http://dx.doi.org/10.5755/j01.eee.19.9.2523 ELEKTRONIKA IR ELEKTROTECHNIKA, ISSN 1392-1215, VOL. 19, NO. 9, 2013

A New Method to Solve Reactive Power Optimization Problems of Power System by Introducing the Branch Current

Tao Yi[1], Chengmin Wang[1], Ning Xie[1], Yang Gao[2]
[1]*Department of Electric Engineering, Shanghai Jiaotong University, Dongchuan Rd. 800, 200240 Shanghai, China*
[2]*Department of Electric Engineering, Shenyang Institute of Engineering, Puchang Rd. 18, 110136 Shenyang, China*
yitao4965@126.com

***Abstract*—The mathematical model used in this paper is not only the traditional node voltage equation but also the introduction of the branch current equation when research the power system reactive power optimization, and so establishes a mathematical model of hybrid power network node voltages and branch currents. The state variables in this model are the node voltages and branch currents, the network flow is explicitly expressed, and they play a key role for the simplification of the solving model. Solving the model will be broken down into two sub-problems, one is a network loss minimization objective augmented Lagrange function, forming the Kuhn-Tucker conditions, and the other is a linear equation. IEEE 30-bus system example shows that the complexity and high dimension of the model solution have been significantly improved, the solving process becomes easier, and the solution is close to the global optimal solution. Compared with traditional optimal power flow algorithm, this algorithm can improve the computational efficiency of reactive power optimization.**

***Index Terms*—Current, optimization methods, power systems, reactive power.**

I. INTRODUCTION

Reactive power optimization of the power network is a dynamic, multi-bound and nonlinear nixed planning which involves the choice of reactive power compensation location, reactive compensation capacity, transformer tap adjustments and the generator terminal voltage tie with other aspects. Reactive power optimization of power system is on the condition of ensuring the system reactive power balance, reduce loss of the entire network, save system operating costs, and improve the voltage quality by regulating the generator bus voltage, on-line tap changer tap stalls, and the capacity of reactive power compensation equipment.

The mathematical model includes the choice of objective function and constraints of the agreement, generally expressed as (1).

In (1), x is the state variable, refers to the node voltage, u is control variable, means that the node injection of reactive power, the functions $f(x,u)$, $g(x,u)$ and $h(x,u)$ respectively represent the network loss (objective function), the equality and inequality constraints

$$\begin{aligned} &\min\ f(x,u) \\ &s.t. \begin{cases} g(x,u)=0, \\ h(x,u)\le 0. \end{cases} \end{aligned} \tag{1}$$

There are several ways to solve (1), including linear programming [1], [2], nonlinear programming (including quadratic programming method [3], [4], interior point methods [5]–[8], evolutionary programming [9]–[11], etc.) and intelligent algorithms [12]–[17]. All these algorithms are based on the node voltages equation, in which the node voltage, nodal injected active or reactive power as variables. Although the node voltage analysis method is effective, but there are some problems and shortcomings:

1) As the most obvious feature of the electricity network, the trend of the amount is not directly reflected, during power network analysis, the success of many network flow theory and not being used;

2) Due to the large number of inequality constraints, there is the problem of the "curse of dimensionality", and therefore it is necessary to improve its computational efficiency.

Power state through the node voltage, node injection active power, node injection reactive power and branch currents reflect these physical quantities. References [18], [19] are based on branch-current model for distribution network, the flow convergence is better, but the model ignores the admittance, and the use of a constant load impedance model, it is not suitable for transmission network.

Therefore, it is necessary to introduce branch current as the state variables in order to overcome the above problems. This paper has established a grounding branch as a current source, and forms the expansion power network equations including node voltage variable and branch current variable. Thus, the objective function can be written as the product of the line current and impedance. The reactive power optimization problem can be decomposed into two sub problems, one is the

Manuscript received September 29, 2012; accepted September 27, 2013.
This research was funded by a grant (No. 51307108) from the Youth Science Fund Project of National Natural Science Foundation of China. This research was performed in cooperation with the Institution.

minimum cost flow model and the other is a linear equation group. The former can be solved by quadratic programming method. The method in this paper has high calculation efficiency of the algorithm, and close to the global optimal solution. It has got better results by analysis IEEE-30 system.

