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# Mathematical Model of the Linear Motor

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## Introduction

Constant improvement of technological processes enables the perfection of the devices which actualize linear motions that appear so often. One of the possible ways of producing linear movements is linear electromagnetic motor (LEM). This type of motor possesses simple structure and signifies itself by many positive characteristics.

LEM is comprised of the original part in which there is situated C type magnetic core, with an air gap [1]. On the magnetic core there has been wounded the excitation core powered from the constant voltage resource. The secondary part (SE – secondary element) is comprised of ferromagnetic saw-edged rail, the teeth of which get on the magnetic air gap between the poles of constant magnetic field. The thread of the teeth and magnetic core poles is not the same. LEM operation is based on the principle of induction. By connecting voltage in sequence to the electromagnetic coils the system of electromagnets (in case when the saw-edge rail is attached) or saw-edge rail (in case when the system of electromagnets is attached) starts moving similarly the same as the stepping motor with reactive rotor [2-4].

The features of the motors of such kind are described in [1], and the analysis of the static characteristics is made in [5].

High technical and economical indexes of LEM prove that this type of motor is a promising one but for their successful implementation it is necessary to carry out comprehensive theoretical and experimental investigations [6-8].

The main reason of this article is to provide the idealized generalized dynamic mathematical model of the linear motor, which could be used in the research of the operation possibilities of it.

## The operation voltage of the motor

The best way to operate the motor is to use unipolar rectangular impulse direct (constant) voltage of power supply, which could change the amplitude and the width of the impulses. The change of the amplitude is evaluated by the voltage factor (coefficient):

$$\gamma = \frac{U}{Um}, \tag{1}$$

where $U$, $Um$ – the direct (constant) voltage of the excitation of a induction coil a motor and this voltage maximum evaluation. The voltage factor (coefficient) can change from 0 till 1.

The duration of the voltage impulse is characterized by the casual power supply voltage impulse duration factor (coefficient):

$$\beta = \frac{t_{imp}}{T} = t_{imp} \cdot f_{imp}, \tag{2}$$

where $t_{imp}$, $T$, $f_{imp}$ – the operation voltage impulse duration period and frequency. The casual power supply voltage impulse duration can change from 0 till 1.

The power supply voltage impulse of a motor is created by an impulse power converter, the output voltage (the motor power supply voltage) moment evaluation expression of which is:

$$u(t) = \gamma \cdot U_m \cdot signk; \tag{3}$$

$$signk = \begin{cases} 1, if \left( \dfrac{\beta}{f_{imp}} + t_k - t \right) > 0, \\ 0, if \left( \dfrac{\beta}{f_{imp}} + t_k - t \right) \le 0; \end{cases} \tag{4}$$

where $t_k = (n-1) \cdot T = \dfrac{n-1}{f_{imp}}$ – the moment of time when the n's communication has happened. At the time of the first communication $t_k = 0$. The time of the communication is equal to nil and the characteristics of the transformer (the active resistance and induction) are taken into consideration in the characteristics of the motor.

The maximum whole number of communications could be made according to this expression:

$$n_{\max} = \frac{L}{\tau/3}, \quad (5)$$

where $L$ – the length of the rectilinear way of the motor; $\tau = 2 \cdot b$ – the polar step of the secondary teethed element; $b$ – the width between the similar tooth and spans of the secondary element.

**The traction force of the motor**

The expression of the useful traction force of the motor along x coordinate can be got using the theory of the change of the electromagnetic energy which is presented in several scientific resources [9-12]. According to this theory to get the extraction of the is necessary to know the law of the change of the induction of the x coordinate. After the fulfillment of experiments, with the physical model of a linear motor the characteristics of which are given in the table 1, was denoted the change of induction of the induction coil of the motor.

