follows

$$\tilde{x}(t) = \sum_{n=0}^{N-1} c(n) g(t - nT_c)\,, \qquad (2)$$

where, $T_c \approx 977.5$ ns denotes a period of one chip in spreading sequence, N = 1023 is length of the code sequence and *g(t)* denotes a chip-shaping pulse [1], [2].

III. MULTIPATH ESTIMATING DELAY LOCK LOOP

The received signal is correlated with $2N_{corr}+1$ locally generated spreading waveform replicas in the following way

$$R(\delta_k) = \frac{1}{T}\int_0^T r(t)\, x(t-\delta_k) dt\,, \qquad (3)$$

where k = $-N_{corr},...,N_{corr}$, δ_k is a delay of the locally generated replica, T is an integration time and *x(t)* is the infinite bandwidth spreading waveform (*g(t)* is the rectangular pulse). The delays of the spreading waveform replicas are distributed evenly over the assumed time-delay interval of the received signal [3]–[5].

Considering that the parameters of the received signal (α_m, τ_m, ϕ_m) are slowly varying, during integration period, the signal after correlation can be written as

$$R(\delta_k) \approx \sum_{m=1}^{M} \alpha_m R_{\tilde{x}x}(\delta_k - \tau_m) e^{j\phi_m}\,, \qquad (4)$$

where $R_{\tilde{x}x}(\cdot)$ is the cross-correlation function defined as

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$$R_{\tilde{x}x}(\delta_k-\tau_m)=\frac{1}{T}\int_0^T\tilde{x}(t-\tau_m)\,x(t-\delta_k)dt\,. \quad (5)$$

Under assumption that the parameters of the received signal are almost constant, the ML seeks for the parameter values that are most likely produced signal $r(t)$ on interval T. Using the ML principle, the MEDLL calculates the parameter estimates as values for which the following log-likelihood function is maximized [3]–[5]

$$L(\alpha_m,\tau_m,\phi_m)=-\int_{t-T}^{t}\left\| r(t)-\sum_{m=1}^{M}\hat{\alpha}_m x(\delta_k-\hat{\tau}_m)e^{j\hat{\phi}_m}\right\|^2 dt\,, \quad (6)$$

where the second term of the integrand is the estimate of LOS plus multipath signals.

In the case of estimating M different signals, the equations for the m-th ($1\le m\le M$) signal are [3]–[5]:

$$\begin{cases}\hat{\tau}_m=\max\limits_{\tau}\Re\left\{\left(R(\tau)-\sum\limits_{k=1,k\neq m}^{M}\hat{\alpha}_k R_{\tilde{x}x}(\tau-\hat{\tau}_k)e^{j\hat{\phi}_k}\right)e^{-j\hat{\phi}_m}\right\},\\ \hat{\alpha}_m=\Re\left\{\left(R(\hat{\tau}_m)-\sum\limits_{k=1,k\neq m}^{M}\hat{\alpha}_k R_{\tilde{x}x}(\hat{\tau}_m-\hat{\tau}_k)e^{j\hat{\phi}_k}\right)e^{-j\hat{\phi}_m}\right\},\\ \hat{\phi}_m=\arg\left\{\left(R(\hat{\tau}_m)-\sum\limits_{k=1,k\neq m}^{M}\hat{\alpha}_k R_{\tilde{x}x}(\hat{\tau}_m-\hat{\tau}_k)e^{j\hat{\phi}_k}\right)e^{-j\hat{\phi}_m}\right\}.\end{cases} \quad (7)$$

The system of equations (7) is solved using a following algorithm [5]:

Step 1 – Calculate the correlation function $R(\delta_k)$ and assign its values to $R^{(1)}(\delta_k)$. Find the maximum of the correlation function, $R^{(1)}(\delta_k)$, and corresponding delay τ_1, amplitude α_1 and phase ϕ_1.

Step 2 – Subtract the contribution of the calculated parameters (τ_1, α_1, ϕ_1) from $R^{(1)}(\delta_k)$ to yield a new approximation of the correlation function

$$R^{(2)}(\delta_k)=R(\delta_k)-\hat{\alpha}_1 R_{\tilde{x}x}(\delta_k-\hat{\tau}_1)e^{j\hat{\phi}_1}. \quad (8)$$

Find the maximum of the correlation function, $R^{(2)}(\delta_k)$, and corresponding delay τ_2, amplitude α_2 and phase ϕ_2.