This paper has the following assumptions:

1) The reactive power optimization inject reactive power to all nodes in the optimal target;

2) The net loss reduce small when adjust transformer tap, so ignore the optimization of the transformer turns ratio;

3) The active power is regarded as a constant, reactive power of all nodes and node voltage are considered variables except slack bus;

4) The power load model is constant;

5) Assumed that the power grid is a three-phase balanced network.

II. POWER SYSTEM NETWORK MODELLING

The electric power system network is typically used by an impedance branch and two grounded branch consisting of a π-type equivalent circuit approximation (Fig. 1). Therefore, electric power network equations make up of the impedance equation and the earthed branch circuit equation. Which, each node load can have a variety of equivalent forms, and the load is equivalent to the voltage source in this paper.

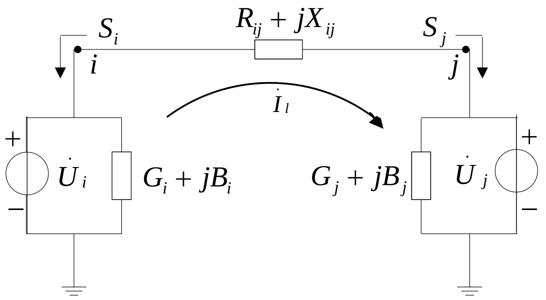

Fig. 1. π-type equivalent circuit 1.

In Fig. 1, $S_i = p_i + \mathrm{j}q_i$, $S_j = p_j + \mathrm{j}q_j$ are the node i, j injection power respectively, assume node i, j voltage vector as $\dot{U}_i = V_i \cos\theta_i + \mathrm{j}V_i \sin\theta_i$, $\dot{U}_j = V_j \cos\theta_j + \mathrm{j}V_j \sin\theta_j$, respectively, branch current is $\dot{I}_l = i_l^a + \mathrm{j}i_l^r$, $R_{ij} + \mathrm{j}X_{ij}$ is the impedance of impedance branch, $G_i + \mathrm{j}B_i$ is the grounded branch admittance value of node i, subscript $i, j = 1, 2, \cdots, N$ is node No., subscript $l = 1, 2, \cdots, L$ is branch No., so the branch equation can be expressed

$$\dot{I}_l (R_{ij} + \mathrm{j}X_{ij}) = \dot{U}_i - \dot{U}_j . \tag{2}$$

The mixed power network branch current - node voltage equations can be obtained by deducing (2):

$$i_l^a R_{ij} - i_l^r X_{ij} - V_i \cos\theta_i + V_j \cos\theta_j = 0, \tag{3}$$

$$i_l^a X_{ij} + i_l^r R_{ij} - V_i \sin\theta_i + V_j \sin\theta_j = 0. \tag{4}$$

For node i, the node injection power equals the product of node voltage and branch current conjugate when the load is treated as voltage source. Node injection current is divided into two parts, one is earth branch and the other is load branch, and then the power equation is

$$\overset{*}{U}_i \left[\sum_{l \in i} \dot{i}_{li} - U_i \sum_{l \in i} (G_l + \mathrm{j}B_l) \right] = p_i - \mathrm{j}q_i , \tag{5}$$

where $\sum_{l \in i} \dot{i}_{li}$ is node injection current, its plural form is $\sum_{l \in i} i_{li} = \sum_{l \in i} i_{li}^a + \mathrm{j}\sum_{l \in i} i_{li}^r$, which means the sum of all branch currents associated with node i. $U_i \sum_{l \in i} (G_l + \mathrm{j}B_l)$ is the sum of ground branch currents associated with node i, and $\sum_{l \in i} (G_l + \mathrm{j}B_l)$ is the sum of ground branch admittances associated with node i. Then the current of load branch equals node injection current minus ground branch current.

