# Peak to Peak Current Minimization by Differential Evolution Optimization

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Abstract— This paper deals with differential evolution optimization of peak to peak current value, which consist of several harmonics with defined amplitudes. The actual amplitude of the current signal is computed as a sum of harmonics with different frequencies and depends on the harmonics amplitudes, phases and actual time. The paper presents the optimization algorithm of harmonic's phases to reach the highest amplitude of all harmonics with the lowest absolute amplitude of resulted current (computed as a sum of all harmonics).

The optimized current waveform is used as a reference signal for analogue hysteresis controller of single phase H-bridge inverter. The inverter works as a current source connected in parallel to auxiliary winding of Petersen coil. The H-bridge converter is used for injecting of selected current harmonics into power network grid. The measured converter voltage responses of injected current harmonics are used for grid resonant frequency identification. This information is used for tuning of Petersen coil.

Index Terms—Power grids; evolutionary computation; genetic algorithms; DC-AC power converters.

# I. INTRODUCTION

Heuristic optimization methods are popular solutions for solving optimization problems which are difficult to solve by traditional methods. The optimization problems can be divided into three categories: discrete problems, continues space problems and hybrid mixed problems. The heuristic optimization algorithms were successfully used to solve many hard and multidimensional problems e.g. [1]–[4]. The peak to peak current minimization by modification of harmonics phases is continues space problem.

The most popular global heuristic optimization methods for continues space problems are particle swarm optimization (PSO), genetic algorithms (GA) and differential evolution (DE) algorithms.

PSO is inspired by swarm behaviour and popular method for continues spaces problems. Special attention was dedicated to GAs and their applications [1], [2]. Genetic algorithms are based on the natural selection and genetics. The GAs are naturally discrete solvers therefore continues space problems must be transformed to discrete.

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Popular continues space solvers are based on DE. The first paper about DE optimization was published by Price and Storn in 1995 [3], [4]. The optimization problem of peak to peak current minimization is naturally continues spaced (real valued) with many local optima. For these types of problems DE based solvers frequently outperform PSO and GAs.

This paper is focused on optimization of inverter reference current waveform for several current harmonics injection. The injector current reference is calculated as a sum of selected harmonic signals with the same amplitude. The H-bridge injector is connected to auxiliary winding of Petersen coil (PC). PCs are variable inductance coils for active earth fault currents compensation [5]–[8]. PCs are connected between the grid neutral point and the ground to increase the total 50/60 Hz grid to ground impedance. The properly tuned PC is in parallel resonance with the grid-to-ground network capacity at fundamental power supply frequency. During the grid faults the PC inject ground current in the opposite phase to grid-to-ground capacitor currents and thus minimize current flowing throw the earth fault (Fig. 1).

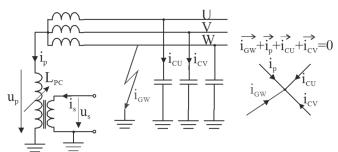


Fig. 1. Scheme of W phase-to-ground fault and Petersen coil with auxiliary winding.

The common PC tuning and grid-to-ground capacitance identification method is based on the pulse method [5]. The novel method based on injecting of two frequencies was explained in [8] and fast-pulse detection method in [9]. The earth localization methods are summarized in [10]. Presented method is based on injecting of twelve selected frequencies, which allows more precise estimation of grid resonant frequency.

The required current waveform signal is generated by microcontroller's D/A converter and connected to analogue hysteresis current controller. Hysteresis current controller controls switching of IGBT transistors [11] directly to inject

controlled current i<sub>s</sub> into auxiliary winding of the PC. The voltage response u<sub>s</sub> of the inverter is measured by microcontroller via 4th degree active Butterworth low pass filter and analysed by several single-point discrete Furrier transforms (DFTs). The results of DFT analyses are used for PC tuning. The basic scheme of H-bridge inverter connected to PC auxiliary winding is in the Fig. 2.

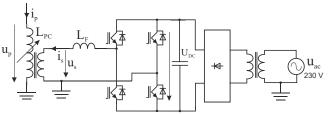


Fig. 2. H-bridge converter for phases-to-ground capacitance identification.