**Table 1** . The characteristics of the physical model of the linear motor

| No. | Title of the parameters | Symbol | Value | Math units |
|---|---|---|---|---|
| 1. | Hignt of pole | $a_c$ | 15 | mm |
| 2. | Width of pole | $b_c$ | 14,5 | mm |
| 3. | Hignt of teeth | $a_z$ | 15 | mm |
| 4. | Width of teeth | $b_z$ | 15 | mm |
| 5. | Thickness of teeth | 2Δ | 16 | mm |
| 6. | Span between tooth | $b_t$ | 15 | mm |
| 7. | Air gap length | δ | 1,1 | mm |
| 8. | Number of induction coils | $S_R$ | 3 | – |
| 9. | Number of coils | w | 2200 | – |
| 10. | Thickness of the wire | d | 0,04 | mm |
| 11. | Active resistance | R | 32,7 | Ω |

Just enough accurate for the engineering calculations the experimental curve could be approximated by this formula:

$$L(x) = L_0 + L_m \cdot \cos\frac{2\pi}{\tau}x, \; -b \le x \le b \;; \quad (6)$$

$$L_0 = \frac{L_1 + L_2}{2}; \quad (7)$$

$$L_m = \frac{L_1 - L_2}{2}; \quad (8)$$

where $L_0$, $L_m$ – the direct (constant) and changeable impulse of excitation induction coil induction;
$L_1$, $L_2$ – the evaluation of the induction of coil, when the teeth and span poles are just at the middle position between the tooth.

The induction change graphic is presented in Fig. 1. According to the theory of electromagnetic change, having in mind that the magnetic system is not fully saturated it is possible to get the expression of the traction force of a linear motor with one induction coil:

$$f_x = -\frac{\pi}{\tau} \cdot L_m \cdot i^2 \cdot \sin\frac{2\pi}{\tau}x, \; -b \le x \le b. \quad (9)$$

Supplying with power all three coils at the same time it is necessary to evaluate the disposition of the coils on the secondary element.

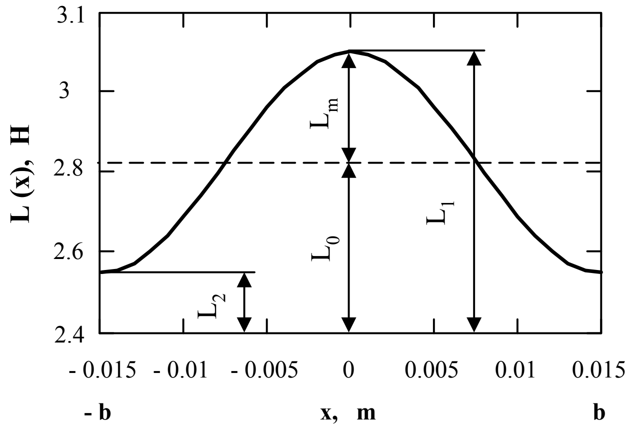


**Fig. 1.** Dependence of inductivity for 100 V direct (constant) power supply voltage

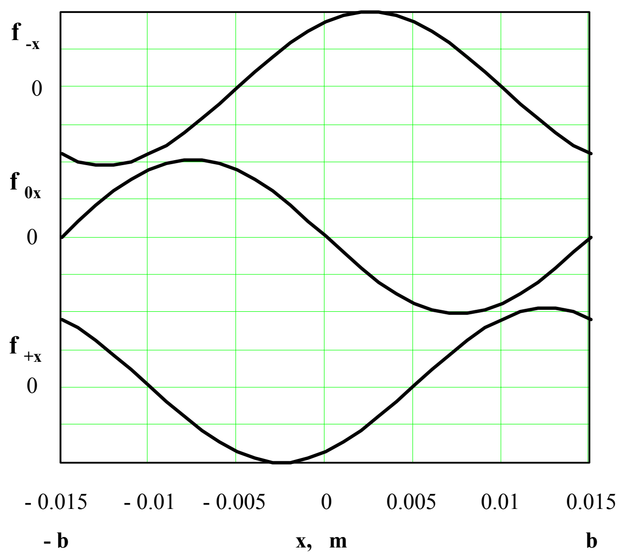


**Fig. 2.** The linear motor traction force diagram

In this case the traction force of each coils:

$$f_{-x} = -\frac{\pi}{\tau} \cdot L_m \cdot i^2 \cdot \sin\left[\frac{2\pi}{\tau}\left(x - \frac{\tau}{3}\right)\right], \; -b \le x \le b; \quad (10)$$

$$f_{0x} = -\frac{\pi}{\tau} \cdot L_m \cdot i^2 \cdot \sin\frac{2\pi}{\tau}x, \; -b \le x \le b; \quad (11)$$

$$f_{+x} = -\frac{\pi}{\tau} \cdot L_m \cdot i^2 \cdot \sin\left[\frac{2\pi}{\tau}\left(x + \frac{\tau}{3}\right)\right], \; -b \le x \le b; \quad (12)$$

where $f_{-x}$, $f_{0x}$, $f_{+x}$ – the traction force of the coil hold in the – x coordinate direction, of the coil hold in the middle, and of the coil hold in the + x coordinate direction.