In order to simplify the equation derivation and calculation, ignore the node ground branch conductance G_l, and suppose $x_i = \sum_{l \in i} i_{li}^a$, $y_i = \sum_{l \in i} i_{li}^r$, $B_{i0} = \sum_{l \in i} B_l$ expand (5):

$$V_i x_i \cos\theta_i + V_i y_i \sin\theta_i = p_i , \tag{6}$$

$$V_i y_i \cos\theta_i - V_i x_i \sin\theta_i + V_i^2 B_{i0} = -q_i . \tag{7}$$

There are the same form equations for the node j. We can obtain (8) by deducing (3) and (4):

$$\begin{cases} i_l^a = -G_{ij}(V_i \cos\theta_i - V_j \cos\theta_j) + \\ \qquad + B_{ij}(V_i \sin\theta_i - V_j \sin\theta_j), \\ i_l^r = -G_{ij}(V_i \sin\theta_i - V_j \sin\theta_j) - \\ \qquad - B_{ij}(V_i \cos\theta_i - V_j \cos\theta_j), \end{cases} \tag{8}$$

where, G_{ij}, B_{ij} is the admittance of branch l. Brings it into (6) and (7) can get the traditional forms of node voltage equation:

$$\begin{cases} p_i - V_i \cos\theta_i \sum_{j \in i} (G_{ij} V_j \cos\theta_j - B_{ij} V_j \sin\theta_j) - \\ -V_i \cos\theta_i \sum_{j \in i} (G_{ij} V_j \sin\theta_j + B_{ij} V_j \cos\theta_j) = 0, \\ q_i - V_i \sin\theta_i \sum_{j \in i} (G_{ij} V_j \cos\theta_j - B_{ij} V_j \sin\theta_j) + \\ +V_i \cos\theta_i \sum_{j \in i} (G_{ij} V_j \sin\theta_j + B_{ij} V_j \cos\theta_j) + V_i^2 B_{i0} = 0. \end{cases} \tag{9}$$

The difference in (9) is that there do not contain ground branch susceptance in the node self-susceptance B_{ii}. This also verified that the introduction of the branch current does not change the nature of the equations. And with the introduction of the branch current the directly observed values increase. It can be directly expressed by branch current like the question of network loss.

Mixed equations of the electricity network are composed by (3), (4), (6), (7), and node voltage and branch current are the state variables. The equations are linear function of the branch current and nonlinear function of node voltage. Figure 2 is a π-type equivalent circuit of current source simulated ground branch, where, $\dot{I}_{Gi}$ is the ground branch current of node i.

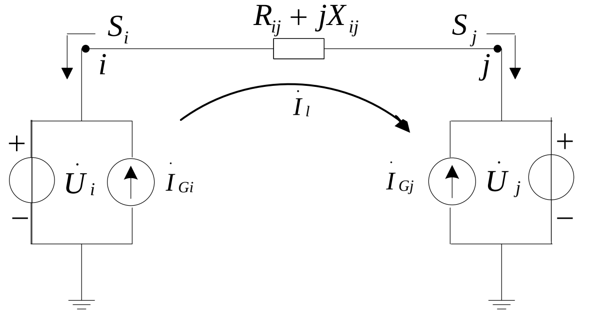

Fig. 2. π-type equivalent circuit 2.

III. Reactive Power Optimization

The global minimum network loss objective function of reactive power optimization can be expressed as

$$f = \min \sum_{l=1}^{L} \left[(i_l^a)^2 + (i_l^r)^2 \right] R_l, \tag{10}$$

where $l = 1, 2, \cdots, L$. Equality constraint are (3), (4), (6), (7), inequality constraints are:

$$V_i^{\min} \le V_i \le V_i^{\max}, \tag{11}$$

$$Q_i^{\min} \le Q_i \le Q_i^{\max}. \tag{12}$$

Equation (13) can be obtained by deducing (6)

$$V_i = \frac{p_i}{x_i \cos\theta_i + y_i \sin\theta_i} \tag{13}$$

and then

$$V_i^{\min} \le \frac{p_i}{x_i \cos\theta_i + y_i \sin\theta_i} \le V_i^{\max} \tag{14}$$

and from (7):

$$Q_i^{\min} \le V_i x_i \sin\theta_i - V_i y_i \cos\theta_i - V_i^2 B_{i0} \le Q_i^{\max}. \tag{15}$$

The reactive power optimization mathematical model is (10), (3), (4), (6), (7), (14) and (15), it has the following characteristics:

1) The target function is a quadratic function of branch current variables;

2) The constraint is the linear function of branch current variables.