The paper is organised as follows: the second section describes mathematical formulation of peak to peak minimization, the third section deals with DE algorithm description, the next section presents optimization results and the last part is conclusion.

#### II. PROBLEM FORMULATION

The minimization of peak absolute value of sum of sinusoidal waveforms over phases with defined amplitudes could be described as

$$\arg\min_{\mathcal{G}\in[0,2\pi)} |\sum_{i=1}^{N} \sin(\omega_{i}t + \mathcal{G}_{i})| \text{ for } t \in R.$$
 (1)

The argument of minima is a vector of phases  $\theta \in [0, 2\pi)$  of the absolute value of sum of N sinusoidal waveforms for  $t \in R$ . In other words, the optimization algorithm must search for solution (vector of N sinusoidal phases) of minimal absolute value of the sum of sinuses for defined selected amplitudes and frequencies of all N harmonics. Solution must be valid for the whole time space.

For signal which contains N harmonics the optimization problem has dimension of N-1. The first harmonic has  $\theta = 0$  and other harmonics phases (position) are optimized to the first one. The optimization process was realized by DE.

## III. DIFFERENTIAL EVOLUTION OPTIMIZATION

DE is a heuristic genetically inspired optimization algorithm for continuous space problems. The DE optimization process uses floating point numbers operations directly. Therefore, the real number problems are not necessary to transform to the fixed point numbers and solved by conventional GAs. Furthermore, the DE contains only simple arithmetic operations such as addition, subtraction and multiplication. The basic DE scheme is in Fig. 3.



Fig. 3. Differential population basic scheme.

The DE optimization starts with N potential solutions with randomly uniformly distributed variables in the expected range of searching space. In our case 11 sinusoidal phases were randomly generated in the range  $\vartheta \in [0,2\pi)$  for 1500 of potential solutions. In the next step, the fitness function, which represents quality of solution, is calculated for all potential candidates. The fitness function results of candidates are saved in the memory. In the selection process, candidate results are compared with previous results and N promising solutions are selected for further modifications and evaluations. The candidate modification is provided by mutation and crossover algorithms. The new candidates are evaluated by the fitness function and the whole process is repeated until the sufficient optima or maximal number of iteration is not reached. In our case the 5000 number of iterations was selected as a compromise between convergences of solutions and consumed computing time.

## A. Initial Population

The initial population consists of 1500 randomly uniformly generated members of solution candidates. Each candidate is a vector of 11 variables representing 11 phases of sinusoidal waveforms in the range of  $\vartheta \in [0,2\pi)$ . Equation (2) describes the initial population

$$X_{init(k=0,j,i)} = \begin{bmatrix} \theta_{j=1,i=1} & \dots & \theta_{j=1,i=11} \\ \vdots & \ddots & \vdots \\ \theta_{j=1500,i=1} & \dots & \theta_{j=1500,i=11} \end{bmatrix} = \begin{bmatrix} \overline{x}_{j=1} \\ \dots \\ \overline{x}_{j=1500} \end{bmatrix}.$$

$$(2)$$

where parameter k is iteration number, j is order of candidate vector in population and i is the number of phase which belongs to defined sinusoidal frequency.

## B. Fitness Function

The fitness function evaluates the candidate solutions. The peak to peak current minimization described in (1) is discretized for numerical problem solution with the time step of 0.0001s (3). The maximum of the signal is calculated for all candidate vectors for time range from 0 to 0.6s which corresponds to common (fundamental) harmonic frequency of sinusoidal waveforms (in our case 1/1.667=0.6s)

$$y_{(k,j)} = max \left( \left| \sum_{i=0}^{11} sin(\omega_i t + \theta_i) \right| \right) \text{ for } t \in \langle 0, 0.6 \rangle.$$
 (3)

# C. Selection

The selection is used for choosing the best candidates of the actually evaluated population by fitness functions and their parent population (totally size 2xN i.e. 3000 candidate vectors) to create the new population of N candidates. Used DE selection replaces the original candidate solution if it has an equal or higher fitness value (in our case lower maximal amplitude than its parent candidate (4)). Otherwise, the original candidate remains in its place for at least one more generation

$$x_{(k+1,j,i)} = \begin{cases} x_{offspr(k,j,i)}, & if(y_{(k,j)} \le y_{(k-1,j)}), \\ x_{(k,j,i)}, & otherwise. \end{cases}$$
(4)