Supplying with the same voltage power all the coils at the same time the summarized traction force theoretically will be equal to nil, but practically due to different technological and technical reasons the force will not be equal to nil.

By choosing the particular power supply voltage to each coil is possible to form the required law of the change of a motor traction force just for a concrete technological process requirements. The solution of this problem could be the aim of the further research work.

The traction force diagram is presented in Fig. 2.

The disposition of the excitation induction coil poles according to the secondary element is presented in Fig. 3.

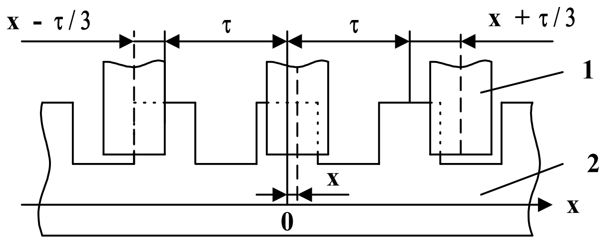


**Fig. 3.** The disposition of the excitation induction coil poles according to the secondary element along x axis: 1 – the excitation induction coil pole; 2 – the secondary element.

## The idealized dynamic mathematical model of the motor

While creating the dynamic mathematic model of the motor it is considered that the stimulating induction coils do not have any magnetic connections within each other. And as well the magnetic system of the motor is not fully saturated that is why the induction of the coils depends on the x coordinate. The dynamic mathematical model of the linear motor is expressed by the equality:

$$u_i(t) = R_i \cdot i_i + L_i(x) \cdot \frac{i_i}{dt}; \qquad (13)$$

$$L_i(x) = L_{0i} + L_{mi} \cdot \cos\frac{2\pi}{\tau}\left(x + l\frac{\tau}{3}\right); \qquad (14)$$

$$f_{xi} = -\frac{\pi}{\tau} \cdot L_{mi} \cdot i_i^2 \cdot \sin\frac{2\pi}{\tau}\left(x + l\frac{\tau}{3}\right); \qquad (15)$$

$$F_x = \sum_{i=1}^{N} f_{xi} = m\frac{d^2x}{dt^2} + k_v\frac{dx}{dt} + k_x \cdot x + f_s; \qquad (16)$$

where $N = 3$ – the number of the induction coils of the motor; $l = -1, 0, +1$ – the factor (coefficient) of the pole directed along the negative x coordinate till 0 position is equal to –1, the middle pole is equal to 0 and the pole directed along the positive x coordinate is equal to +1; $F_x$ – the summarized traction force of the motor along the x coordinate; $u_i(t)$, $R_i$, $i_i$, $L_i(x)$, $f_{xi}$ – the power supply of an i-stimulating induction coil, active resistance, current, induction and the traction force; $u_i(t)$ voltage characteristics are presented in (1-5); $m$, $k_v$, $k_x$, $f_s$, $x$ – the mass of the moving parts of the motor, the force of resistance dependant on the speed and way proportion coefficent, static resistance force and the way coordinates.

The formulas of the mathematical model (13-16) could be used in the x coordinate change:

$$-b \le x \le b$$

where $b$ – the width of tooth and spans of the secondary toothed ferromagnetic element.

According to created mathematical model the impulse operating method is realized by changing the width amplitude and frequency of the operating impulse.

## Conclusions

1. After the examination of the law of the change of the induction of the linear motor the mathematical expression of the law was obtained.
2. By applying the theory of the change of the electromagnetic energy, the analyzed expression of the traction force of the idealized motor was created.
3. By the choice of the particular power supply voltage of the motor coils it is possible to form the required summary traction force of the motor that could be applied to the concrete technological process requirements.
4. Idealized dynamic mathematical model of the linear motor which realizes the impulse operation by changing the width, amplitude and frequency of the voltage impulse was created.
5. The obtained mathematical model could be used in the examination of the operation possibilities of the linear motors working as an electrical transmission system.