The injection active power is a constant value for all nodes in reactive power optimization model, and variables are divided into branch current, node voltage and reactive power injection. Branch currents and node voltages represent the network status, they are considered as state variables, and node injected reactive power is the control variable. The augmented Lagrange function of reactive power optimization model is

$$\begin{aligned} L(U, I, Q) = {} & \sum_{l=1}^{L} \left[(i_l^a)^2 + (i_l^r)^2 \right] R_l + \\ & + \alpha^a (i_l^a R_{ij} - i_l^r X_{ij} - V_i \cos\theta_i + V_j \cos\theta_j) + \\ & + \alpha^r (i_l^a X_{ij} + i_l^r R_{ij} - V_i \sin\theta_i + V_j \sin\theta_j) + \\ & + \beta^p (p_i - V_i \cos\theta_i \sum_{l\in i} i_{li}^a + V_i \sin\theta_i \sum_{l \in i} i_{li}^r) + \\ & + \beta^q (q_i + V_i y_i \cos\theta_i + V_i x_i \sin\theta_i - V_i^2 B_{i0}) + \\ & + \gamma^{u1} \left[\frac{p_i}{x_i \cos\theta_i + y_i \sin\theta_i} - V^{\max} \right] + \\ & + \gamma^{u2} \left[V^{\min} - \frac{p_i}{x_i \cos\theta_i + y_i \sin\theta_i} \right] + \\ & + \lambda^{q1} (Q_i^{\min} - V_i x_i \sin\theta_i - V_i y_i \cos\theta_i - V_i^2 B_{i0}) + \\ & + \lambda^{q2} (V_i x_i \sin\theta_i - V_i y_i \cos\theta_i - V_i^2 B_{i0} - Q_i^{\max}). \end{aligned} \tag{16}$$

Due to the partial derivatives $\partial L / \partial q_i = \beta^q = 0$ of the control variable q_i, and q_i is only in (7), so (7) can be omitted, and the optimal solution has nothing to do with the q_i. So, the reactive power optimization model can be simplified by the (10), (3), (4), (6), (14) and (15), and optimize only for state variables $\left[\dot{I}_l, \dot{U}_i \right]^T$. Eventually optimal node inject reactive power q_i can be calculated by (7).

IV. Model Solution

We assume that $E = V_i \cos\theta_i - V_j \cos\theta_j$ and $F = V_i \sin\theta_i - V_j \sin\theta_j$ from (3) and (4), the (17) can be obtained

$$\begin{bmatrix} E \\ F \end{bmatrix} = \begin{bmatrix} R_{ij} & -X_{ij} \\ X_{ij} & R_{ij} \end{bmatrix} \begin{bmatrix} I_l^a \\ I_l^r \end{bmatrix}. \tag{17}$$

Equations (17) are linear, and each node voltage value can be calculated if known the real part value and the imaginary part value of branch current.

Reactive power optimization model (call the problem A) composes by (10), (6), (14), (15) and (17). It can be broken down into two sub-problems: one is (10), (14), (15) named S, and the other is composed of linear equations (6) and (17).

Problems A can be described as (1), where u is expressed as a real part and an imaginary part of the branch current vector, x is expressed in amplitude and phase angle of nodal voltage vector. $h(x, u) \le 0$ is the inequality constraints, in particular are (14) and (15). $g(x,u)=0$ is the equality constraints, in particular (6) and (17). So the Kuhn-Tucker condition of problems A is:

$$\begin{cases} f_x + h_x^T \alpha + g_x^T \beta = 0, \\ f_u + h_u^T \alpha + g_u^T \beta = 0, \\ h(x,u) \le 0, \\ \alpha \ge 0, \\ h(x,u)\alpha^T = 0, \\ g(x,u) = 0. \end{cases} \tag{18}$$

In (18), α and β are the Lagrange factor. Because there is no node voltage vector in the objective function, $f_x = 0$, so (18) can be written as:

$$\begin{cases} f_u + h_u^T \alpha - g_u^T (g_x^T)^{-1} h_x^T \alpha = 0, \\ h(x,u) \le 0, \\ \alpha \ge 0, \\ h(x,u)\alpha^T = 0, \\ g(x,u) = 0. \end{cases} \tag{19}$$

Similarly, sub-problem S can be described as:

$$\begin{cases} \min f(x,u), \\ s.t.\ h(x,u) \le 0. \end{cases} \tag{20}$$

Its Kuhn-Tucker condition is:

$$\begin{cases} f_u + h_u^T \alpha = 0, \\ h(x,u) \le 0, \\ \alpha \ge 0, \\ h(x,u)\alpha^T = 0. \end{cases} \tag{21}$$

Comparing (19) with (21) we find that the difference between them lies in $g(x,u)=0$ and (22)

$$\Delta u' = -g_u^T (g_x^T)^{-1} h_x^T \alpha. \tag{22}$$

Obviously, the solutions calculated by solving the sub-problem S and linear equations (6) and (17) are not equivalent with that of problem A because of $\Delta u'$. In order to obtain the exact solution of the reactive power optimization, the variable u must be modified according to (22) after solving the sub-problem S.