# D. Mutation and Crossover

Mutation and crossover is used to modify the population to create new candidate vectors. The selected form of mutation adds the weighted difference vector of two randomly chosen candidate (parent) vectors to the third individual (5)

$$\overline{x}_{mut(k,j)} = \overline{x}_{(k,j)} + K\left(\overline{x}_{(k,r1)} - \overline{x}_{(k,r2)}\right). \tag{5}$$

This process is repeated for whole population to create the mutated population of N members. It forms a new mutated vector of parameters. K is the mutation constant. Results for various K are in the Section IV

In DE, crossover is complementary operation to mutation. During the crossover, the parts of parent vectors and mutated (child) vectors are mixed together to create a new population. The crossover constant Cr controls speed of the offspring evolution. The mutation constant K and crossover constant Cr is statistically analysed in section IV. Generally speaking, the larger mutation and crossover constants increase the speed of convergence. On the other hand process is greedier and the chance to find sufficient optima is reduced. Process of crossover can be described as

$$x_{offspr(k,j,i)} = \begin{cases} x_{mut(k,j,i)}, & if (rand_i \le Cr), \\ x_{(k,j,i)}, & otherwise. \end{cases}$$
 (6)

## IV. OPTIMIZATION RESULTS

The optimization algorithm was tested on the signal which contains 12 sinusoidal waveforms with frequencies: [20, 25, 30, 33.33, 41.67, 48.33, 51.67, 60, 75, 83.33, 100, 125] Hz and common fundamental frequency of 1.667Hz. The all signal components have the same amplitude. The frequency range is common resonant frequency of the Petersen coil and phase-to-grid capacitance. The individual frequencies were selected to fulfil following equation

$$2500 \cong f(i) \times f(12 - i + 1) \text{ for } i = (1:6).$$
 (7)

For properly tuned PC, the injected current signal which contains the components with the same amplitude and properly selected frequencies (7), the voltage response is symmetric. In other words, the voltage amplitude for the first frequency (20 Hz) has the same value as the last one (frequency of 125 Hz). More details about the frequency selection could be found in [8].

The current signal containing 12 components with all phases equal to zero and amplitudes equal to 1A is shown in Fig. 4. The current peak is up to 8.95 A (the worst case is 12 A). After optimization process of the harmonic component phases, peak current decreased down to 4.62 A (Fig. 5). The example of the typical progress of the best (minimal) amplitude of DE algorithm for 4 initial populations could be found in Fig. 6.

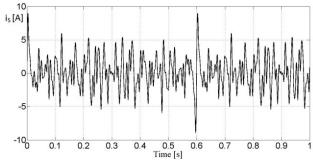


Fig. 4. Current reference waveform *before* optimization (all harmonics phases are equal to zero).

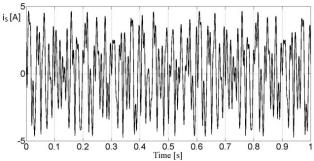


Fig. 5. Current reference waveform *after* optimization (harmonics phases were optimized by DE).

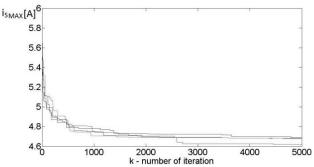


Fig. 6. The progress of DE optimization of maximum amplitude for K = 0.3, Cr = 0.1 for 4 selected initial populations.

The statistical results of DE optimization for different initial populations are in the Table I. The first column represents crossover constant Cr settings. The second column is the mutation constant settings. The third column is the best result (minimal peak to peak amplitude) for all populations with the same DE setting. The last two columns are average best results of DE and its standard deviance. For 1500 members of candidate vectors, both DE coefficient settings K and Cr don't significantly affect the results. The main reason is quite large population in comparison with problem dimension and chosen algorithm of selection. These properties prevent population degradation. The finally selected constants were Cr=0.2 and K=0.45 with the best average results and low deviance.

