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# **Study of Frequency Modulated Boost Converter Operating in Discontinuous Conduction Mode**

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#### Introduction

Switch-mode power converters (SMPC) are very popular today mainly because of higher efficiency and smaller size. They are used in switch-mode power supplies, battery chargers, active power factor correctors, etc [1-3]. SMPC can operate in both continuous and discontinuous conduction modes (CCM and DCM respectively). The main mode of operation is usually CCM, in which power inductor current never falls to zero. DCM sometimes is chosen to be the main mode of operation of SMPC, mainly, due to feedback loop stability considerations [4, 11, 12]. At light loads SMPC can also enter DCM.

The main disadvantage of SMPC is high electromagnetic interference (EMI) [3, 5]. In recent years spread spectrum method which is usually based on the use of switching frequency modulation (FM) has been of great interest for EMI reduction [5, 6, 9]. However the use of this method can substantially increase output voltage ripples (OVR) in both CCM and DCM [5-9]. In [8] effect of FM on OVR of closed-loop buck converter operating in DCM was examined in details, however effect of FM on OVR of other SMPC topologies operating in DCM has not been investigated yet.

In this paper OVR of FM boost converter operating in DCM will be considered and influence of modulation parameters on OVR will be examined. Experimental verification will also be performed.

#### Theoretical analysis of OVR of FM boost SMPC

Closed-loop FM boost converter (shown in Fig.1) will be used for the analysis. Boost converter is chosen here because it is one of the most popular SMPC topologies [11]. It is assumed that the converter operates in DCM. As it is proved in [8] for buck SMPC operating in DCM, in general the ripples consist of HF switching ripples with switching frequency  $f_{sw}$  and LF ripples with modulation frequency  $f_m$ . The same can also be concluded for boost

SMPC, as it can also be seen in Fig.2, where simulated in SIMULINK output voltage waveforms for the unmodulated and FM boost converter are shown. Peak-to-peak output voltage ripples of FM SMPC in DCM can be calculated using a general expression from [8] as follows

$$V_{ofmp-p} = \max\{\tilde{v}_{LF}(t) + A_{HF}(t)\} - \min\{\tilde{v}_{LF}(t) - A_{HF}(t)\}, (1)$$

where  $\tilde{v}_{LF}(t)$  is the LF ripples;  $A_{HF}(t)$  is the envelope of the HF ripples, which can be derived by substituting instantaneous switching frequency  $f_{sw}(t)$  into the expression for the unmodulated SMPC peak-to-peak output voltage ripples  $V_{p-p}$ . Assuming that usually OVR are mainly due to an equivalent series resistance (ESR) of the output capacitor, it can be obtained that

$$A_{HF}(t) = \frac{r_{coul}V_{in}D}{2Lf_{sw}(t)},$$
(2)

where L is the power inductor inductance;  $r_{cout}$  is the output capacitor ESR; D is the average duty ratio;  $V_{in}$  is DC input voltage;  $f_{sw}(t)$  is the instantaneous switching frequency according to

$$f_{sw}(t) = f_{sw} + \Delta f_{sw} m(t) \quad , \tag{3}$$

where  $\Delta f_{sw}$  is the switching frequency deviation; m(t) is the modulating signal (e.g. sine, triangular, sawtooth, etc) with unitary amplitude;  $f_{sw}$  is the nominal switching frequency. Note that for more precise calculations of  $A_{HF}(t)$ , OVR due to finite output capacitor capacitance ( $C_{out}$ ) should also be considered.

OVR of FM boost converter in DCM increase not only due to increase in HF switching ripples but also because the LF ripples  $\tilde{v}_{LF}(t)$  are induced in output voltage. In Fig. 2(b) clear presence of the LF component in output voltage for closed-loop FM boost converter can be seen. In [7, 8] it is found out that the LF variations of output voltage of FM buck SMPC can be evaluated by averaging current and voltage waveforms. It is obvious, that the cause of the LF ripples in FM boost converter operating in DCM is the same as in FM buck converters and other FM SMPC: output voltage averaged over switching period ( $\langle v_{out} \rangle$ ) depends on the switching frequency which changes in time with modulation frequency  $f_m$ .



Fig. 1. Schematic of closed-loop boost converter



**Fig. 2.** Simulated closed-loop boost converter OVR in DCM: (a) without FM; (b) with sinusoidal FM. (Parameters:  $f_m$ =1kHz  $f_{sw}$ =80kHz;  $\Delta f_{sw}$ =30kHz  $C_{out}$ =330 $\mu$ F; L=40 $\mu$ H;  $r_{cout}$ =0.035 $\Omega$ ;  $R_{load}$ =120 $\Omega$ ;  $f_{cut}$ =4kHz;  $V_{in}$ =7V;  $V_{out}$ =19V)

Initially LF ripples of open-loop FM boost converter will be considered. For this purpose averaged circuit model for boost converter in DCM from [10] can be used (see Fig.3). It follows from this model that LF ripples for open-loop boost converter in operator form are

$$\widetilde{v}_{LF}(s) = \widetilde{i}_d(s)H_{out}(s) , \qquad (4)$$

where  $\tilde{i}_d(s)$  is small-signal AC component of time-averaged diode current;  $H_{out}(s)$  is output voltage to diode current