Solution of the whole model iteration steps are as follows:

1) Assuming $k=0$ set the initial value to $U^{(k)}$;

2) To $I^{(k)}$, $U^{(k)}$ as state variables for solving sub-problem S, get branch current $I^{(k+1)}$;

3) If $\left\| I^{(k+1)} - I^{(k)} \right\| \le \varepsilon$ (ε is a small positive number) finish iteration and go to step 5), otherwise continue;

4) The node voltages can be calculated by using (6) and (17), $k=k+1$ and return to step 2);

5) The node reactive power injection will be calculated by using (7).

The solution of sub-problem S is global optimal because it is about convex quadratic programming problem of line resistance, and (6), (17) are linear equations which have the only solution. Therefore, the optimal solution obtained finally is closely enough to the global optimal solution.

V. EXAMPLE ANALYSIS

The case study is made at IEEE-30 system with active set arithmetic solved the sub-problem S. The upper limit of voltage at node 10 is set as 1.0421 and the upper limit of voltage at node 24 is set as 1.0261 while upper limit of voltage is set as 1.1 and lower limit of voltage is set as 0.95 at all nodes.

The final calculating results are listed in Table I, and the reactive powers at node 10 and 24 are 0.144558 and 0.0914809 calculated by the nodal injective current.

TABLE I. IEEE-30 SYSTEM NODE CALCULATION RESULTS.

Node Number	**Voltage Magnitude**	**Voltage Single**	**Real Part of Nodal Injective Current**	**Imaginary Part of Nodal Injective Current**
1	1.0012	-12.867	0.0994	-0.0422
2	1.0341	-2.773	-0.3471	0.0024
3	1.0313	-4.746	0.0243	0.0118
4	1.0261	-5.686	0.0778	-0.0470
5	1.0062	-9.054	0.6936	-0.0466
6	1.0219	-6.561	0.0075	-0.1056
7	1.0067	-8.112	0.2114	-0.1204
8	1.0232	-6.565	-0.0430	0.0520
9	1.0540	-8.392	-0.0030	0.0770
10	1.0390	-10.181	0.0914	0.1781
11	1.0911	-6.854	-0.1456	0.1647
12	1.0532	-9.537	0.1044	0.0277
13	1.0888	-8.830	-0.1198	0.2407
14	1.0382	-10.440	0.0560	-0.0260
15	1.0341	-10.550	0.0736	-0.0382
16	1.0404	-10.084	0.0301	-0.0229
17	1.0342	-10.361	0.0754	-0.0708
18	1.0237	-11.121	0.0290	-0.0146
19	1.0211	-11.267	0.0848	-0.0508
20	1.0242	-11.053	0.0197	-0.0108
21	1.0279	-10.674	0.1471	-0.1386
22	1.0302	-10.675	-4.9789×10-5	8.0926×10-5
23	1.0268	-11.021	0.0277	-0.0213
24	1.0258	-11.299	0.0787	0.0085
25	1.0241	-11.141	-6.6480×10-6	1.9351×10-5
26	1.0060	-11.557	0.0296	-0.0294
27	1.0322	-10.783	0.0090	0.1032
28	1.0180	-6.980	0.0011	-0.0788
29	1.0118	-11.996	0.0214	-0.0136
30	1.0502	0	-1.3347	0.0185

The thirteen iterations are needed with step 3.24 listed in Table II. The upper limit of voltage at node 10 is violated in iteration 4, then the search direction is changed and the violation is eliminated with Lagrange factor 0.0178555. Up to iteration 12, the upper limit of voltage at 24 is violated and it is eliminated with Lagrange factor 0.0079243. The final network losses are reduced from 0.0879016 to 0.0873921.