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**E. Matkevičius, L. Radzevičius. Mathematical Model of the Linear Motor // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 5(77). – P. 11–14.**

The creation of the idealized mathematic model of the linear motor is presented. It denoted the law of the change of the motor coil induction dependant on the motion along the x coordinate, which has a rather accurate approximated analytic engineering expression. The formula of the change of the induction is obtained having in mind that the motor magnetic system is not saturated and there are no any magnetic connections between the coils. There was obtained the analytic expression of each coil traction force that is summarized into the general traction force of the motor. These expressions could be used to form the law of the change of the summary traction force and are the main reason for the further research work. Applying the theory of the change of the electromagnetic energy the idealized dynamic mathematic model of the linear motor which realizes the impulse operation was created. For the impulse operation of the motor stimulating coils unipolar rectangular voltage impulse is used. For the operation impulses could be changed in width, amplitude and frequency. The characteristics of the impulse converter (active resistance, induction) are evaluated while changing the active resistance and induction of the coils. The mathematic model evaluates the static loading of the motor dependant on the x coordinate that is why it could be used for the research work of the transmission with the linear motors. Ill.3, bibl. 12 (in English; summaries in English, Russian and Lithuanian).

**Э. Маткявичюс, Л. Радзявичюс. Математическая модель линейного двигателя // Электроника и электротехника. – Каунас: Технология, 2007. – № 5(77). – С. 11–14.**

Представлено составление идеализированной математической модели линейного двигателя. Установлен закон изменения индуктивности катушки возбуждения двигателя в зависимости от перемещения вдоль x координаты, который с приемлемой для инженерной практики точностью, аппроксимируется аналитическим выражением. Формула изменения индуктивности получена при допущении, что магнитная система ненасыщена и между отдельными катушками двигателя нет магнитной связи. Получены аналитические выражения отдельных составляющих сил, представляющих общую силу тяги двигателя. Эти выражения могут быть использованы для формирования закона изменения общей силы тяги и являются одной из задач дальнейшего исследования. Применяя теорию преобразования электромеханической энергии, составлена идеализированная динамическая математическая модель линейного двигателя, реализующая импульсное управление. Во время импульсного управления, напряжения в виде однополярных прямоугольных импульсов подаются в катушки возбуждения двигателя. Может изменяться ширина импульсов, частота их следования и амплитуда постоянного напряжения питания. Параметры (активное сопротивление, индуктивность) такого преобразователя мощности учитываются путем изменения на соответствующую величину активного сопротивления и индуктивности катушки возбуждения двигателя. Математическая модель учитывает статическую нагрузку, а также составляющие нагрузки двигателя, зависящие от перемещения вдоль x координаты и скорости, поэтому модель может быть использована для исследования приводов с линейными двигателями. Ил. 3, библ. 12 (на английском языке; рефераты на английском, русском и литовском яз.).

**E. Matkevičius, L. Radzevičius. Tiesiaeigio variklio matematinis modelis // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 5(77). – P. 11–14.**

Pateikiamas tiesiaeigio variklio idealizuoto matematinio modelio sudarymas. Nustatytas variklio ritės induktyvumo kitimo priklausomai nuo poslinkio išilgai x koordinatės dėsnis, kuris inžinerinei praktikai pakankamu tikslumu aproksimuojamas analizine išraiška. Induktyvumo kitimo formulė gauta, padarius prielaidą, kad variklio magnetinė sistema neįsotinta ir tarp ričių nėra magnetinio ryšio. Gautos atskirų variklio ričių jėgų dedamųjų, sudarančių visuminę variklio traukos jėgą, analizinės išraiškos. Šios išraiškos gali būti panaudotos formuojant visuminės traukos jėgos kitimo dėsnį ir yra vienas iš tolesnio tyrimo uždavinių. Pritaikant elektromechaninės energijos keitimo teoriją, sudarytas idealizuotas dinaminis tiesiaeigio variklio matematinis modelis, realizuojantis impulsinį valdymą. Impulsinio valdymo metu į variklio žadinimo rites įjungiama vienpolių stačiakampių impulsų pavidalo įtampa. Gali būti keičiamas nuolatinės maitinimo įtampos impulso plotis, impulsų dažnis ir amplitudė. Šio impulsinio galios keitiklio parametrai (aktyvioji varža, induktyvumas) įvertinami atitinkamai keičiant variklio ritės aktyviąją varžą bei induktyvumą. Matematinis modelis įvertina variklio statinę apkrovą bei apkrovos dedamąsias, priklausančias nuo poslinkio išilgai x koordinatės ir greičio, todėl gali būti naudojamas pavaroms su tiesiaeigiais varikliais tirti. Il.3, bibl. 12 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).