Step 3 – Subtract the contribution of calculated parameters (τ_2, α_2, ϕ_2) from $R(\delta_k)$ and find a new approximation of the correlation function

$$R^{(1)}(\delta_k)=R(\delta_k)-\hat{\alpha}_2 R_{\tilde{x}x}(\delta_k-\hat{\tau}_2)e^{j\hat{\phi}_2}. \quad (9)$$

Step 4 – *S*tep 2 and Step 3 are repeated, until a certain criterion of convergence is met.

IV. PARTICLE FILTER

An objective of particle filter (PF), as the Bayesian estimator, is to approximate a conditional posterior probability density function, $p(\mathbf{x}(k)|\mathbf{Z}_k)$, where $\mathbf{x}(k)$ is vector of states at time instant k, and $\mathbf{Z}_k=\{\mathbf{z}(1),,...,\mathbf{z}(k)\}$ is set of observations until time instant k. The solution is found by recursively solving the equations based on the Bayes' rule. The recursive prediction equation and update equation are:

$$p(\mathbf{x}(k+1)|\mathbf{Z}_k)=\int p(\mathbf{x}(k+1)|\mathbf{x}(k))\,p(\mathbf{x}(k)|\mathbf{Z}_k)\,d\mathbf{x}(k)\,, \quad (10)$$

$$p(\mathbf{x}(k)|\mathbf{Z}_k)=\frac{p(\mathbf{z}(k)|\mathbf{x}(k))\,p(\mathbf{x}(k)|\mathbf{Z}_{k-1})}{p(\mathbf{z}(k)|\mathbf{Z}_{k-1})}. \quad (11)$$

From (11) is can be seen that the posterior density, $p(\mathbf{x}(k)|\mathbf{Z}_k)$, is product of a prior density, $p(\mathbf{x}(k)|\mathbf{Z}_{k-1})$, and a likelihood, $p(\mathbf{z}(k)|\mathbf{x}(k))$ [6].

The particle filter approximates the posterior density by a set of particles, $\{\mathbf{x}^{(i)}(k)\}_{i=1}^{P}$, where each particle has assigned relative weight, $\{w_i(k)\}_{i=1}^{P}$, in the following discrete summation form [1], [6]

$$p(\mathbf{x}(k)|\mathbf{Z}_k)\approx\sum_{i=1}^{P}w_i(k)\,\delta(\mathbf{x}(k)-\mathbf{x}_i(k))\,, \quad (12)$$

where $\delta(t)$ is a Dirac delta function. The weights are positive numbers and their sum is equal to one [6].

Ideally, particles are generated directly from the posterior density. Since this density is mostly unavailable, a density that is simple to sample from is defined before sampling. This density is called *importance sampling distribution*, $\pi(\mathbf{x}(k)|\mathrm{X}_{k-1},\mathbf{Z}_k)$, where $\mathrm{X}_{k-1}=\{\mathbf{x}(0),...,\mathbf{x}(k-1)\}$ [6].

The location and weight of each particle describes the posterior density. Recursively, the particle filter updates the particles location and the corresponding weights, with each new observation [6]. The importance weights can be computed sequentially as

$$\tilde{w}_i(k)=\frac{p(\mathbf{z}(k)|\mathbf{x}^{(i)}k))\,p(\mathbf{x}^{(i)}(k)|\mathbf{x}^{(i)}(k-1))}{\pi(\mathbf{x}^{(i)}(k)|\mathbf{X}_{k-1}^{(i)},\mathbf{Z}_k)}\tilde{w}_i(k-1)\,. \quad (13)$$

The importance weights are normalized using following equation

$$w_i(k)=\frac{\tilde{w}_i(k)}{\sum_{j=1}^{P}\tilde{w}_j(k)},\; i=1,...,P\,. \quad (14)$$

This approach, often leads to divergence, where at some point, all the weights are tending to zero. Using resampling step this problem can be handled [6].

The estimation process is initiated with known distribution $p(\mathbf{x}(0)|\mathbf{Z}_0)=p(\mathbf{x}(0))$ ($\mathbf{Z}_0$ is empty set of observation) [1], [6].