TABLE I. DE PARAMETER SELECTION.

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Cross. Cr	Mutation K	Best Y	Best <sub>AV</sub> Y	Best <sub>STDV</sub> Y
0.1	0.15	4.623	4.662	5.77e-4
0.1	0.3	4.621	4.667	7.07e-4
0.1	0.45	4.641	4.658	2.07e-4
0.2	0.15	4.652	4.666	1.43e-4
0.2	0.3	4.661	4.681	2.74e-4
0.2	0.45	4.641	4.676	4.22e-4

The resulted DE signal was tested on simplified model of the PC with current source connected to the auxiliary winding. The scheme is presented in Fig. 7. The resulted auxiliary winding voltage  $u_{\rm S}$  for optimized current  $i_{\rm S}$  is in Fig. 8. The frequency spectrum of optimized current is in Fig. 9. All harmonics have current amplitudes equals to 1. The parallel resonant frequency of the circuit with primary PC inductance  $L_{\rm PC}=1$  H, PC voltage transform ratio p=22000 V/650 V and grid to ground capacitance  $C=10.13~\mu F$  was 50 Hz (i.e. the PC is properly tuned). This frequency is dominant in the voltage response frequency spectrum (Fig. 10). This frequency was generated by the start-up transient of the current source. All other harmonics amplitudes should be the symmetric around the 50 Hz.

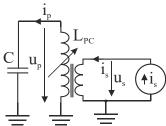


Fig. 7. Simplified scheme with current source and 50 Hz parallel resonance circuit ( $L_{PC}$  = 1 H, C = 10.13  $\mu$ F, p = 22000 V/650 V).

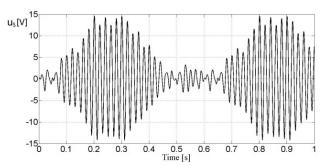


Fig. 8. Voltage response  $u_S$  for 50 Hz resonance circuit (L  $_{PC}$  = 1 H, C = 10.13  $\mu F,\,p$  = 22000 V/650 V) and current presented in Fig. 5.

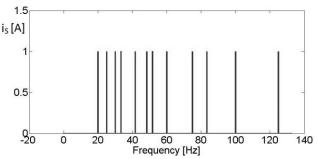


Fig. 9. Frequency spectra of transformer secondary winding current  $i_S$  presented in Fig. 5 for 50 Hz resonance circuit ( $L_{PC}$  = 1 H, C = 10.13  $\mu$ F, p = 22000 V/650 V).

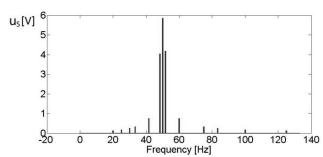


Fig. 10. Frequency spectra of voltage response (Fig. 8.) on the secondary winding of transformer  $u_s$  for 50 Hz resonance circuit ( $L_{PC}=1$  H,  $C=10.13~\mu F$ , p=22000~V/650~V) and current presented in Fig. 5.

## V. CONCLUSIONS

This paper presents the algorithm for minimization of peak to peak value of waveform, which contains several harmonic signals, by modification of signal harmonic component phases. The selected real value optimization algorithm was differential evolution for its simplicity and effective implementation.

The DE algorithm was used for optimization of reference current of H-bridge inverter to maximize amplitude of all required current components with limited actual maximal current of inverter. The maximal current peak was reduced more than two times to the worst case. The maximal current affects transistor selection and inverter price.

The DE algorithm was statistically analysed for different settings. The algorithm was implemented in C++ in Visual Studio Express 2013 for optimization of different sets of current harmonic frequencies. The average time of optimization for 5000 iterations and 1500 candidate vectors was 4 minutes and 12 s for notebook with i7 4712HQ Intel processor.

#### APPENDIX A

Twelve selected harmonics for optimization: [20, 25, 30, 33.33, 41.67, 48.33, 51.67, 60, 75, 83.33, 100, 125] Hz.

The best found phases by DE: [0, 0.745, 0.616, 0.288, 3.473, 4.650, 0.737, 3.787, 1.547, 1.302, 3.280, 1.225] rad.

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