TABLE II. CALCULATING PROCESS INFORMATION.

Iterations	Network losses	Lagrange Factors	Voltage Magnitude at Node 10	Voltage Magnitude at Node 24
1	0.0879016	0	1.04077	1.02017
2	0.0878057	0	1.04146	1.02195
3	0.0877322	0	1.04190	1.02334
4	0.0876737	0.0178555	1.04214	1.02441
5	0.0876258	0	1.03724	1.01923
6	0.0876102	0	1.03812	1.02109
7	0.0875532	0	1.03873	1.02254
8	0.0875103	0	1.03913	1.02367
9	0.0874768	0	1.03936	1.02454
10	0.0874499	0	1.03947	1.02521
11	0.0874275	0	1.03949	1.02573
12	0.0874085	0.0079243	1.03943	1.02612
13	0.0873921	0	1.03662	1.02269

The comparing results of proposed approach with Newton method and reduced gradient algorithm are listed in Table III while the inequality constraints are ignored. It can be seen that optimization effect of proposed approach is better than Newton method and reduced gradient algorithm.

TABLE III. COMPARING RESULTS WITH OTHERS.

Arithmetic	Proposed in This Paper	Newton Method	Gradient Method
Network Losses	0.0852368	0.0854694	0.0854816
Injective Reactive Power at Node 10	0.1746810	0.2937780	0.2872100
Injective Reactive Power at Node 24	0.0157019	0.1408180	0.1534660

The comparing results of voltage magnitude with Newton method and reduced gradient algorithm are listed in Table IV. The results show that the node voltage magnitude is more closed to the standard data calculated by the proposed algorithm in this paper, so the error of voltage magnitude is smaller while optimizing the reactive power.

TABLE IV. COMPARING NODE VOLTAGE MAGNITUDE WITH OTHERS.

Arithmetic	Standard Data	Proposed in this Paper	Newton Method	Gradient Method
Voltage Magnitude at Node 10	1.0339	1.0390	1.0402	1.0399
Voltage Magnitude at Node 24	1.0108	1.0258	1.0262	1.0264

VI. DISCUSSION AND OUTLOOK

The hybrid electric power network equations composed of node voltage and branch current can also provide useful ideas to solve practical problems of power system in several other areas in addition to the excellent performance in the reactive power optimization.

A. *Explicit expression of node voltage high and low solutions*

Deducing (5) can obtain:

$$\begin{cases} e_i = (p_i - f_i y_i)/x_i, \\ f_i = \dfrac{\left(\begin{array}{l}\left[2B_{i0} p_i y_i - x_i (x_i^2 + y_i^2)\right] \pm \\ \pm x_i \sqrt{(x_i^2 + y_i^2)^2 + 4B_{i0} q_i (x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2}\end{array}\right)}{2B_{i0}(x_i^2 + y_i^2)}, \end{cases} \quad (23)$$

where $\dot{U}_i = e_i + \mathrm{j} f_i$ is the voltages of node i expressing as rectangular coordinate, and the node ground branch conductance G_l has been ignored.

Equation (23) is the node voltage analytical expression represented by the branch current. It represents the high and low solutions of node voltage, and then the problem of multiple solutions of power system can be analyzed by using (23).

B. *Voltage instability region (unstable round)*

If (23) have solutions the following condition must be met

$$\begin{aligned} &(x_i^2 + y_i^2)^2 + 4B_{i0} q_i (x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2 \geq 0 \Rightarrow \\ &\Rightarrow \left[(x_i^2 + y_i^2) + 2B_{i0} q_i\right]^2 \geq (2B_{i0}\sqrt{p_i^2 + q_i^2})^2 \Rightarrow \\ &\Rightarrow x_i^2 + y_i^2 \geq -2B_{i0} q_i + 2B_{i0}\sqrt{p_i^2 + q_i^2}. \end{aligned} \quad (24)$$

The meaning of (24) is:

1) When the square of the amplitude of the injection current of nodes is out of the circle of which the center is $2B_{i0} q_i$ and the radius is $2B_{i0}\sqrt{q_i^2 + p_i^2}$, that is only '>' condition has been met in (24), the high and low solutions of (23) exist, and the system is stable;

2) When the square of the amplitude of the injection current of nodes is in the circle, that is only '<' condition has been met in (24), no solutions of (23) exist, so the system is unstable;

3) Equation (23) has a unique solution and the solution is on the circle when '=' condition has been met in (24), so the unique solution is the stable margin of system. The voltage collapse point can be found if calculate the power flow equations under this conditions.