The reason for approximating the posterior density is that it enables computing of state estimates with respect to any criterion. One approach is to find the Maximum A Posteriori

(MAP) estimate, which is the state value that maximizes the probability of posterior distribution [1], [6]

$$\hat{\mathbf{x}}^{MAP}(k) = \arg\max_{\mathbf{x}(k)} p\left(\mathbf{x}^{(i)}(k)\middle|\mathbf{Z}_k\right). \quad (15)$$

V. DELAY ESTIMATION USING PF-MEDLL ALGORITHM

The state vector $\mathbf{x}(k)$ contains only delays for the LOS and the multipath signals. The basic PF-MEDLL algorithm for the LOS and one multipath signal can be summarized as following:

Step 1 – Initialize the particles for the LOS signal and the multipath signal delay:

$$\begin{cases} \tau_1^{(i)}(0) \sim Unif\left(-\frac{T_u}{2}, \frac{T_u}{2}\right), \\ \tau_2^{(i)}(0) \sim Unif\left(\tau_1^{(i)}(0), \tau_1^{(i)}(0) + \tau_{mp}\right), \end{cases} \quad (16)$$

where T_u is the LOS signal uncertainty, $\left[\tau_1(0), \tau_1(0) + \tau_{mp}\right]$ is the interval of interest for the multipath signal delay and $i = 1, \ldots, P$.

Step 2 – For k=1,2,... repeat step 3, step 4 and step 5.

Step 3 – Calculate estimates using MEDLL algorithm ($i = 1, \ldots, P$):

1. Calculate the correlation functions $R_1^{(1)}\left(\tau_1^{(i)}(k)\right) = R_1\left(\tau_1^{(i)}(k)\right)$ and $R_2^{(1)}\left(\tau_2^{(i)}(k)\right) = R_2\left(\tau_2^{(i)}(k)\right)$. Find the maximum of the correlation function, $R_1^{(1)}\left(\tau_1^{(i)}(k)\right)$, and corresponding estimates of delay $\hat{\tau}_1(k)$, amplitude $\hat{\alpha}_1(k)$ and phase $\hat{\phi}_1(k)$.

2. Subtract the contribution of the $\left(\hat{\tau}_1(k), \hat{\alpha}_1(k), \hat{\phi}_1(k)\right)$ from $R_2^{(1)}\left(\tau_2^{(i)}(k)\right)$ to yield a new approximation of the correlation function

$$R_2^{(2)}\left(\tau_2^{(i)}(k)\right) = R_2^{(1)}\left(\tau_2^{(i)}(k)\right) - \\ - \hat{\alpha}_1(k) R_{\tilde{x}x}\left(\tau_2^{(i)}(k) - \hat{\tau}_1(k)\right) e^{j\hat{\phi}_1(k)}. \quad (17)$$

Find the maximum of the correlation function, $R_2^{(2)}\left(\tau_2^{(i)}(k)\right)$, and corresponding estimates of delay $\hat{\tau}_2(k)$, amplitude $\hat{\alpha}_2(k)$ and phase $\hat{\phi}_2(k)$.

3. Subtract the contribution of calculated parameters $\left(\hat{\tau}_2(k), \hat{\alpha}_2(k), \hat{\phi}_2(k)\right)$ from $R_1^{(1)}\left(\tau_1^{(i)}(k)\right)$ and find a new approximation of the correlation function

$$R_1^{(1)}\left(\tau_1^{(i)}(k)\right) = R_1\left(\tau_1^{(i)}(k)\right) - \\ - \hat{\alpha}_2(k) R_{\tilde{x}x}\left(\tau_1^{(i)}(k) - \hat{\tau}_2(k)\right) e^{j\hat{\phi}_2(k)}. \quad (18)$$

4. Repeat 2 and 3, until a certain criterion of convergence is met.

Step 4 – Calculate the particle weights using equation

$$\tilde{w}_i(k) = \left\| R_1\left(\tau_1^{(i)}(k)\right) - \hat{\alpha}_2(k) R_{\tilde{x}x}\left(\tau_1^{(i)}(k) - \hat{\tau}_2(k)\right) e^{j\hat{\phi}_2(k)} \right\| \times \\ \times \left\| R_2\left(\tau_2^{(i)}(k)\right) - \hat{\alpha}_1(k) R_{\tilde{x}x}\left(\tau_2^{(i)}(k) - \hat{\tau}_1(k)\right) e^{j\hat{\phi}_1(k)} \right\|, \quad (19)$$

where $i = 1, \ldots, P$, normalize them using (14) and calculate the elements on the main diagonal of the posterior estimate covariance matrix with the following equation

$$\sigma_j^2(k) \approx \sum_{i=1}^{P} w_i(k)\left(\tau_j^{(i)}(k) - \hat{\tau}_j(k)\right)^2, \; j = 1, 2. \quad (20)$$