Fig. 3. Open-loop boost SMPC averaged circuit model in DCM

transfer ratio according to

$$H_{out}(s) = \frac{\widetilde{v}_{out}(s)}{\widetilde{i}_d(s)} = \frac{R_{load}(sr_{cout}C_{out}+1)}{s(R_{load}C_{out}+r_{cout}C_{out})+1}.$$
 (5)

From boost converter averaged circuit model it can be obtained that time-averaged diode current is [10]

$$\left\langle i_d \right\rangle = \frac{d^2 V_{in}^2}{2L f_{sw}(t) \left( \left\langle v_{out} \right\rangle - V_{in} \right)} , \qquad (6)$$

where *d* is instantaneous duty ratio. Assuming that d=D, output DC voltage  $V_{out} >> \tilde{v}_{out}$ , and using the first-order-Taylor-series-approximation for (6) it can be obtained that averaged diode current LF component is as follows

$$\widetilde{i}_{d}(t) \approx -\frac{\Delta f_{sw}}{f_{sw}^{2}} m(t) \frac{D^{2} V_{in}^{2}}{2L(V_{out} - V_{in})}.$$
(7)

Now let us consider LF ripples of closed-loop FM boost converter. For this purpose small-signal averaged FM boost converter model is used as shown in Fig.4. The model is slightly modified version of unmodulated boost converter small-signal averaged model obtained from [11]. It follows from this model that LF ripples for closed-loop boost converter in operator form are

$$\widetilde{v}_{LF}(s) = \widetilde{i}_d(s) \frac{H_{out}(s)}{1 + T(s)} , \qquad (8)$$

where T(s) is boost converter open-loop gain in DCM according to [4, 11]

$$T(s) = H_{div}(s)H_c(s)H_{co}(s)H_{PWM}(s),$$
(9)

where  $H_{PWM}(s)$  is PWM gain;  $H_{div}(s)$  and  $H_c(s)$  are voltage divider and compensation circuit transfer functions respectively;  $H_{co}(s)$  is boost SMPC control-to-output transfer function in DCM as follows [12]

$$H_{co}(s) = \frac{2V_{out}(M-1)(1+s/z_1)(1-s/z_2)}{D(2M-1)(1+s/p_1)(1+s/p_2)},$$
 (10)

where 
$$M = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{R_{load}D^2}{2Lf_{sw}}};$$
  $z_1 = \frac{1}{r_{cout}C_{out}};$ 

$$z_2 = \frac{R_{load}}{M^2 L}; \ p_1 = \frac{2M - 1}{(M - 1)R_{load}C_{out}}; \ p_2 = 2f_{sw} \left(\frac{1 - 1/M}{D}\right)^2.$$



Fig. 4. Small-signal averaged closed-loop FM boost SMPC model



**Fig. 5.** Closed-loop boost FM SMPC transfer functions in DCM. (Parameters:  $C_{out}$ =330 $\mu$ F; L=40 $\mu$ H;  $r_{cout}$ =0.035 $\Omega$ ;  $R_{load}$ =120 $\Omega$ ;  $f_{cut}$ =4kHz;  $V_{in}$ =7V)

As an example the transfer functions for the closed-loop boost SMPC are shown in Fig.5.

To derive the steady-state LF ripples  $\tilde{v}_{LF}(t)$  in time domain, the inverse Laplace transform of the Eq. (8) should be applied. The LF ripples in the time domain for sinusoidal FM can be easily obtained from (8) as follows

$$\begin{split} \widetilde{v}_{LF}(t) &\approx -\frac{\Delta f_{sw}}{f_{sw}^2} \frac{D^2 V_{in}^2}{2L(V_{out} - V_{in})} \Big| H_{full}(j2\pi f_m) \Big| \times \\ &\times \cos(2\pi f_m t + \arg(H_{full}(j2\pi f_m))), \end{split}$$
(11)

where  $H_{full}(j2\pi f_m) = H_{out}(j2\pi f_m)/(1+T(j2\pi f_m))$ . Total FM boost converter OVR  $V_{ofmp-p}$  for different periodic modulating signals m(t) can be calculated using (1), (2) and (8). As an example calculated, simulated in Simulink and experimental  $V_{ofmp-p}$  for comparison reasons are summarized in Table 1 for FM closed-loop boost converter in DCM. The difference between the results is not high; therefore the expressions derived can be used for the analysis and calculations of OVR.