C. *Voltage stability critical condition*

When the equality of (24) meets the two solution curves of node voltage intersect, and reach the voltage stability critical point. So, the voltage stability critical condition is

$$(x_i^2 + y_i^2) = 2B_{i0}\gamma_i, \quad (25)$$

where $\gamma_i = q_i + \sqrt{p_i^2 + q_i^2}$. Equation (25) is called the characteristics of the voltage stability critical point. When (25) is met at any node of power system the voltage instability will occur.

The corresponding node voltage changes to (26), and it is single solution but not multiple solutions:

$$\begin{cases} e_i = \dfrac{p_i x_i - y_i \gamma_i}{2B_{i0}\gamma_i}, \\ f_i = \dfrac{p_i y_i + x_i \gamma_i}{2B_{i0}\gamma_i}. \end{cases} \tag{25}$$

When the branch current and node voltage are as variables the expressive information is more abundant, and can solve the problem difficult to resolve in the past. So the hybrid electric network equations can be applied to research in different fields of power system.

VII. CONCLUSIONS

The dimension of network equations increases while introduce node voltage and branch current as a state variable during power system reactive power optimization. At the same time the computational complexity increases because need to consider the constraint of node voltage and node injection power. So it brings some problems and difficulties compared with the conventional node voltage equations. But the above issues are handled through the improvement of the algorithm. Ultimately, conclude as follows:

1) The power network equations can be expressed as a mixed form based on node voltages and branch currents, and the mixed equations contain more abundant information to improve the observability of the power system, and the calculation efficiency enhances also because the branch power flow can be calculated in the same computing cycle with node voltage;

2) In order to simplify the solution of reactive power optimization the problem has been decomposed into two easy solving sub-problems, and then reduce the complexity of the problem and the dimension of the equations. So the computational efficiency has been improved.

3) After comparing with the traditional method the proposed algorithm is closer to the global optimal solution because of the further correction to the state variable, and the node voltage error is smaller, too.

4) Because the convergence of the proposed algorithm is good the mixed equation mathematical model based on node voltages and branch currents proposed in this paper can be applied to other optimization problems in the power system.

REFERENCES

[1] D. Thukaram, G. Yesuratnam, “Optimal reactive power dispatch in a large power system with AC-DC and FACTS controllers”, *IET Generation, Transmission & Distribution*, vol. 2, pp. 71–81, Jan. 2008. [Online]. Available: http://dx.doi.org/10.1049/iet-gtd:2007 0163

[2] Yuwei Yang, Renjun Zhou, Xiaohong Ran, “Robust optimization with box set for reactive power optimization in wind power integrated system”, in *Proc. IEEE Power and Energy Society General Meeting*, San Diego, Canada, July 2012, pp. 1–6. [Online]. Available: http://dx.doi.org/10.1109/PESGM.2012.6345174

[3] Wang Ke, Wang Le, “Improved sequential quadratic programming algorithm of reactive voltage optimization”, in *Proc. Asia-Pacific Power and Energy Engineering Conf.*, Wuhan, China, Mar. 2011, pp. 1–4. [Online]. Available: http://dx.doi.org/10.1109/ APPEEC. 2011.5748938

[4] Yongfei Ma, Xianmin Wang, “Application of Sequential Quadratic Programming Algorithm based on Region Search Method in Reactive Power Optimization”, in *Proc. 3rd Int. Conf. Electrical and Electronics Engineering*, Veracruz, Mexico, Sept. 2006, pp. 1–5. [Online]. Available: http://dx.doi.org/ 10.1109/ICEEE.2006.251913

[5] W. D. Rosehart, C. A. Canizares, V. H. Quintana, “Multiobjective optimal power flows to evaluate voltage security costs in power networks”, *IEEE Trans. Power Systems*, vol. 18, pp. 578–587, May 2003. [Online]. Available: http://dx.doi.org/10.1109/TPWRS. 2003.810895