Step 5 – Generate new particles using Gaussian distribution for the LOS signal and truncated Gaussian distribution for the multipath component [5]

$$\begin{cases} \tau_1^{(i)}(k) \sim N\left(\hat{\tau}_1(k), \sigma_1^2(k)\right), \\ \tau_2^{(i)}(k) \sim \tau_1^{(i)}(k) + \left|N\left(\hat{\tau}_2(k) - \tau_1^{(i)}(k), \sigma_2^2(k)\right)\right|, \end{cases} \quad (21)$$

where $i = 1, \ldots, P$.

VI. SIMULATION AND RESULTS

The multipath mitigation techniques capability is compared by analysis of the estimation error for different multipath delays of signal with one multipath component in a static channel scenario [7]. In Fig. 1, the root mean square error (RMSE) is shown for the proposed PF-MEDLL estimator with P = 5 particles. The required number of correlators for PF-MEDLL is M·P. For comparison, the performance of conventional MEDLL (Ncorr = 10, 21 correlators) is also shown. The GPS signal is simulated using received signal model (1) with the bandlimited rectangular pulse defined as follows

$$g(t) = \frac{1}{\pi T_c}\left[Si\left(2\pi b t / T_c\right) - Si\left(2\pi b\left(t / T_c - 1\right)\right)\right], \quad (22)$$

where $b = BT_c/2\pi$ describes the location of the received signal cut-off frequency and $Si(t)$ is the sine integral. The bandwidth of the resulting navigation signal is B = 6 MHz. On Fig. 1, PF-MEDLL and MEDLL estimators with a two-path model and a single path model are shown for comparison. The phase difference between the LOS and the multipath component for simulated signal is 0° (constrictive phase) and adopted signal-to-multipath ratio ($SMR = 20\log\frac{a_1}{a_2}$) is 6 *dB*. RMSEs of single-path estimators are similiar and there is considerable error for estimators when multipath delay is larger or equall then 0.20T_c. The PF-MEDLL with two paths successfully mitigates the multipath bias for delays greater than 0.3T_c. On the other side, the MEDLL algorithm with two paths successfully removes multipath influence for delays greater than 0.4T_c. For smaller delays (≤0.25T_c) *RMSE* for two-path model of the PF-MEDLL is larger then the MEDLL.

Fig. 2 and Fig. 3 compare the performance of MEDLL and PF-MEDLL estimators in the presence of additive white Gaussian noise. Assumed integration time is T = 600 ms. This integration time corresponds to the duration of 30 GPS navigational bits. After correlation, this integration time yields to a *SNR* higher than 30 dB.

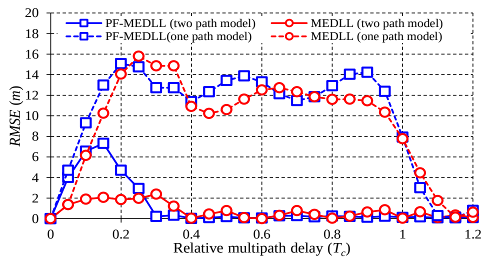

Fig. 1. RMSE of the LOS signal delay versus the relative multipath delay.

Fig. 2 and Fig. 3 show the *RMSE* of the estimation error, $e = \hat{\tau}_1 - \tau_1$, as a function of the *SNR* at the input of a satellite navigation receiver. The *SNR* values between −30 and −14 dB are typical at the input of a satellite navigation receiver [8]. On Fig. 2 RMSEs for the PF-MEDLL and the MEDLL algorithms when the received signal contains only the LOS component is shown. From the figure it is evident that RMSE of the PF-MEDLL is smaller than RMSE of the MEDLL for all SNR values.

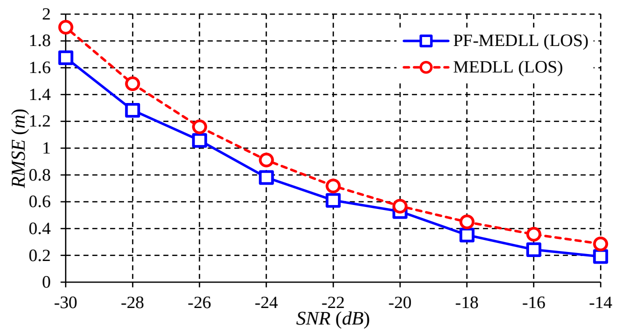

Fig. 2. RMSE of the LOS signal delay versus SNR in the multipath-free case.