**Table 1.** Comparison of the calculated, simulated and experimental peak-to-peak OVR for closed-loop boost FM SMPC with  $f_m=1$  kHz;  $f_{sw}=80$  kHz;  $f_{cut}=4$  kHz;  $R_{load}=120 \ \Omega$ 

m(t)	∆f <sub>sw</sub> , kHz	$V_{\mathit{ofmp-p}},\mathrm{mV}$		
		Calcu- lated	Simula- ted	Experi- mental
Unmo- dulated	0	39	41	42
sine	10	44.2	47.6	49
	30	62.6	66.3	65
sawtooth	10	43.8	45.5	47
	30	61.3	64	62

#### **Experimental verification**

For the experimental verification of the theoretical and simulated results FM closed-loop boost converter was designed and tested. The converter nominal output voltage is 19 V and nominal  $f_{sw}$ =80 kHz. As PWM control circuit SG2524 is used. Simplified schematic diagram is shown in Fig. 6. For modulating the switching frequency approach proposed in [14] is applied by inducing modulation signal

m(t) via RC circuit in timing resistor pin RT of the control circuit.



Fig. 6. Simplified schematic of the experimental setup

Peak-to-peak OVR were measured using a digital oscilloscope. The measurement results are shown in Table 1 and in Fig.7. As it can be seen the difference between the theoretical and experimental results is lower than 8%. The differences in Fig. 7 are mainly due to the digital oscilloscope measurement accuracy and due to the fact that the theoretical  $V_{ofmp-p}$  is calculated assuming that HF OVR are only due to ESR of the output capacitor.

#### Influence of modulation parameters on OVR



**Fig. 7.** Closed-loop boost FM SMPC peak-to-peak OVR versus  $f_{sw}$ ,  $f_{m}$ ,  $\Delta f_{sw}$ . (Parameters:  $C_{out}$ =330 μF; L=40 μH;  $r_{cout}$ =0.035 Ω;  $R_{load}$ =120 Ω;  $f_{cut}$ =4 kHz;  $V_{in}$ =7 V)

In order to allow a designer to simplify choice of proper values of modulation parameters ( $f_{sw}$ ,  $f_m$ ,  $\Delta f_{sw}$ ), their influence on Vofmp-p should be examined. In Fig.7 Vofmp-p versus  $f_{sw}$ ,  $f_m$ ,  $\Delta f_{sw}$  are shown both using (1), (2), (12) and experimental measurements. As it can be seen from the results increasing  $\Delta f_{sw}$  leads to increasing  $V_{ofmp-p}$ . For rather small  $\Delta f_{sw}$  (<20kHz) the increase is almost linear. This however can also be concluded after simplification of the derived expressions ((2) and (12)) and using the first-order-Taylor-series-approximation of (2)).  $V_{ofmp-p}$  decreases as  $f_{sw}$ increases.  $V_{ofmp-p}$  depends also on  $f_m$ , mainly because LF ripples depend on FM boost converter transfer function  $H_{full}(j2\pi f_m)$  as it can also be deduced from (8), (12) and Fig. 5. It should be noted that changing  $f_m$  has minor effect on  $V_{ofmp-p}$  in closed-loop FM SMPC (see Fig. 7(c)), however in open-loop FM SMPC the effect is much more appreciable.

#### Conclusions

Output voltage ripples of closed-loop boost SMPC in DCM increase due to the use of FM. OVR can be simply analyzed and calculated using the expressions derived in the paper. Theoretical analysis of OVR of boost FM converter shows that OVR consist of both HF ripples which also present in unmodulated SMPC and LF ripples with  $f_m$ . HF ripples can be calculated using expression of OVR for unmodulated SMPC, but LF ripples can be calculated using boost SMPC averaged circuit model.

FM boost converter peak-to-peak OVR depend on modulation signal and its parameters:  $f_m$ ,  $\Delta f_{sw}$ . The higher  $\Delta f_{sw}$  is the higher peak-to-peak OVR are, so  $\Delta f_{sw}$  should be chosen not only from EMI attenuation point of view but also OVR should be considered. The results presented in the paper can be helpful when designing FM SMPC operating in DCM.

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## D. Stepins, J. Jankovskis. Study of Frequency Modulated Boost Converter Operating in Discontinuous Conduction Mode // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 6(122). – P. 41–44.

In the paper output voltage ripples (OVR) of switching frequency modulated (FM) boost switch-mode power converter (SMPC) operating in discontinuous conduction mode (DCM) are examined thoroughly. Useful expressions to calculate OVR for closed-loop boost SMPC operating in DCM are derived and verified both using simulations in SIMULINK and experimentally. Influence of parameters of modulation and control signals on OVR is also examined. The results presented in the paper can be helpful when designing FM SMPC operating in DCM. Ill. 7, bibl. 12, tabl. 1 (in English; abstracts in English and Lithuanian).

# D. Stepins, J. Jankovskis. Dažniu moduliuoto aukštinančiojo keitiklio, veikiančio nenutrūkstamo laidumo režimu tyrimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 6(122). – P. 41–44.

Ištirtos dažniu moduliuoto aukštinančiojo keitiklio veikiančio nenutrūkstamo laidumo režimu išėjimo įtampos pulsacijos (IĮP). Išvestos ir modeliuojant SIMULINK bei eksperimentiškai patikrintos išraiškos, skirtos minėtų keitiklių išėjimo įtampos pulsacijoms skaičiuoti. Taip pat ištirta moduliacijos parametrų ir valdymo signalų įtaka IĮP. Pateikti rezultatai gali būti naudingi projektuojant tokio tipo keitiklius. II. 7, bibl. 12, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).