[6] A. Mahmoudabadi, M. Rashidinejad, M. Mohammadian et al, “An application of CHA to concurrent short-term transmission expansion & reactive power planning”, in *Proc. IEEE Trondheim PowerTech*, Trondheim, Norway, June 2011, pp. 1–6. [Online]. Available: http://dx.doi.org/10.1109/PTC.2011.6019283

[7] M. B. Liu, S. K. Tso, Y. Cheng, “An extended nonlinear primal-dual interior-point algorithm for reactive-power optimization of large-scale power systems with discrete control variables”, *IEEE Trans. Power Systems*, vol. 17, pp. 982–991, Jan. 2002. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2002.804 922

[8] M. B. Liu, Y. Cheng, S. H. Lin, “For reactive power optimization interior point linear and nonlinear programming methods”, *Automation of electric power systems*, vol. 26, pp. 22–26, May 2002.

[9] C. Jiang, C. Wang, “Improved evolutionary programming with dynamic mutation and metropolis criteria for multi-objective reactive power optimisation”, in *Proc. IEE Generation, Transmission and Distribution*, vol. 152, pp. 291–294, Apr. 2005. [Online]. Available: http://dx.doi.org/10.1049/ip-gtd:20045007

[10] C. H. Liang, Z. Y. Zhong, J. B. Huang, et al, “An improved evolutionary programming method and its use in power system reactive power optimization”, *Power System Technology*, vol. 30, pp. 16–20, May 2006.

[11] C. H. Liang, C. Y. Chung, K. P. Wong, et al, “Parallel Optimal Reactive Power Flow Based on Cooperative Co-Evolutionary Differential Evolution and Power System Decomposition”, *IEEE Trans. on Power Systems*, vol. 22, pp. 249–257, Jan. 2007. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2006.887 889

[12] S. Durairaj, B. Fox, “Evolutionary computation based reactive power optimization”, in *Proc. IET-UK International Conf. Information and Communication Technology in Electrical Sciences*, Chennai, Tamilnadu, India, Dec. 2007, pp. 120–125.

[13] M. Y. Ali, K. Raahemifar, “Reactive power optimization based on hybrid particle swarm optimization algorithm”, in *Proc. 25th IEEE Canadian Conf. Electrical & Computer Engineering*, Montreal, QC, Canada, Apr. 2012, pp. 173–177. [Online]. Available: http://dx.doi.org/10.1109/APWCS.2010.50

[14] R. H. Liang, Y. S. Wang, “Fuzzy-Based Reactive Power and Voltage Control in a Distribution System”, *IEEE Trans. Power Delivery*, vol. 18, pp. 610–618, Apr. 2003. [Online]. Available: http://dx.doi.org/10.1109/TPWRD.2003.809740

[15] W. Yan, S. Lu, D. C. Yu, “A Novel Optimal Reactive Power Dispatch Method Based on an Improved Hybrid Evolutionary Programming Technique”, *IEEE Trans. Power System*, vol. 19, pp. 913–918, May 2004. [Online]. Available: http://dx.doi.org/ 10.1109/TPWRS.2004.826716

[16] Junmin Zhang, Tinghua Huang, Honggang Zhang, “The Reactive Power Optimization of Distribution Network Base on an Improved Genetic Algorithm”, in *Proc. IEEE/PES Asia and Pacific Transmission and Distribution Conf. and Exhibition*, Dalian, China, 2005, pp. 1–4. [Online]. Available: http://dx.doi.org/ 10.1109/TDC.2005. 1546960

[17] H. Mantawy, M. S. AI Ghamdi, “A New Reactive Power Optimization Algorithm”, in *Proc. IEEE Bologna PowerTech Conf.*, Bologna, Italy, June 2003, [Online]. Available: http://dx.doi.org/10.1109/PTC.2003. 1304768

[18] M. Pau, P. A. Pegoraro, “Efficient Branch-Current-Based Distribution System State Estimation Including Synchronized Measurements”, *IEEE Trans. Instrumentation and Measurement*, vol. 62, pp. 2419–2429, Sept. 2013. [Online]. Available: http://dx.doi.org/10.1109 /TIM.2013.2272397

[19] Hui Li, Ming-hao Yang, “A branch-current-based state estimation for distribution systems non-measurement loads”, in *Proc. IEEE Power Engineering Society General Meeting*, June 2004, pp. 440–444. [Online]. Available: http://dx.doi.org/10.1109/PES.2004. 1372835