On Fig. 3 the two curves show the *RMSE* for PF-MEDLL and MEDLL methods in case of an additional reflected path delayed by $0.25T_c$ and a *SMR* of 6 dB. The figure shows that RMSE of the PF-MEDLL is better than the RMSE of the MEDLL algorithm for all SNR values.

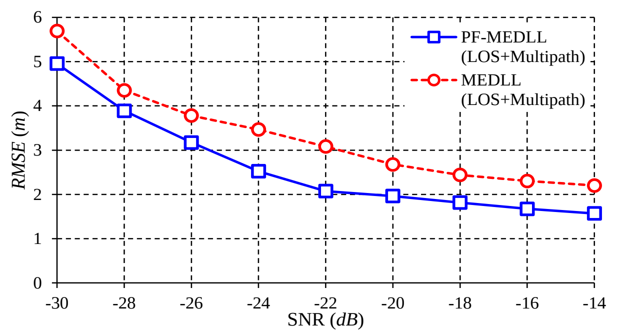

Fig. 3. Tracking RMSE versus SNR for the signal with the LOS and one multipath component.

VII. CONCLUSIONS

In this paper a Bayesian modification of MEDLL algorithm has been proposed to mitigate multipath influence in the estimation of the LOS signal delay. The proposed algorithm takes into account multipath and estimates delays of the LOS and the multipath signal components. The PF-MEDLL algorithm is computationaly efficient and obtains accurate estimates of delays under different multipath conditions.

REFERENCES

[1] P. Closas, C. Fernandez-Prades, J. A. Fernandez-Rubio, "Bayesian DLL for Multipath Mitigation in Navigation Systems Using Particle Filters", in *Proc. of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 06)*, 2006.

[2] M. Djogatović, M. Stanojević, "Multipath mitigation of GPS signal using sequential Monte-Carlo filter", in *Proc. of the 9th International Conference on Telecommunication in Modern Satellite, Cable, and Broadcasting Services (TELSIKS 09)*, vol. 1, 2009, pp. 229–232. [Online]. Available: http://dx.doi.org/10.1109/TELSKS.2009.5339420

[3] R. D. J. van Nee, "The Multipath Estimating Delay Lock Loop", in *Proc. of the IEEE Second International Symposium on Spread Spectrum Techniques and Applications (ISSSTA 92)*, 1992, pp. 39–42 [Online]. Available: http://dx.doi.org/10.1109/ISSSTA.1992.665623

[4] R. D. J. van Nee, J. Siereveld, P.C. Fenton, B. R. Townsend, "The multipath estimating delay lock loop: approaching theoretical accuracy limits", in *Proc. of the IEEE Position Location and Navigation Symposium (PLANS 94)*, 1994, pp. 246–251. [Online]. Available: http://dx.doi.org/10.1109/PLANS.1994.303320

[5] N. Delgado, F. Nunes, "Theoretical Performance of the MEDLL Algorithm for the New Navigation Signals", in *Proc. of the Seventh Conference on Telecommunications (ConfTele 09)*, vol. 1, 2009, pp.1–4.

[6] P. M. Djurić, J. H. Kotecha, J. Yhang, Y. Huang, T. Ghirmai, M. F. Bugallo, J. Miguez, "Particle Filtering", *IEEE Signal Processing Magazine*, vol. 20, no. 5, pp.19–38, 2003. [Online]. Available: http://dx.doi.org/10.1109/MSP.2003.1236770

[7] M. Lentmaier, B. Krach, P. Robertson, "Bayesian Time Delay Estimation of GNSS Signals in Dynamic Multipath Environments", *International Journal of Navigation and Observation*, 2008 [Online]. Available: http://dx.doi.org/10.1155/2008/372651

[8] R. Bischoff, R. Häb-Umbach, N. S. Ramesh, "Multipath-resistant Time of Arrival Estimation for Satellite Positioning", *International Journal of Electronics and Communications*, vol. 58, no. 1, pp. 3–12, 2004. [Online]. Available: http://dx.doi.org/10.1078/1434-8411-54100200